Name: ___________________________

Indian Institute of Technology, Bombay
Department of Mechanical Engineering
Admission Test for Ph.D. Candidates

Time Limit: 90 minutes

Note:

1. This test has several applied mathematics problems and they do not require any specialized field knowledge. It is designed only to test elementary comprehension and simple math skills. There are no complicated formulae to be used from memory as all the required information is given in the problem statements. So, read your problems carefully and you should be able to solve all of them.
2. All questions carry equal marks.
3. No calculators are allowed
4. Please write in the space provided after each problem and use the back side of the page if necessary.

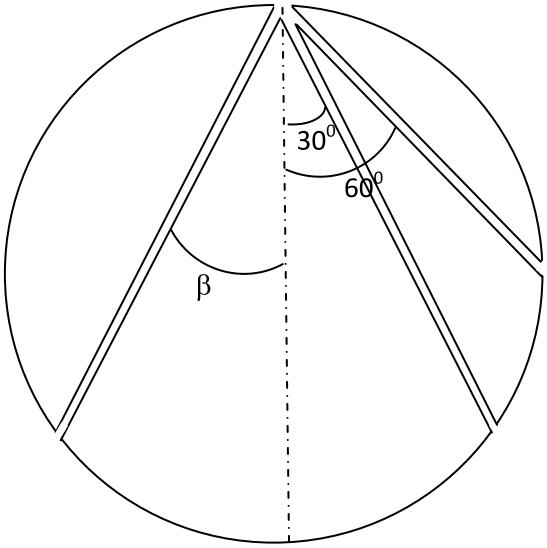
DO NOT WRITE IN THE SPACE BELOW

Problem	Marks
1	
2	
3	
4	
5	
6	
Total	

1. Write which of the following functions are discontinuous at zero and which have slope discontinuity at zero? Find the LHS and RHS derivatives of them at zero.

Function f(x)
x-|x|
$x-x^2$
x^2-|x|
sin|x| - sin(x)
sin|x| - cos(x)

2. A circular disk of diameter ***L*** has channels cut at different angles as shown below. The disk is placed vertically so that its axis is horizontal. Calculate the time for a frictionless ball (initially at rest) to travel the three channels under gravity.



3. Eigenvalues and the associated eigenvectors of a matrix A are important tools when one wants to analyze the behaviour of a matrix **A**. An eigenvector of a matrix **A** is a non-zero vector which simply scales by a factor equal to the eigenvalue when it is multiplied by **A** i.e. there the vector does not rotate sideways though it may reverse its direction. Eigenvalues and the associated eigenvectors of a matrix are found by solving the eigenvalue equation given by

$$\mathbf{Ax} = \lambda\, \mathbf{x}.$$

Here λ is the eigenvalue and $\mathbf{x}$ is the associated eigenvector. Given this background find the two eigenvalues and the associated eigenvectors of the following 2 × 2 matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

Now consider a 2 × 1 vector

$$\mathbf{y} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

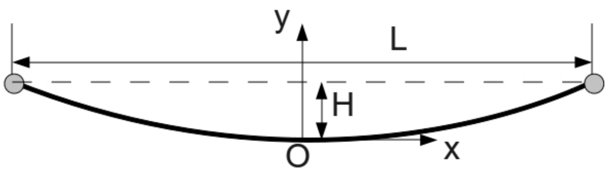
Evaluate $\mathbf{A}^{100}\mathbf{y}$. Note that $\mathbf{A}^2 = \mathbf{A} \times \mathbf{A}$, $\mathbf{A}^3 = \mathbf{A} \times \mathbf{A} \times \mathbf{A}$ and so on. **Hint**: Express the vector **y** as a linear combination of the eigenvectors of **A**, i.e. if $\mathbf{x}_1$ and $\mathbf{x}_2$ denote the two eigenvectors of **A**, then write $\mathbf{y} = C1{*}\mathbf{x}_1 + C2{*}\mathbf{x}_2$ and then carry out the multiplication. Note that you will have to find the constants C1 and C2.

4. The shape of a flexible string hanging under the action of gravity is given by the equation,

$$\frac{d^2 y}{dx^2} = C\sqrt{1+\left(\frac{dy}{dx}\right)^2},$$

where, y is the deflection and C is a constant. The coordinate system is located at midway between the supports and is as shown in the figure. The above equation is a non-linear equation and is difficult to solve analytically. In such cases, one looks for obtaining approximate solutions. Consider the case of a string supported between two points separated by a distance L. Answer the following:

1. What is the deflection and slope at x = 0.
2. If you are allowed to assume that the absolute value of the slope anywhere in the actual string is much smaller than 1 (this is true when H is much smaller than L), can you simplify the above equation and solve it to obtain the expression for deflection in terms of x, C only? Note that you need to use results from part (a). Now using the deflection at x = L/2, express the constant C in terms of H and L. Finally express the deflection y in terms of x, L and H. What is the shape of the string.



5. Consider a physical situation where a spherical liquid drop is evaporating thereby decreasing its diameter with time.

   (a) If it can be assumed that the rate of change of volume of the drop is directly proportional to its diameter (D), formulate an ordinary differential equation that will govern the variation of the drop diameter with time. You can take the proportionality constant to be C.

   (b) If the initial diameter of the drop is D_0, get an expression for the variation of the drop diameter with time

   (c) Plot the variation of D^2 vs time.

6. One of the basic laws in Fluid Mechanics is the law of Conservation of Mass. For a control volume with a number of one-dimensional inlets and exits, the Conservation of Mass, for a general control volume CV can be written as:

$$\int_{CV} \frac{\partial \rho}{\partial t} dV + \sum_j \left(\rho_j A_j v_j\right)\Big|_{out} - \sum_k \left(\rho_k A_k v_k\right)\Big|_{in} = 0$$

The first term is the unsteady term, which would go to zero in cases where the flow is steady or when the flow is incompressible, meaning, density ρ is a constant. The second term refers to the sum of the mass flow rates leaving the control volume through 'j' one-dimensional exits. The third term is the sum of the mass flow rates entering the control volume through 'k' one-dimensional inlets.

Volumetric flow rate, Q, is given by the product of the flow area and the velocity of flow, i.e. $Q = Av$, where Q is in m^3/s, A is the cross sectional area of flow, in m^2 and v is the average velocity of the fluid in m/s. In general, if the velocity varies with the flow area, then we can write $Q = \int_A v dA$

Now, consider the flow of an incompressible fluid past an impermeable flat plate as shown in the Figure below. The inlet velocity is uniform, i.e. $v = U_o$ and the exit velocity profile is a cubic polynomial given by: $v = U_o\left(\frac{3\eta - \eta^3}{2}\right)$ where, $\eta = \frac{y}{\delta}$. Starting with the Conservation of Mass equation given above, and the definition of volumetric flow rate, compute the volumetric flow rate across the top surface of the control volume, which is shown by dotted lines in the figure.