Automatic Controls ME 309 Course Project

AIM : Design a position controller such that the position of center of a sphere (of mass m and radius R restricted to move in the vertical direction) from the ground level, x, is maintained at a specified reference value, x_{ref} . A proportional-integral-derivative (PID) control action should be designed using only physical elements.

(a) PID Gains Computation for Time Domain Specifications:

$$u(t) - m\ddot{x} = K_P\, e(t) + K_D\, \dot{e}(t) + K_I \int e(t)\, dt$$

$$u(t) - m\ddot{x} = K_P\, (x - x_{ref}) + K_D\, (\dot{x} - \dot{x}_{ref}) + K_I \int (x - x_{ref})\, dt$$

$$u(t) + K_P\, x_{ref} + K_D\, \dot{x}_{ref} + K_I \int x_{ref}\, dt = m\ddot{x} + K_P\, x + K_D\, \dot{x} + K_I \int x\, dt$$

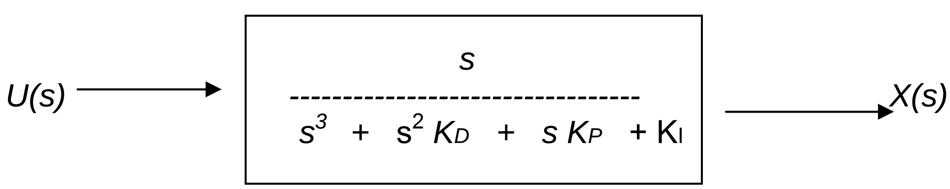
Taking Laplace transform on both sides we get the transfer functions as

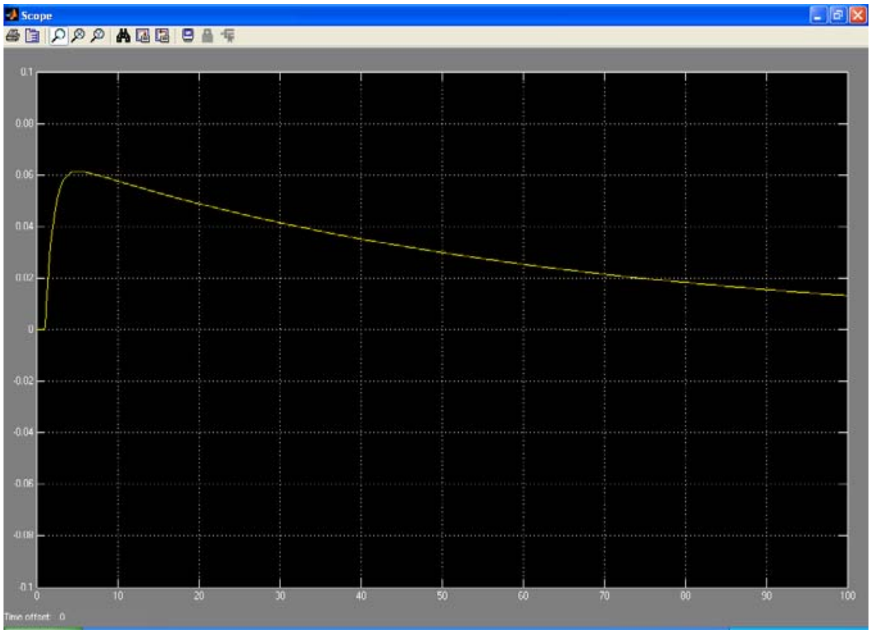
$$U(s) + K_P X_{ref}(s) + sK_D X_{ref}(s) + K_I X_{ref}(s)/s = s^2 mX(s) + K_P X(s) + s K_D X(s) + K_I X(s)/s$$

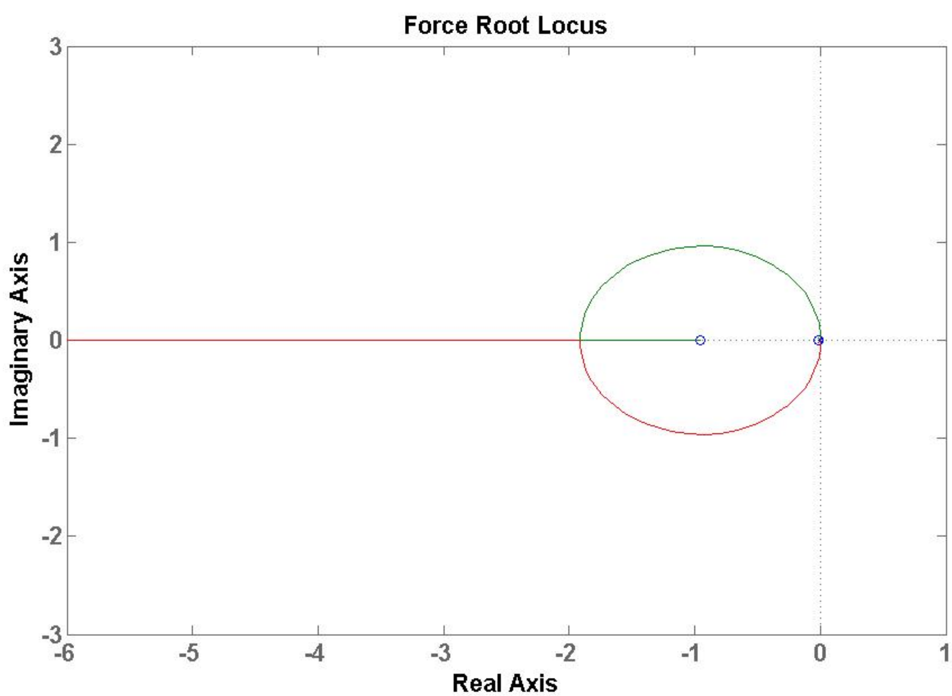
Rearranging this we get

$$X(s) = \frac{1}{s^2 + K_P + s K_D + K_I/s}\, U(s) + \frac{K_P + s K_D + K_I/s}{s^2 + K_P + s K_D + K_I/s}\, X_{ref}(s)$$

$$X(s) = \frac{s}{s^3 + s^2 K_D + s K_P + K_I}\, U(s) + \frac{s^2 K_D + s K_P + K_I}{s^3 + s^2 K_D + s K_P + K_I}\, X_{ref}(s)$$

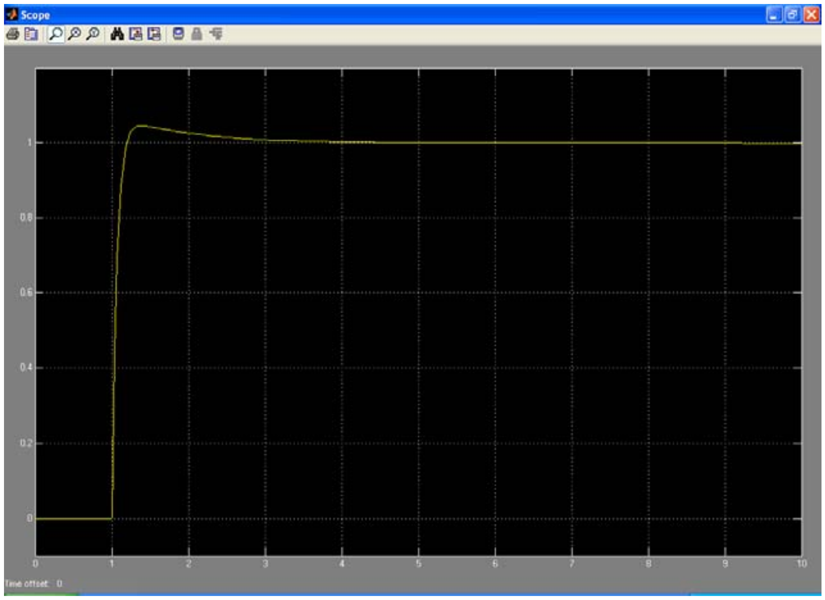


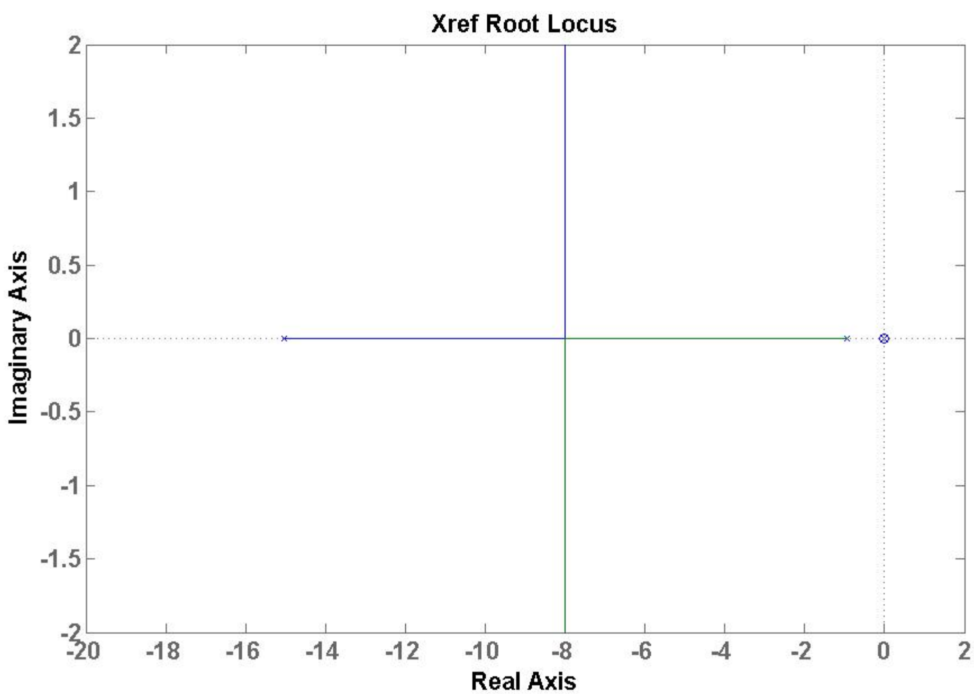




The system is BIBO stable as seen from the Root-Locus plots below.

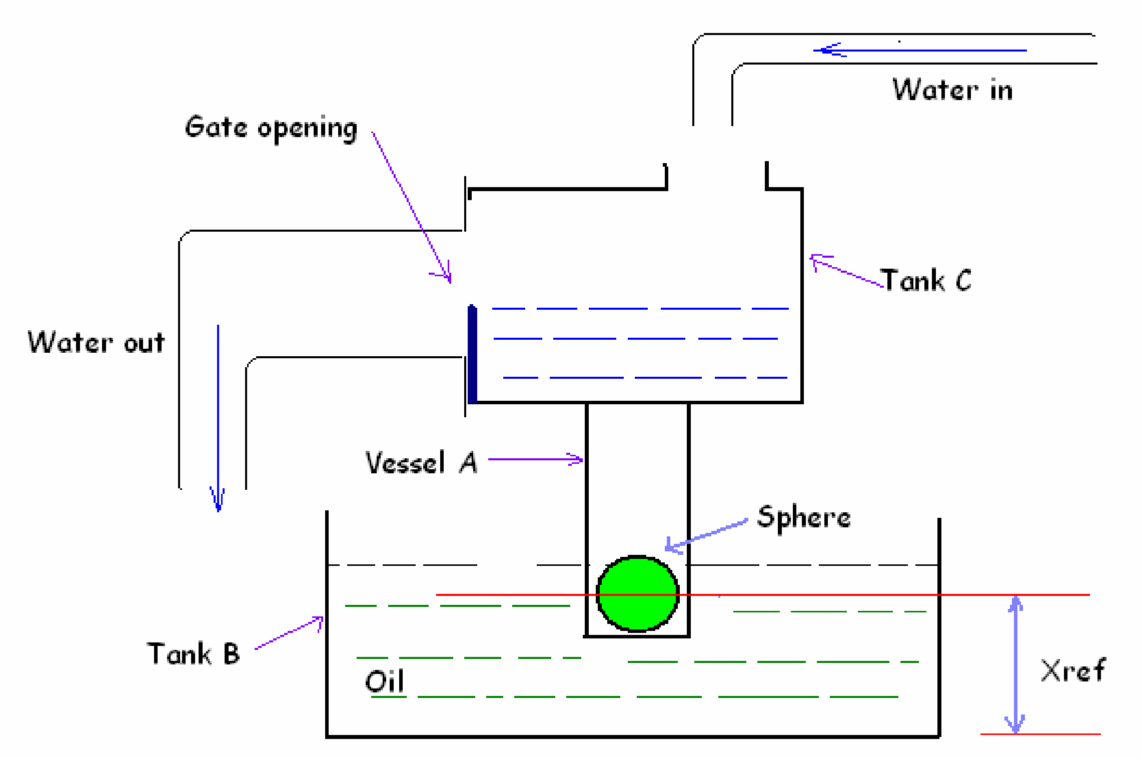
$$X_{ref}(s) \longrightarrow \boxed{\frac{s^2 K_D + s K_P + K_I}{s^3 + s^2 K_D + s K_P + K_I}} \longrightarrow X(s)$$





The system is BIBO stable as seen from the Root-Locus plots below.

(b) PID Controller Realization:



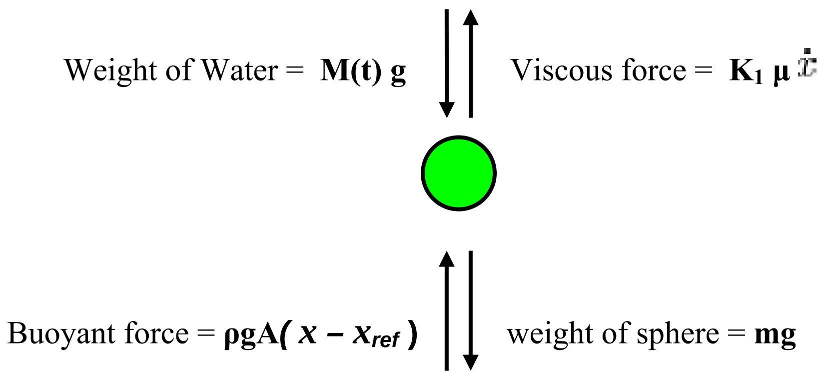
The sphere to be position controlled is put in a vessel A. The vessel A is kept in a tank B filled with oil. The vessel A is also connected to a Tank C above in which water is continuously flowing in and flowing out through gate opening.

The gate opening is such that at equilibrium position the gate is open exactly so that water coming in is equal to water flowing out. When the system is moved a little downwards the gate opens up more so that flow of water outwards increases and if the system is moved a little upwards the gate closes up so that flow of water outwards decreases.

The Xref position of the sphere is specified by the level of oil in the tank B.

(c) Modeling:

Proportional-integral-derivative (PID) control action



Forces

Proportional $= \rho g A e = K_p e = K_p (x - x_{ref})$

Integral $= M(t) g = \int q (x - x_{ref}) dt \times g = K_i \int (x - x_{ref}) dt$

Derivative $= K_1 \mu (\dot{x} - \dot{x}_{ref}) = K_D (\dot{x} - \dot{x}_{ref})$

Now at Xref position the forces are all balanced

$$Mg + mg = \rho g A h$$

At any position X we have

$$u(t) - m\ddot{x} = K_p (x - x_{ref}) + K_i \int (x - x_{ref}) dt + K_D (\dot{x} - \dot{x}_{ref})$$

(d) Linearization:

Governing equation for unit step force

$$\acute{u}(t) - m(x''') = \frac{d}{dt}\{ K_p (x - x_{ref}) + K_i \int (x - x_{ref}) dt + K_D (\dot{x} - \dot{x}_{ref})\}$$

Case 1

State variables

$x_1 = x$

$\dot{x}_1 = x_2$

$x_3 = u(t)$
$\dot{x}_3 = x_4$
$\dot{x}_2 = x_5$
$x_{ref} = const.$

Substituting above state variables in governing equation

$$x_4 - m(\dot{x}_5) = K_p (x_2 - x_{ref}) + K_i (x_1) + K_D (x_5)$$

$$\dot{x}_5 = (x_4 - K_p (x_2 - x_{ref}) - K_i (x_1) - K_D (x_5)) / m$$

$\delta\dot{x}_1 = \delta x_2$
$\delta\dot{x}_3 = \delta x_4$
$\delta\dot{x}_2 = \delta x_5$
$\delta\dot{x}_5 = (\delta x_4 - K_p (\delta x_2) - K_i (\delta x_1) - K_D (\delta x_5)) / m$

Case 2
State variables
$x_1 = x$
$\dot{x}_1 = x_2$
$\dot{x}_2 = x_5$
$x_3 = x_{ref}$
$\dot{x}_3 = x_4$
$\dot{x}_4 = x_6$
$u(t) = 0$

Substituting above state variables in governing equation

$$- m(\dot{x}_5) = K_p (x_2 - x_4) + K_i (x_1 - x_3) + K_D (x_5 - x_6)$$

$$\dot{x}_5 = - (K_p (x_2 - x_4) + K_i (x_1 - x_3) + K_D (x_5 - x_6)) / m$$

$\delta\dot{x}_1 = \delta x_2$
$\delta\dot{x}_3 = \delta x_4$
$\delta\dot{x}_2 = \delta x_5$
$\delta \dot{x}_4 = \delta x_6$
$\delta\dot{x}_5 = - (K_p (\delta x_2 - \delta x_4) + K_i (\delta x_1 - \delta x_3) + K_D (\delta x_5 - \delta x_6)) / m$

(e) PID Controller Synthesis:

Dimensions of the system elements :

Kp = 15.5 = Aρg / m

m = mass of sphere + vessel = 0.1 + 2 =2.1 kg
density of oil $\rho = 100 \text{ kg} / m^3$
this gives Area at the bottom of vessel = A = 0.0174 m^2

$\mathbf{K_D}$ = 16 = $\mu A_2 L / m$

Area at the sides of vessel $A_2 = 2 \; m^2$
Viscosity of oil μ = 6 kg/ms
We get distance between vessel 'A' and Tank 'B' walls L = 1.3 m

$\mathbf{K_I}$ = 0.25 = $V b \rho_{water} g / m$

Velocity of incoming water V = 0.01 m/ s
$\rho_{water} g = 9810 \text{ kg} / m^2s^2$
We get breadth of gate opening = b = 2.5 mm

(f) Saturation Limits and Anti-windup Strategy:

The designed system attains saturation under 2 conditions:

1. First saturation limit is attained in the Proportional force when the vessel 'A' touches the bottom of the tank 'B'. In this case no more volume of oil can be displaced by the vessel 'A'.

2. The second saturation limit is attained in the Integrative force when the tank 'C' is filled with water upto its maximum capacity or it is completely emptied. In this case no more water mass can add up or can be removed from the integral term.

(g) Higher Order Controller Realization:

To achieve higher order control systems we need to design forces which are dependent on higher order derivatives of position. Physically these forces are difficult to design and here we appreciate the advantage that microprocessor gives us.