Mathematical Modeling of Systems + Control

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Recap

- Modeling of systems
  - Need ??
  - Basis and criteria for modeling
  - Some examples with block diagram representation: simple mass, spring mass, liquid level

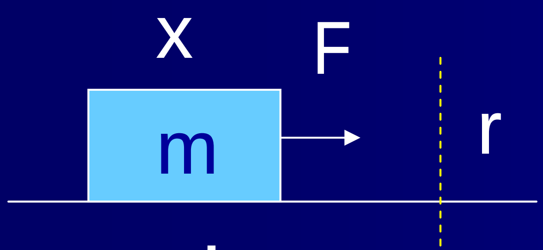
Today

- Modeling Example
  - *Spring mass system is equivalent to simple mass regulation with PD control*
  - Motor: rotary version of the above with additional dynamics due to electrical part

Steps in Modeling

- Identify the most important physical phenomenon
- Assess their complexity to be represented using equations governing physics
- Take decision regarding I/O model vs. physics
- Apply corresponding laws to get the final equations
- Verify models by matching model simulation and experimental results

Example 1: PD control



$$m\ddot{x} = F$$

Let control Law be

$$F = \underbrace{-k_p(x-r)}_{\text{Proportional part}} \underbrace{-k_d(\dot{x}-0)}_{\text{Derivative part}}$$

Substitute

$$m\ddot{x} = -k_p(x-r) - k_d(\dot{x}-0)$$

$$m\ddot{e} + k_d\dot{e} + k_p e = 0$$

$$e = x - r$$

$$\dot{e} = \dot{x}$$

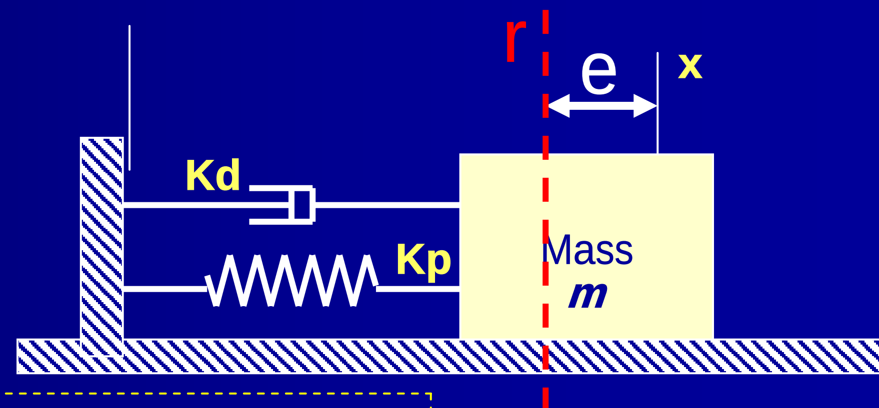
$$\ddot{e} = \ddot{x}$$

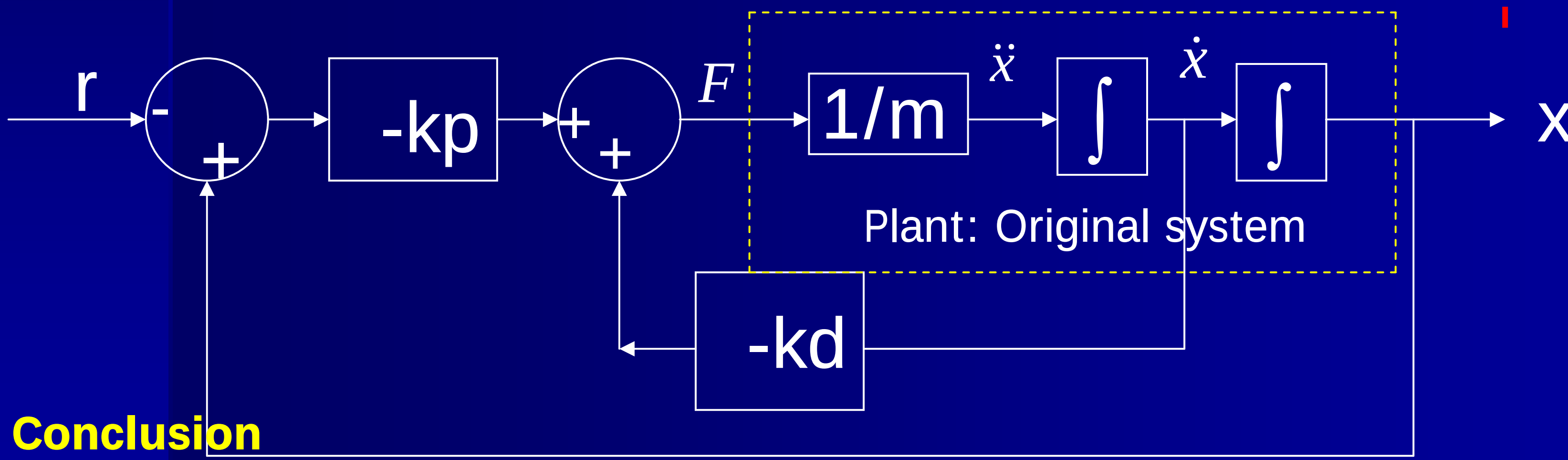
Does this click anything

Example 1: PD control

What physical system the equation corresponds to

$$m\ddot{e} + k_d \dot{e} + k_p e = 0$$





Conclusion

PD control for a mass moving on surface is equivalent to a virtual Simple spring mass system with spring const kp and damping kd

Example: PID control

Concept of integral action

ss error in integral action

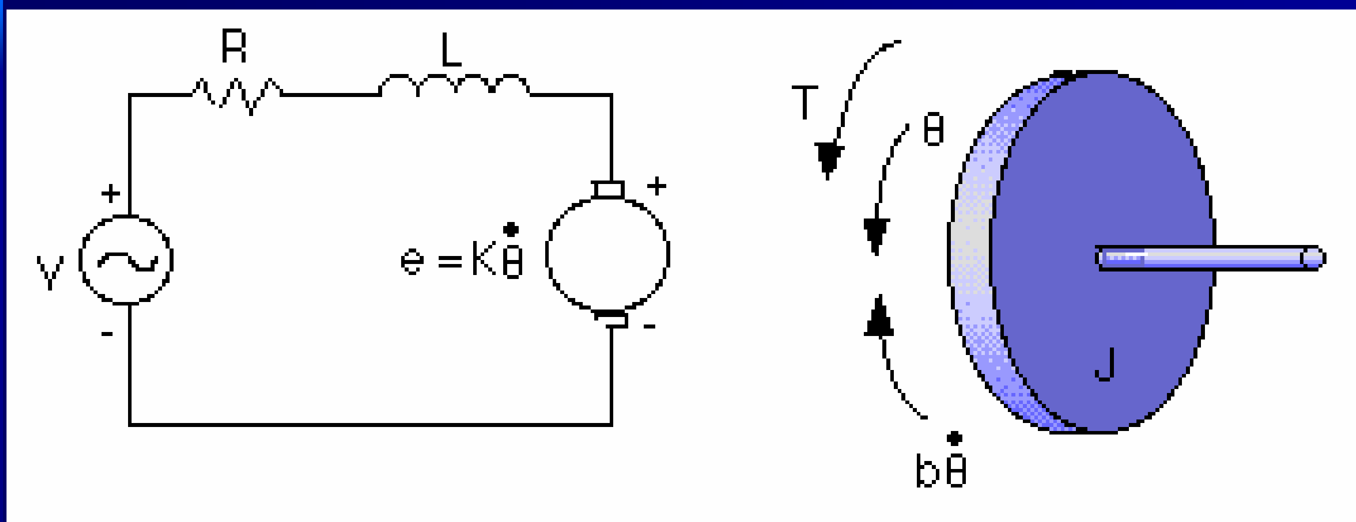
$$F = \underbrace{-k_p(x-r)}_{\text{Proportional part}} \underbrace{-k_d(\dot{x}-0)}_{\text{Derivative part}} \underbrace{-k_i\int(x-r)dt}_{\text{Integral part}}$$

$$= -k_p e - k_d \dot{e} - k_i \int e dt$$

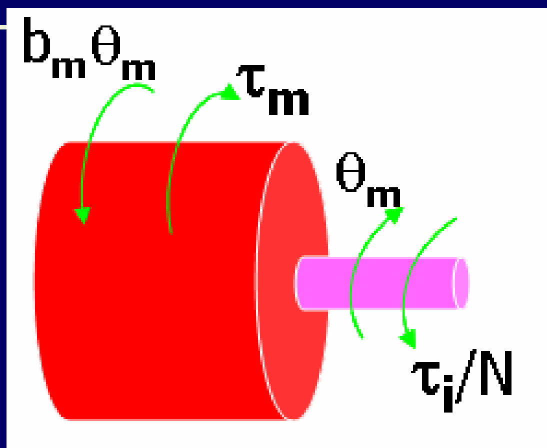
$$m\ddot{e} + k_d \dot{e} + k_p e + k_i \int e dt = 0$$

Example: DC Motor

Simple model of a DC motor



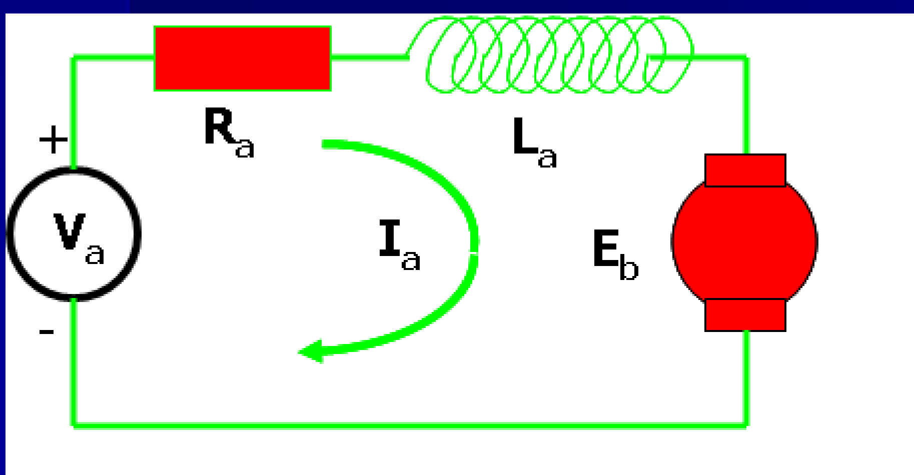
Motor Dynamics



$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m + \frac{\tau_i}{N} = \tau_m$$

$$\tau_m = K_t\phi I_a$$

Armature Free Body Diagram



$$V_a = I_a R_a + E_b + L\frac{dI_a}{dt}$$

$$E_b \propto \phi\omega$$

$$E_b = K\phi\omega$$

Electrical circuit Diagram

Next Class

- Control: Proportional, Derivative, Integral control actions (PID controller: popular in industry)
- Linearity LTI