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EN634 Nuclear Reactor Thermal Hydraulics

Modelling of Two-phase Flows

Homogeneous Equilibrium Model

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Homogeneous Equilibrium Model

- Homogeneous refers to complete dispersal. This leads to the definition of average properties.
- The term equilibrium refers to thermodynamic equilibrium between phases.
- By virtue of complete dispersal, we can assume that there is no slip between gas and liquid locally. For plug flow model, this will imply there is no global slip between average velocities.

$$\dot{m} = \dot{m}_g + \dot{m}_l = \rho_g A_g u_g + \rho_l A_l u_l$$

$$= A u_H \left(\rho_g \alpha + \rho_l (1-\alpha)\right) = A u_H \rho_H$$

Consistent with single-phase flow

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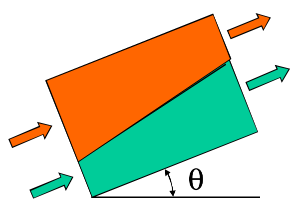
Homogeneous No-Slip Model

- Conservation of Mass

Gas $\dfrac{\partial(\rho_g A_g)}{\partial t} + \dfrac{\partial(\dot{m}_g)}{\partial s} = \Gamma'_g$

liquid $\dfrac{\partial(\rho_l A_l)}{\partial t} + \dfrac{\partial(\dot{m}_l)}{\partial s} = \Gamma'_l$

Pipe $\dfrac{\partial\left(A[\rho_l(1-\alpha)+\rho_g\alpha]\right)}{\partial t} + \dfrac{\partial(\dot{m})}{\partial s} = 0$



Note $\Gamma'_g = -\Gamma'_l$

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Momentum Equation-I

For Single-Phase

$$\frac{\partial(\rho A u)}{\partial t} + \frac{\partial(\rho A u^2)}{\partial s} = -A\frac{\partial p}{\partial x} - \tau_w P - \rho A g \mathrm{Sin}\theta$$

For Two-Phase

$$\frac{\partial(\rho_H A u_H)}{\partial t} + \frac{\partial(\rho_H A u_H{}^2)}{\partial s} = -A\frac{\partial p}{\partial x} - \tau_w P - \rho_H A g \mathrm{Sin}\theta$$

Note $\rho_H = \rho_g \alpha + \rho_l(1-\alpha)$; $u_H = \dot{m}/\rho_H A$

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Momentum Equation-II

- Rearranging

$$-\frac{\partial p}{\partial x}=\frac{1}{A}\frac{\partial(\dot{m})}{\partial t}+\frac{1}{A}\frac{\partial(\rho_H A u_H^{\,2})}{\partial s}+\tau_w\frac{P}{A}+\rho_H g \mathrm{Sin}\theta$$

temporal | accelerational | frictional | gravitational

Pressure gradient

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Momentum Equation-III

- We can get non-conservative form by using mass conservation equation

$$\rho_H A\frac{\partial u_H}{\partial t}+\rho_H A u_H\frac{\partial u_H}{\partial s}=-A\frac{\partial p}{\partial x}-\tau_w P-\rho_H A g \mathrm{Sin}\theta$$

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Mechanical Energy Equation

Multiplication of momentum equation with velocity will give mechanical energy equation

$$\rho_H A u_H\frac{\partial u_H}{\partial t}+\rho_H A u_H^{\,2}\frac{\partial u_H}{\partial s}=-u_H A\frac{\partial p}{\partial x}-u_H\tau_w P$$
$$-u_H\rho_H A g \mathrm{Sin}\theta$$

$$\rho_H A\frac{\partial u_H^{\,2}}{\partial t}+\rho_H A\frac{\partial u_H^{\,2}}{\partial s}=-u_H A\frac{\partial p}{\partial x}-u_H\tau_w P-u_H\rho_H A g \mathrm{Sin}\theta$$

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I-Law of Thermodynamics

Definitions

Specific Flow Energy

$$e=h+u^2/2+gZ$$

Specific Energy

$$i=h-p/\rho+u^2/2+gZ$$

Internal energy

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I-Law - II

$$\dot{E}_{CV} = \dot{Q} - \dot{W} + \dot{m}(e_{inlet} - e_{exit})$$

- For homogeneous flow

$$\dot{E}_{CV} = \frac{\partial\left(\rho_g A_g \Delta s\left(e_g - {p}/{\rho_g}\right) + \rho_l A_l \Delta s\left(e_l - {p}/{\rho_l}\right)\right)}{\partial t}$$

$$= A\Delta s \frac{\partial\left(\rho_g \alpha e_g + \rho_l(1-\alpha)e_l - p\right)}{\partial t}$$

$$= A\Delta s \frac{\partial\left(\rho_H e_H - p\right)}{\partial t}$$

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I-Law - III

$$\dot{E}_{CV} = A\Delta s \frac{\partial\left(\rho_H e_H - p\right)}{\partial t}$$

- Here the definition of $\rho_H e_H$ should be noted

$$\rho_H e_H = \rho_g \alpha e_g + \rho_l(1-\alpha)e_l$$

- On similar lines we can derive the following

$$\dot{m}_{inlet} e_{inlet} - \dot{m}_{inlet} e_{exit} = -\frac{\partial(\dot{m}_g e_g + \dot{m}_l e_l)}{\partial s}\Delta s$$

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I-Law - IV

$$-\frac{\partial(\dot{m}_g e_g + \dot{m}_l e_l)}{\partial s}\Delta s = -\frac{\partial(\rho_g A_g u_H e_g + \rho_l A_l u_H e_l)}{\partial s}\Delta s$$

$$= -A\Delta s u_H \frac{\partial(\rho_g \alpha e_g + \rho_l(1-\alpha)e_l)}{\partial s}$$

$$= -A\Delta s u_H \frac{\partial(\rho_H e_H)}{\partial s}$$

- We can write

$$\dot{W} = \tau_w P\Delta s u_H + W'_{CV}\Delta s \qquad \dot{Q} = q'' P\Delta s$$

Axial conduction neglected

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I-Law - V

- In view of the above

$$\frac{\partial(\rho_H A e_H)}{\partial t} + \frac{\partial(\rho_H A u_H e_H)}{\partial s} = A\frac{\partial p}{\partial t} + q''P - \tau_w P u_H - W'$$

- Using mass conservation we can write

$$\rho_H A\frac{\partial(e_H)}{\partial t} + \rho_H A u_H \frac{\partial(e_H)}{\partial s} = A\frac{\partial p}{\partial t} + q''P - \tau_w P u_H - W'$$

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I-Law - VI

- Subtracting both sides of mechanical energy equation from I – Law, we can write the thermal energy equation:

$$\rho_H A\frac{\partial(h_H)}{\partial t}+\rho_H A u_H\frac{\partial(h_H)}{\partial s}=A\frac{\partial p}{\partial t}+q''P-W'$$

- Here we have assumed that shear work is lost out to the surroundings. For insulated systems this gets ploughed back and hence we have to add this term.

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Final Set of Equations

Mass Balance

$$\frac{\partial(A\rho_H)}{\partial t}+\frac{\partial(\dot{m})}{\partial s}=0$$

Momentum Balance

$$-\frac{\partial p}{\partial x}=\frac{1}{A}\frac{\partial(\dot{m})}{\partial t}+\frac{1}{A}\frac{\partial(\rho_H A u_H{}^2)}{\partial s}+\tau_w\frac{P}{A}+\rho_H g Sin\theta$$

Energy Balance

$$\rho_H A\frac{\partial(h_H)}{\partial t}+\rho_H A u_H\frac{\partial(h_H)}{\partial s}=A\frac{\partial p}{\partial t}+q''P-W'$$

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Solution Strategy-I

Variables	$\rho_H, u_H, p, \tau_w, h_H$
Equations	3
Closing relations required	2

$$\rho_H=\rho_H(p,\alpha)$$

$$h_H=h_H(p,\alpha)$$

If $\tau_w=\tau_w(p,\alpha,u_H)$ then the system is closed

Since for no slip flow $\alpha=\alpha(x)$ we can also choose (p,x,u_H) system

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Solution Strategy-II

$v_H=v_f+xv_{fg}$ and $\rho_H=\dfrac{1}{v_H}$

$h_H=h_f+xh_{fg}$

If $\tau_w=\tau_w(p,x,u_H)$, then the system is closed

Closure for τ_w

- Two-phase multipliers are used to close
- These are empirical, though some theoretical justification exists

Solution Strategy-III

Homogeneous Multiplier

$$\tau_{w-2\phi} = \tau_{w-1\phi}\phi_{lo}^2$$

$$\phi_{lo}^2 = \left[1 + x\left(\frac{\rho_f}{\rho_g} - 1\right)\right]\left[1 + x\left(\frac{\mu_f}{\mu_g} - 1\right)\right]^{-0.2}$$

- The single-phase wall shear is computed assuming total mass flow rate is equal to single-phase mass flow rate.
- The second term in two-phase multiplier is sometimes neglected with no loss of accuracy

Closure for Friction-I

In single-phase flow

$$\tau_{w-1\varphi} = 0.5\rho u^2 C\left(\frac{\rho u d_h}{\mu}\right)^{-n} = \frac{0.5\dot{m}^2}{\rho A^2} C\left(\frac{\rho u d_h}{\mu}\right)^{-n}$$

If we choose to extend

$$\tau_{w-2\varphi} = 0.5\rho_H u_H{}^2 C_H\left(\frac{\rho_H u_H d_{hyd}}{\mu_H}\right)^{-n} = 0.5\frac{\dot{m}^2}{\rho_H A^2} C_H\left(\frac{\rho_H u_H d_{hyd}}{\mu_H}\right)^{-}$$

If we compare the results for same $\dot{m}$, and we assume C and n are identical in both equations, dividing one equation by the other we get

$$\frac{\tau_{w-2\varphi}}{\tau_{w-1\varphi}} = \phi_{lo}{}^2 = \frac{\rho_f}{\rho_H}\left(\frac{\mu_f}{\mu_H}\right)^{-n} = \frac{v_H}{v_f}\left(\frac{\mu_f}{\mu_H}\right)^{-n}$$

Closure for Friction-II

If we choose

$$\frac{1}{\mu_H} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f}$$

We get the desired multiplier

$$\phi_{lo}^2 = \left[1 + x\left(\frac{\rho_f}{\rho_g} - 1\right)\right]\left[1 + x\left(\frac{\mu_f}{\mu_g} - 1\right)\right]^{-0.2}$$

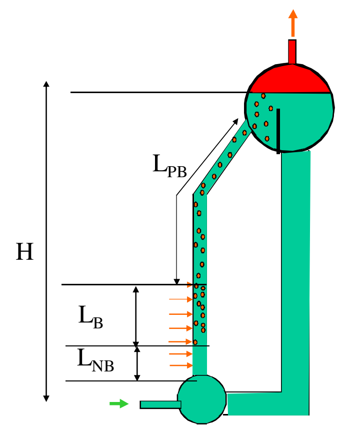
Note n is chosen as –0.2. Thus from definitions

$$\left.\frac{dp}{ds}\right|_{2\varphi} = \left.\frac{dp}{ds}\right|_{1\varphi}\phi_{lo}^2$$

Illustrative Application

- To illustrate the application let us consider circulation in a boiler
- The downcomer is considered large so as to neglect frictional effects

$$p_{feeder} - p_{drum} = \rho_f g H \quad (1)$$



Application-I

Mass Balance

$$\cancel{\frac{\partial(A\rho_H)}{\partial t}} + \frac{\partial(\dot{m})}{\partial s} = 0$$

Mass low rate is constant along the duct

Energy Balance

$$\cancel{\rho_H \frac{\partial(h_H)}{\partial t}} + \dot{m}\frac{\partial(h_H)}{\partial s} = \cancel{\frac{\dot{m}}{\rho_H}\frac{\partial p}{\partial s}} + \cancel{A\frac{\partial p}{\partial t}} + q''P$$

$$-\cancel{W} + \cancel{\frac{\tau_w P}{A}\frac{\dot{m}}{\rho_H}}$$

Application-II

$$\frac{d(h_H)}{ds} = \frac{q''P}{\dot{m}}$$

Integration in Non-boiling region

$$\int_{h_{in}}^{h_f} d(h_H) = \frac{q''P}{\dot{m}}\int_0^{L_{NB}} ds \Rightarrow h_f - h_{in} = \frac{q''P}{\dot{m}} L_{NB}$$

$$\Rightarrow L_{NB} == \frac{\dot{m}(h_f - h_{in})}{q''P} \qquad 2$$

Energy Balance in feeder

$$\dot{m}h_{in} = \dot{m}_{feed}h_{feed} + (\dot{m} - \dot{m}_{feed})h_f$$

Application-III

$$h_{in} = \frac{\dot{m}_{feed}}{\dot{m}}h_{feed} + \frac{(\dot{m} - \dot{m}_{feed})}{\dot{m}}h_f \qquad 3$$

Integration in Boiling region

$$\int_{h_f}^{h_{exit}} d(h_H) = \frac{q''P}{\dot{m}}\int_0^{L_B} ds \Rightarrow h_e - h_f = \frac{q''P}{\dot{m}} L_B$$

$$h_f + x_e h_{fg} - h_f = \frac{q''P}{\dot{m}} L_B$$

$$x_e = \frac{q''P}{\dot{m}h_{fg}} L_B \qquad 4$$

Application-III

Integration in Boiling region upto any arbitrary s, where quality is x can similarly be shown as

$$x = \frac{q''P}{\dot{m}h_{fg}} s \qquad 5$$

In Non-boiling region

$$x = x_e \qquad 6$$

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Application-IV

Momentum Balance

$$-\frac{\partial p}{\partial s}=\frac{1}{A}\frac{\partial(\cancel{\dot m})}{\cancel{\partial t}}+\frac{1}{A}\frac{\partial(\rho_H A u_H^{\ 2})}{\partial s}+\tau_w\frac{P}{A}+\rho_H g Sin\theta \quad (7)$$

We shall integrate term by term

Integration from feeder to drum

$$-\int_{feeder}^{drum}\frac{\partial p}{\partial s}ds=(p_{feeder}-p_{drum})=\rho_f gH \quad \text{[From Eq. (1)]}$$

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Application-V

v = constant

$$\int_{feeder}^{drum}\frac{1}{A}\frac{\partial\left(\dot m^2/A\rho_H\right)}{\partial s}ds=\frac{\dot m^2}{A^2}\left(\cancel{\int_{NB}\frac{\partial v_H}{\partial s}ds}+\int_B\frac{\partial v_H}{\partial s}ds+\cancel{\int_{PB}\frac{\partial v_H}{\partial s}ds}\right)$$

$$-\Delta p_{acc}=\frac{\dot m^2}{A^2}\left(v_{exit}-v_f\right)=\frac{\dot m^2}{A^2}\left(v_f+x_e v_{fg}-v_f\right)$$

$$\therefore -\Delta p_{acc}=\frac{\dot m^2}{A^2}x_e v_{fg} \quad (8)$$

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Application-VI

$$\int_{feeder}^{drum}\frac{\tau_w P}{A}ds=\int_{NB}\frac{\tau_w P}{A}ds+\int_B\frac{\tau_w P}{A}ds+\int_{PB}\frac{\tau_w P}{A}ds$$

$$\int_{NB}\frac{\tau_w P}{A}ds=\frac{4\dot m^2 f_{lo}}{2\rho_f d_{hyd}A^2}L_{NB} \quad (9)$$

$$\int_B\frac{\tau_w P}{A}ds=\int_B\frac{\tau_{wlo}P}{A}\phi_{lo}^2 ds=\frac{4\dot m^2 f_{lo}}{2\rho_f d_{hyd}A^2}\int_B\phi_{lo}^2 ds$$

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Application-VII

$$=\frac{4\dot m^2 f_{lo}}{2\rho_f d_{hyd}A^2}\int_B\left(1+x\left(\frac{\rho_f}{\rho_g}-\cancel{1}\right)\right)\left(1+x\left(\cancel{\frac{\mu_f}{\mu_g}-1}\right)\right)^{-0.2}ds$$

small

$$=\frac{4\dot m^2 f_{lo}}{2\rho_f d_{hyd}A^2}\int_B\left(1+x\frac{\rho_f}{\rho_g}\right)ds$$

Substituting for variation of x as given by Eq. (5) and integration gives

$$\int_B\frac{\tau_w}{A}Pds=\frac{4\dot m^2 f_{lo}}{2\rho_f d_{hyd}A^2}L_B\left(1+\frac{q''P}{2\dot m h_{fg}}\frac{\rho_f}{\rho_g}L_B\right) \quad (10)$$

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Application-VIII

In post boiling region x is constant hence integration is straight forward

$$\int_{PB} \frac{\tau_w}{A} P ds = \frac{4\dot{m}^2 f_{lo}}{2\rho_f d_{hyd} A^2} \int_{PB} \left(1 + x_e \left(\frac{\rho_f}{\rho_g} - 1 \right) \right) ds$$

$$\int_{PB} \frac{\tau_w}{A} P ds = \frac{4\dot{m}^2 f_{lo}}{2\rho_f d_{hyd} A^2} L_{PB} \left(1 + x_e \frac{\rho_f}{\rho_g} \right) \quad (11)$$

Thus

$$-\Delta p_{fric} = \frac{4\dot{m}^2 f_{lo}}{2\rho_f d_{hyd} A^2} \left[L_{NB} + L_{NB} \left(1 + \frac{x_e}{2} \frac{\rho_f}{\rho_g} \right) + L_{PB} \left(1 + x_e \frac{\rho_f}{\rho_g} \right) \right] \quad (12)$$

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Application-IX

$$\int_{feeder}^{drum} \rho_H g Sin\theta ds = \int_{feeder}^{drum} \rho_H g dH = \int_{NB} \rho_H g dH + \int_{B} \rho_H g dH + \int_{PB} \rho_H g dH$$

$$\int_{NB} \rho_H g dH = \rho_f g (\Delta H)_{NB} \quad (13)$$

$$\int_{B} \rho_H g dH = \int_{B} \frac{1}{v_H} g ds = \int_{B} \frac{1}{v_f + x v_{fg}} g ds$$

Substituting for variation of x as given by Eq. (5) and integration gives

$$\int_{B} \rho_H g dH = \frac{L_B g}{v_f} \left[\frac{ln\left(1 + \frac{x_e v_{fg}}{v_f} \right)}{\frac{x_e v_{fg}}{v_f}} \right] \quad (14)$$

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Application-X

In post boiling region density is constant hence integration is straight forward

$$\int_{B} \rho_H g dH = \rho_e g (\Delta H)_{PB} = g (\Delta H)_{PB} \frac{1}{v_f + x_e v_{fg}} \quad (15)$$

Thus

$$-\Delta p_{grav} = \rho_f g \left\{ \begin{array}{l} (\Delta H)_{NB} + L_B \left[\dfrac{ln\left(1 + \frac{x_e v_{fg}}{v_f} \right)}{\frac{x_e v_{fg}}{v_f}} \right] \\ + (\Delta H)_{PB} \dfrac{1}{1 + \frac{x_e v_{fg}}{v_f}} \end{array} \right\} \quad (16)$$

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Application-XI

Substituting all the terms in Eq. (7), we get

$$\rho_f g H = \frac{\dot{m}^2}{A^2} x_e v_{fg} - \Delta p_{fric} - \Delta p_{grav} \quad (17)$$

For a given H, q'', system pressure, feed flow and enthalpy, we can compute the circulating mass flow rate as follows

1. Assume mass flow rate
2. Obtain h_{in} using Eq. (3)
3. Obtain L_{NB} using Eq. (2), hence get L_B
4. Obtain x_e using Eq. (4)
5. Obtain acceleration (Eq. (8)), friction (Eq. (12)) and gravitational (Eq. (16)) components of pressure drop.
6. Check if Eq. (17) is satisfied, else iterate with new mass low rate till convergence is achieved