EN634 Nuclear Reactor Thermal Hydraulics

Separated Flow Model

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Limitations of Homogeneous Equilibrium Model

- Homogeneous refers to complete dispersal. This leads to the definition of average properties.
- This model is invariably used to get first cut results and is often adequate at high pressures within 20% accuracy and can be tuned
- However, at low pressures and low qualities, dispersal is not natural and hence better models are sought
- The most general model one can construct for a two-field (vapour continuous and gas continuous) flow is the Separated Flow Model

Separated Flow Model

- In most general Separated Flow Model, Mechanical and Thermal equilibrium are not considered
- Thus, we need to solve 6 equations, 3 (mass, momentum and energy) for one phase and 3 for the other
- Though such a model looks more accurate, several closure equations have to be generated and this results in too many empirical equations to be employed. Hence it needs sufficient expertise to succeed in getting satisfactory results
- We shall just get a feel for it before proceeding to use simpler models

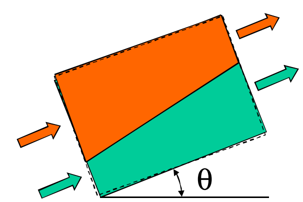
Mass Conservation for Phases

- Gas Phase

$$\frac{\partial(\rho_g A_g)}{\partial t} + \frac{\partial(\rho_g A_g u_g)}{\partial s} = \Gamma'_g$$

- Dividing by pipe area

$$\frac{\partial(\alpha\rho_g)}{\partial t} + \frac{\partial(\alpha\rho_g u_g)}{\partial s} = \Gamma'''_g \quad (1)$$



- Similarly for liquid phase

$$\frac{\partial((1-\alpha)\rho_l)}{\partial t} + \frac{\partial((1-\alpha)\rho_l u_l)}{\partial s} = \Gamma'''_l \quad (2)$$

Note

$$\Gamma'''_l = -\Gamma'''_g$$

Mixture Mass Conservation

- Adding the two mass equations, we get

$$\frac{\partial\left(\rho_l(1-\alpha)+\rho_g\alpha\right)}{\partial t}+\frac{\partial\left(\rho_l u_l(1-\alpha)+\rho_g u_g\alpha\right)}{\partial s}=0 \quad (3)$$

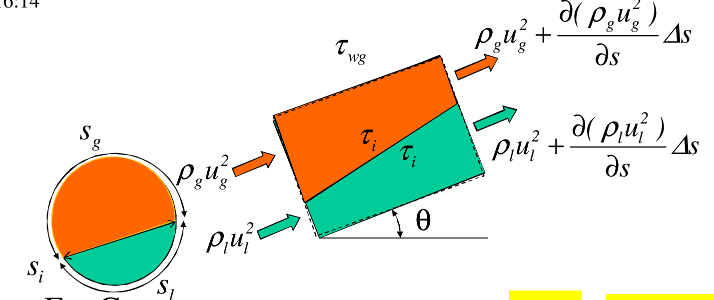
- To make the equations less menacing, we define the following

$$\rho_m=\rho_l(1-\alpha)+\rho_g\alpha \qquad \rho_m u_m=\left(\rho_l u_l(1-\alpha)+\rho_g u_g\alpha\right) \quad (4)$$

- With the above definitions, we can rewrite the mixture mass balance as,

$$\frac{\partial\rho_m}{\partial t}+\frac{\partial\rho_m u_m}{\partial s}=0 \quad (5)$$

Momentum Equation-I



Gas Shear

Interfacial Shear

$$\frac{\partial(\rho_g A_g u_g)}{\partial t}+\frac{\partial(\rho_g A_g u_g^{\,2})}{\partial s}=-A_g\frac{\partial p_g}{\partial x}-\tau_{wg}s_g-\tau_{wi}s_i$$

Mom. Source

$$-\rho_g A_g g Sin\theta+\Gamma_g' u^*$$

u^* donor vel

Momentum Equation-II

Dividing by pipe area

$$\frac{\partial(\alpha\rho_g u_g)}{\partial t}+\frac{\partial(\alpha\rho_g u_g^{\,2})}{\partial s}=-\alpha\frac{\partial p_g}{\partial x}-\frac{\tau_{wg}s_g}{A}-\frac{\tau_{wi}s_i}{A} \quad (6)$$

$$-\alpha\rho_g g Sin\theta+\Gamma_g''' u^*$$

Similarly for Liquid

$$\frac{\partial((1-\alpha)\rho_l u_l)}{\partial t}+\frac{\partial((1-\alpha)\rho_l u_l^{\,2})}{\partial s}=-(1-\alpha)\frac{\partial p_l}{\partial x}-\frac{\tau_{wl}s_l}{A}+\frac{\tau_{wi}s_i}{A}$$

$$-(1-\alpha)\rho_l g Sin\theta+\Gamma_l''' u^* \quad (7)$$

Momentum Equation-III

Adding the two equations, we get

$$\frac{\partial(\alpha\rho_g u_g+(1-\alpha)\rho_l u_l)}{\partial t}+\frac{\partial(\alpha\rho_g u_g^{\,2}+(1-\alpha)\rho_l u_l^{\,2})}{\partial s}=-\alpha\frac{\partial p_g}{\partial x}$$

$$-(1-\alpha)\frac{\partial p_l}{\partial x}-\frac{\tau_{wg}s_g}{A}-\frac{\tau_{wl}s_l}{A}-\left(\alpha\rho_g+(1-\alpha)\rho_l\right)g Sin\theta \quad (8)$$

Note that the interfacial terms cancel. This includes the source term

Mechanical Energy Equation-I

- Using gas mass balance (Eq. (1)) and momentum balance (Eq. (2)), we can write

$$\alpha\rho_g \frac{\partial u_g}{\partial t} + \alpha\rho_g u_g \frac{\partial u_g}{\partial s} = -\Gamma_g''' u_g - \alpha\frac{\partial p_g}{\partial s} - \frac{\tau_{wg} s_g}{A} - \frac{\tau_{wi} s_i}{A}$$

Comes from mass balance

$$-\alpha\rho_g g Sin\theta + \Gamma_g''' u^* \quad (9)$$

- Multiplying the above equation by u_g gives

$$\alpha\rho_g \frac{\partial}{\partial t}\left(\frac{u_g^{\ 2}}{2}\right) + \alpha\rho_g u_g \frac{\partial}{\partial s}\left(\frac{u_g^{\ 2}}{2}\right) = -\Gamma_g''' u_g^{\ 2}$$

$$-\alpha u_g \frac{\partial p_g}{\partial s} - \frac{\tau_{wg} s_g u_g}{A} - \frac{\tau_{wi} s_i u_g}{A} - \alpha\rho_g u_g g Sin\theta + \Gamma_g''' u_g u^* \quad (10)$$

Mechanical Energy Equation-II

- Similarly, for the liquid phase, we can write

$$(1-\alpha)\rho_l \frac{\partial}{\partial t}\left(\frac{u_l^{\ 2}}{2}\right) + \alpha\rho_l u_l \frac{\partial}{\partial s}\left(\frac{u_l^{\ 2}}{2}\right) = -\Gamma_l''' u_l^{\ 2} \quad (11)$$

$$-(1-\alpha)u_l \frac{\partial p_l}{\partial s} - \frac{\tau_{wl} s_l u_l}{A} + \frac{\tau_{wi} s_i u_l}{A} - (1-\alpha)\rho_l u_l g Sin\theta + \Gamma_l''' u_l u^*$$

Total Energy Equation-I

$$\frac{\partial\left(\rho_g A_g\left(e_g - {p_g}/{\rho_g}\right)\right)}{\partial t} + \frac{\partial\left(\rho_g A_g u_g e_g\right)}{\partial s} = \Gamma' e^*$$

$$-\tau_i s_i (u_g - u_l) - \tau_g s_g u_g + q_g' - W_g' - \frac{\partial}{\partial s}\left(q''_{axial} A_g\right)$$

- Dividing by area of the pipe, we get

$$\frac{\partial(\alpha\rho_g e_g)}{\partial t} + \frac{\partial(\alpha\rho_g u_g e_g)}{\partial s} = \frac{\partial(\alpha p_g)}{\partial t} + \Gamma''' e^*$$

$$-\frac{\tau_i s_i (u_g - u_l)}{A} - \frac{\tau_g s_g u_g}{A} + q_g''' - W_g''' - \frac{\partial}{\partial s}\left(\alpha q''_{axial}\right) \quad (12)$$

Total Energy Equation-II

$$\alpha\rho_g \frac{\partial(e_g)}{\partial t} + \alpha\rho_g u_g \frac{\partial(e_g)}{\partial s} = -\Gamma''' e_g + \frac{\partial(\alpha p_g)}{\partial t} + \Gamma''' e^*$$

$$-\frac{\tau_i s_i (u_g - u_l)}{A} - \frac{\tau_g s_g u_g}{A} + q_g''' - W_g''' - \frac{\partial}{\partial s}\left(\alpha q''_{axial}\right)$$

- Subtracting mechanical energy (eq. (10)) (13)

$$\alpha\rho_g \frac{\partial(h_g)}{\partial t} + \alpha\rho_g u_g \frac{\partial(h_g)}{\partial s} = \Gamma'''\left(e^* - e_g + u_g^{\ 2} - u_g u^*\right) + \frac{\partial(\alpha p_g)}{\partial t}$$

$$-\frac{\tau_i s_i u_l}{A} + q_g''' - W_g''' - \frac{\partial}{\partial s}\left(\alpha q''_{axial}\right) + u_g \alpha \frac{\partial p}{\partial s}$$

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Thermal Energy Equation-I

- While wall shear can be assumed to be ploughed in gas, interfacial shear has to be accommodated carefully.

$$\alpha\rho_g \frac{\partial(h_g)}{\partial t} + \alpha\rho_g u_g \frac{\partial(h_g)}{\partial s} = \Gamma_g'''\left(e^* - e_g + u_g^{\,2} - u_g u^*\right) + \frac{\partial(\alpha p_g)}{\partial t}$$
$$+ \frac{\tau_g s_g u_g}{A} + \frac{\tau_i s_i u_l}{A} + q_g''' - W_g''' - \frac{\partial}{\partial s}\left(\alpha q''_{axial}\right) + u_g \alpha \frac{\partial p_g}{\partial s} \quad (14)$$

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Thermal Energy Equation-II

- Similarly, we can get an equation for liquid phase

$$(1-\alpha)\rho_l \frac{\partial(h_l)}{\partial t} + \alpha\rho_l u_l \frac{\partial(h_l)}{\partial s} = \Gamma_l'''\left(e^* - e_l + u_l^{\,2} - u_l u^*\right) + \frac{\partial(\alpha p_l)}{\partial t}$$
$$+ \frac{\tau_l s_l u_l}{A} - \frac{\tau_i s_i u_g}{A} + q_l''' - W_l''' - \frac{\partial}{\partial s}\left((1-\alpha) q''_{axial}\right) + u_l (1-\alpha) \frac{\partial p_l}{\partial s} \quad (15)$$

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Thermal Energy Equation-III

- Summation of Eqs. (14) and (15) gives

$$\alpha\rho_g \frac{\partial(h_g)}{\partial t} + (1-\alpha)\rho_l \frac{\partial(h_l)}{\partial t} + \alpha\rho_g u_g \frac{\partial(h_g)}{\partial s} + (1-\alpha)\rho_l u_l \frac{\partial(h_l)}{\partial s} =$$
$$\Gamma_l'''\left(e^* - e_g + u_g^{\,2} - u_g u^*\right) + \left(e^* - e_l + u_l^{\,2} - u_l u^*\right) + \alpha \frac{\partial(p_g + p_l)}{\partial t}$$
$$+ (p_g - p_l)\frac{\partial(\alpha)}{\partial t} + \alpha u_g \frac{\partial(p_g)}{\partial s} + (1-\alpha) u_l \frac{\partial(p_l)}{\partial s} + \frac{\tau_l s_l u_l}{A} + \frac{\tau_g s_g u_g}{A}$$
$$+ q_l''' - W_l''' - \frac{\partial}{\partial s}\left((1-\alpha) q''_{l-axial} + \alpha q''_{g-axial}\right) \quad (16)$$

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Solution Strategy-I

Variables $\alpha, \rho_g, \rho_l, u_g, u_l, p_g, p_l, \tau_g, \tau_l, \tau_i, s_g, s_l, s_i$
$h_g, h_l, q''_{g-ax}, q''_{l-ax}, T_g, T_l, \Gamma_g''', \Gamma_l'''$ **21**

Equations **6** **Closing relations required** **15**

$T_g = T_g(p_g, h_g)$ **1** $T_l = T_l(p_l, h_l)$ **1**

$\rho_g = \rho_g(p_g, h_g)$ **1** $\rho_l = \rho_l(p_l, h_l)$ **1**

$(\alpha, geo) \Rightarrow s_g, s_l, s_i$ **3** $model\ for \Rightarrow \Gamma_g''',\ \Gamma_g''' = -\Gamma_l'''$ **2**

$model\ for \Rightarrow \tau_g, \tau_l, \tau_i$ **3** $model\ for \Rightarrow p_g \Leftrightarrow p_l$ **1**

$q''_{g-ax} = f(\nabla T_g), q''_{l-ax} = f(\nabla T_l)$ **2**