EN634 Nuclear Reactor Thermal Hydraulics

Slip Flow Model

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Limitations of Separated Flow Model

- Separated flow model requires a large number of closing relations.
- Many of these are difficult to be specified universally
- Simpler concepts to account for slip and empirically closing for slip is often practiced and is called slip flow model
- $p_l = p_g$ is invoked, implying mechanical equilibrium
- Better models have been tried for wall shear representation
- We shall look at these.

Mixture Mass Conservation

- We had derived the mass balance as

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial \rho_m u_m}{\partial s} = 0 \quad (1)$$

- As derived earlier, the definitions of mixture quantities are given by

$$\rho_m = \rho_l(1-\alpha) + \rho_g \alpha \qquad \rho_m u_m = \left(\rho_l u_l (1-\alpha) + \rho_g u_g \alpha\right) \quad (2)$$

- It may be noted that $\rho_m u_m = \frac{\dot{m}}{A} = G$ (2)

Momentum Equation-I

- The simplified momentum equation comes from the mixture momentum equation

$$\frac{\partial(\alpha\rho_g u_g + (1-\alpha)\rho_l u_l)}{\partial t} + \frac{\partial(\alpha\rho_g u_g^2 + (1-\alpha)\rho_l u_l^2)}{\partial s} = -\alpha\frac{\partial p_g}{\partial x}$$

$$-(1-\alpha)\frac{\partial p_l}{\partial x} - \frac{\tau_{wg} S_g}{A} - \frac{\tau_{wl} S_l}{A} - \left(\alpha\rho_g + (1-\alpha)\rho_l\right) g Sin\theta \quad (3)$$

- This can be written as

$$\frac{\partial \rho_m u_m}{\partial t} + \frac{\partial(\alpha\rho_g u_g^2 + (1-\alpha)\rho_l u_l^2)}{\partial s} = -\frac{\partial p}{\partial x} - \frac{\tau_w P}{A} - \rho_m g Sin\theta \quad (4)$$

- The term, $\alpha\rho_g u_g^2 = \frac{\alpha\rho_g \dot{m}_g^2}{(A\alpha\rho_g)^2} = \frac{\dot{m}^2}{A^2}\frac{x^2}{\alpha\rho_g}$ (5)

15:13 Momentum Equation-II 5/9

- Similarly, $(1-\alpha)\rho_l u_l^2 = \frac{\dot{m}^2}{A^2}\frac{(1-x)^2}{(1-\alpha)\rho_l}$ 6
- The second term can now be written as

$$\frac{\partial(\alpha\rho_g u_g^2 + (1-\alpha)\rho_l u_l^2)}{\partial s} = \frac{\partial}{\partial s}\left(\frac{\dot{m}^2}{A^2}\left(\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_l}\right)\right) = \frac{\partial}{\partial s}\left(\frac{\dot{m}^2}{A^2\rho'}\right)$$

7

- Where, $\left(\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_l}\right) = \frac{1}{\rho'}$ Momentum Density
- Thus momentum equation can be written as

$$\frac{\partial\rho_m u_m}{\partial t} + \frac{\partial}{\partial s}\left(\frac{(\rho_m u_m)^2}{\rho'}\right) = -\frac{\partial p}{\partial x} - \frac{\tau_w P}{A} - \rho_m g Sin\theta$$ 8

15:13 Total Energy Equation-I 6/9

- Assuming shear and pressure work is negligible in the system and neglecting shaft work, we can write

$$\frac{\partial\left(\rho_g A_g\left(e_g - p/\rho_g\right) + \rho_l A_l\left(e_l - p/\rho_l\right)\right)}{\partial t} + \frac{\partial(\dot{m}_g e_g + \dot{m}_l e_l)}{\partial s} = q'$$

$$\frac{\partial(\rho_g \alpha e_g + \rho_l(1-\alpha)e_l)}{\partial t} + \frac{\partial(\dot{m}_g e_g + \dot{m}_l e_l)}{\partial s} = A\frac{\partial p}{\partial t} + q'$$

$$\Rightarrow \frac{\partial(\rho_g \alpha e_g + \rho_l(1-\alpha)e_l)}{\partial t} + \frac{\partial\dot{m}(xe_g + (1-x)e_l)}{\partial s} = A\frac{\partial p}{\partial t} + q'$$

15:13 7/9

Total Energy Equation-II

- Defining $\rho_m e_m = (\rho_l e_l(1-\alpha) + \rho_g e_g \alpha)$ 9

$$\frac{\partial(\rho_m e_m)}{\partial t} + \frac{\partial\dot{m}(xe_g + (1-x)e_l)}{\partial s} = \frac{\partial p}{\partial t} + q'$$

$$\Rightarrow \frac{\partial(\rho_m e_m)}{\partial t} + \frac{\partial\dot{m}(xh_g + (1-x)h_l)}{\partial s} + \frac{\partial\dot{m}\left(x\frac{u_g^2}{2} + (1-x)\frac{u_l^2}{2} + H\right)}{\partial s} = A\frac{\partial p}{\partial t} + q'$$ 10

15:13 8/9

Summary

$$\frac{\partial\rho_m}{\partial t} + \frac{\partial\rho_m u_m}{\partial s} = 0$$

$$\frac{\partial\rho_m u_m}{\partial t} + \frac{\partial}{\partial s}\left(\frac{(\rho_m u_m)^2}{\rho'}\right) = -\frac{\partial p}{\partial x} - \frac{\tau_w P}{A} - \rho_m g Sin\theta$$

$$\Rightarrow \frac{\partial(\rho_m e_m)}{\partial t} + \frac{\partial\dot{m}(xh_g + (1-x)h_l)}{\partial s} + \frac{\partial\dot{m}\left(x\frac{u_g^2}{2} + (1-x)\frac{u_l^2}{2} + H\right)}{\partial s} = A\frac{\partial p}{\partial t} + q'$$

Variables $\dot{m}, x, \alpha, \rho_g, \rho_l, p, \tau_w, h_g, h_l, q'$ 10

Equations 3

Closing relations required 7

$\rho_g = \rho_g(p)$ 1

$\rho_l = \rho_l(p)$ 1

$h_g = h_g(p)$ 1

$h_l = h_l(p)$ 1

model for τ_w 1

model for α *or for slip* 1

q' *Modelled or externally specified* 1