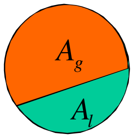
Closure for Wall Shear

- In single phase wall shear is closed by friction factor, which is modelled as a function of a non-dimensional number called Reynolds number
- Search for a single parameter such as Re has been on for a long time
- Lockhart and Martinelli were the first one to obtain success. They obtained this empirically
- Many justification for this parameter has been provided heuristically
- One such justification is by Wallis and is known as the Wallis separate cylinder model

Wallis Separate Cylinder Model-I

- Wallis treated the two phases to be flowing as two fictitious cylinders of areas A_g and A_l respectively



$$A_g = \pi r_g^2 \qquad A_l = \pi r_l^2 \qquad A = \pi R^2 \qquad (1)$$

$$\alpha = \frac{A_g}{A} = \frac{r_g^2}{R^2} \qquad 1-\alpha = \frac{A_l}{A} = \frac{r_l^2}{R^2}$$

- If we assume $\displaystyle \frac{dp_g}{dx} = \frac{dp_l}{dx} = \frac{dp_{2\phi}}{dx}$

Wallis Separate Cylinder Model-II

- We know that

$$\frac{dp_g}{dx} = \frac{4\dot{m}_g^2}{2\rho_g A_g^2}\frac{f_g}{d_g} = \frac{4\dot{m}_g^2}{2\rho_g \pi^2 r_g^4}\frac{f_g}{2r_g} = \frac{\dot{m}_g^2}{\rho_g \pi^2}\frac{f_g}{r_g^5}$$

- Expressing

$$f_g = C_g\, Re_g^{-n} = C_g\left(\frac{\dot{m}_g}{\rho_g A_g}\frac{\rho_g d_g}{\mu_g}\right)^{-n} = C_g\left(\frac{2}{\pi}\frac{\dot{m}_g}{\mu_g r_g}\right)^{-n}$$

- Therefore

$$\frac{dp_g}{dx} = C_g\left(\frac{2}{\pi}\frac{\dot{m}_g}{\mu_g r_g}\right)^{-n}\left(\frac{\dot{m}_g^2}{\rho_g r_g^5 \pi^2}\right) = \frac{dp_{2\phi}}{dx}$$

Wallis Separate Cylinder Model-III

- Similarly

$$\left.\frac{dp}{dx}\right|_{gs} = C_l\left(\frac{2}{\pi}\frac{\dot{m}_g}{\mu_g R}\right)^{-n}\left(\frac{\dot{m}_g^2}{\rho_g R^5 \pi^2}\right)$$

- Defining

$$\phi_g^2 = \frac{\left.\frac{dp}{dx}\right|_{2\phi}}{\left.\frac{dp}{dx}\right|_{gs}} = \left(\frac{R}{r_g}\right)^{5-n} \qquad (2)$$

- Since

$$\frac{r_g}{R} = \alpha^{0.5} \qquad \Rightarrow \phi_g^2 = \left(\frac{1}{\alpha}\right)^{\frac{5-n}{2}} \qquad (3)$$

Wallis Separate Cylinder Model-IV

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- Similarly Defining

$$\phi_l^2 = \frac{\left.\frac{dp}{dx}\right|_{2\phi}}{\left.\frac{dp}{dx}\right|_{ls}} \Rightarrow \phi_l^2 = \left(\frac{1}{1-\alpha}\right)^{\frac{5-n}{2}} \quad (4)$$

- Eqs. (3) and (4) imply

$$\frac{1}{\alpha} = (\phi_g)^{\frac{4}{5-n}} \quad (5) \qquad \frac{1}{1-\alpha} = (\phi_l)^{\frac{4}{5-n}} \quad (6)$$

- Eqs. (5) and (6) imply

$$\frac{1}{(\phi_g)^{\frac{4}{5-n}}} + \frac{1}{(\phi_l)^{\frac{4}{5-n}}} = 1 \quad (7)$$

Wallis Separate Cylinder Model-V

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- Defining

$$\chi^2 = \frac{\left.\frac{dp}{dx}\right|_{ls}}{\left.\frac{dp}{dx}\right|_{gs}} = \frac{\left.\frac{dp}{dx}\right|_{ls} \Big/ \left.\frac{dp}{dx}\right|_{2\phi}}{\left.\frac{dp}{dx}\right|_{gs} \Big/ \left.\frac{dp}{dx}\right|_{2\phi}} = \frac{\phi_g^2}{\phi_l^2}$$

- This implies

$$\chi\,\phi_l = \phi_g \quad (8)$$

- Eqs. (7) and (8) imply

$$\frac{1}{(\chi\phi_l)^{\frac{4}{5-n}}} + \frac{1}{(\phi_l)^{\frac{4}{5-n}}} = 1 \quad \Rightarrow \frac{1}{(\phi_l)^{\frac{4}{5-n}}}\left(\frac{1}{(\chi)^{\frac{4}{5-n}}} + 1\right) = 1$$

Wallis Separate Cylinder Model-VI

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$$\Rightarrow \phi_l = \left(\frac{1}{(\chi)^{\frac{4}{5-n}}} + 1\right)^{\frac{5-n}{4}}$$

$$\Rightarrow \phi_l^2 = \left(\frac{1}{(\chi)^{\frac{4}{5-n}}} + 1\right)^{\frac{5-n}{2}} = \left(\frac{1}{1-\alpha}\right)^{\frac{5-n}{2}}$$

Wallis Separate Cylinder Model-VII

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- For n = 1

$$\Rightarrow \phi_l^2 = \left(\frac{1}{\chi} + 1\right)^2 = \left(\frac{1}{1-\alpha}\right)^2$$

- For n = 0.25

$$\Rightarrow \phi_l^2 = \left(\left(\frac{1}{\chi}\right)^{\frac{16}{19}} + 1\right)^{\frac{19}{8}} = \left(\frac{1}{1-\alpha}\right)^{\frac{19}{8}}$$

- The relations above imply

$$\phi_l^2 = f(\chi),\ \alpha = f(\chi)$$

Lockhart-Martinelli Curves-I

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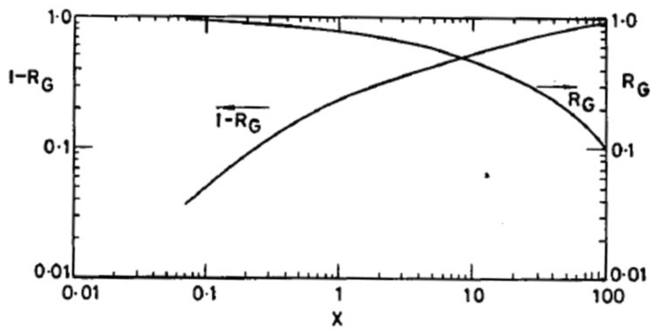
- Lockhart and Martinelli (1949) correlated experimental data for ϕ_l^2 and α with χ.



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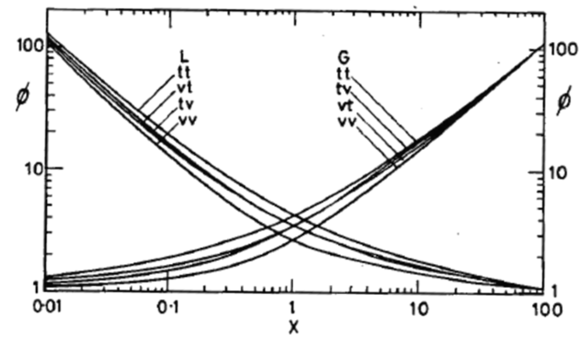

- Chisholm and Laird (1958) fitted the L-M curves as

$$\phi_l^2 = 1 + \frac{C}{\chi} + \frac{1}{\chi^2} = \frac{1}{(1-\alpha)^2}$$

Liq-Gas	C
Tur-Tur	20
Lam-Tur	12
Tur-Lam	10
Lam-Lam	5

Martinelli-Nelson Model-I

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- The above discussions were for air-water flow
- The same was extended to vapourising flows by Martinelli and Nelson (1948)
- Using basic definitions it can be shown that

$$\chi^2 = \left(\frac{1-x}{x}\right)^{2-n} \left(\frac{\mu_l}{\mu_g}\right)^n \frac{\rho_g}{\rho_l}$$

- For n = 0.25

$$\chi = \left(\frac{1-x}{x}\right)^{0.875} \left(\frac{\mu_l}{\mu_g}\right)^{0.125} \left(\frac{\rho_g}{\rho_l}\right)^{0.5}$$

Martinelli-Nelson Model-II

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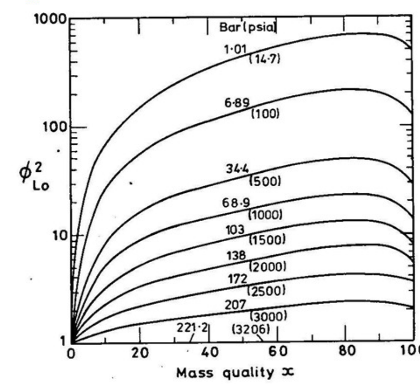
- It can also be shown that for n = 0.25

$$\phi_{lo}^{\ 2} = \phi_l^{\ 2}(1-x)^{1.75}$$

- The variation of two-phase multiplier is shown



Martinelli-Nelson Model-IV

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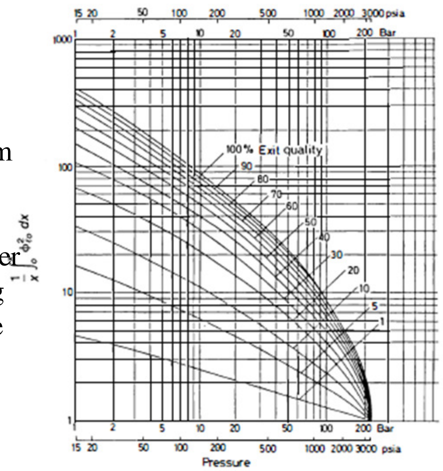
- Using the Chisholm and Laird’s fit, the value of average two-phase multiplier for quality varying from 0 to x_e can be obtained



Baroczy-Chisholm Model-I

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- The problem with Martinelli and Nelson curves was that it could not account for the influence of mass velocity
- It was Baroczy in 1965 came up with a complex set of curves and the same was modelled by Chisholm and fitted into a very useful correlation and is claimed to be more accurate than Baroczy curves

$$\phi_{lo}^{2} = 1 + (\Gamma^{2} - 1)\left[Bx^{0.875}(1-x)^{0.875} + x^{1.75}\right]$$

- Where $$\Gamma^{2} = \frac{v_g}{v_l}\left(\frac{\mu_g}{\mu_l}\right)^{0.2}$$

The value of B is given in next slide

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Baroczy-Chisholm Model-II

Γ	**G (kg/m²-s)**	**B**
≤ 9.5	≤ 500	4.8
	$500 \leq G < 1900$	2400/G
	≥ 1900	$55/G^{0.5}$
$9.5 \leq \Gamma < 28$	≤ 600	$520/(\Gamma G^{0.5})$
	> 600	$21/\Gamma$
≥ 28		$15000/(\Gamma^2 G^{0.5})$