EN634 Nuclear Reactor Thermal Hydraulics

Drift Flux Model and Associated Concepts

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Drift Flux Model

- We had seen the development of slip flow model
- It involved inclusion of a closure for slip and newer definition and correlation for two-phase multiplier.
- Another way of writing the same equations in a different form is called the Drift Flux Model
- This model is cast in a form that corrects homogeneous model.
- A closure is found for α rather than s
- We shall look at these.

Recollection of Definitions

- Drift Velocity

$$u_d = u_{gj} = u_g - j$$

- Drift Flux

Volume flux of gas relative a surface moving with a velocity j

$$j_{gl} = \alpha u_d = \alpha(u_g - j)$$

$$= \alpha u_g - \alpha j = j_g - \alpha j$$

$$\Rightarrow j_g = \alpha j + j_{gl} \qquad (1)$$

The above is interpreted as, the volume flux of gas = conc. of gas x average vol. flux + relative velocity correction

Drift Flux Concepts-I

Zuber compared this with multi-component diffusion and interpreted that flux of gas has convective and diffusive components and suggested that the drift component is similar to the diffusive flux.

Wallis has an interesting suggestion that for all variables slip flow can be viewed as the homogeneous component plus a correction

$$\Rightarrow \alpha = \frac{j_g}{j} - \frac{j_{gl}}{j}$$

$$\Rightarrow \rho_m = \frac{\rho_g j_g + \rho_l j_l}{j} + (\rho_l - \rho_g)\frac{j_{gl}}{j} \qquad (2)$$

Closure for α-I

Before we proceed to drift flux governing equations, let us see some developments on the fundamental plane and experimental evidences

- Volume fraction

$$\beta = \frac{Q_g}{Q_g + Q_l}; \quad = \frac{j_g}{j_g + j_l}$$

$$= \frac{A_g u_g}{A_g u_g + A_l u_l} \quad = \frac{\alpha s}{\alpha s + (1-\alpha)} \qquad (3)$$

Note that β = α when s = 1

Closure for α-II

Armand in 1946 fitted from experimental data

$$\frac{\alpha}{\beta} = 0.833 = C_A \qquad \text{For } \beta < 0.9 \qquad (4)$$

Massena in 1960 extended this fit for $\beta > 0.9$ as

$$\frac{\alpha}{\beta} = [C_A + (1 - C_A)x]\beta$$

While few other correlations for β/α exist as summarised in Todreas and Kazimi, Zuber and Findlay approached this issue from a fundamental angle and we shall concentrate in this direction

Closure for α-III

From basic definitions,

$$\Rightarrow j_g = \frac{\dot{m} x v_g}{A} = G x v_g \qquad (5)$$

$$j = \frac{Q_g + Q_l}{A} = \frac{\dot{m}}{A}\left(x v_g + (1-x)v_l\right) = G\left(x v_g + (1-x)v_l\right)$$

Thus,

$$\frac{j_g}{j} = \beta = \frac{x v_g}{\left(x v_g + (1-x)v_l\right)} \qquad (6)$$

$$\frac{j_g}{\alpha j} = \frac{\beta}{\alpha} = \frac{x v_g}{\alpha\left(x v_g + (1-x)v_l\right)} = \frac{u_g}{j} \qquad (7)$$

Closure for α-IV

Thus,

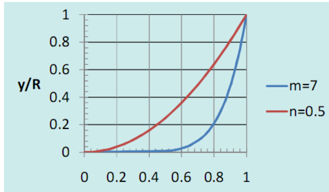
$$\frac{\beta}{\alpha} = \frac{u_g}{j} = 1 + \frac{u_d}{j} \qquad \because u_d = u_g - j \qquad (8)$$

- ❑ The above implies that *β/α* should tend to 1 as *j* increases, which is contrary to experiments which indicates it to be $1/C_A$
- ❑ The possible reason for this was provided by Bankoff (1960) using his variable density model
- ❑ He postulated varying profiles for u and *α* and explained the same. Let us take a brief look at that

Bankoff's Model-I

Bankoff assumed the profiles for velocity and void as,

$$\frac{u}{u_{CL}} = \left(\frac{y}{R}\right)^{\frac{1}{m}} \qquad \frac{\alpha}{\alpha_{CL}} = \left(\frac{y}{R}\right)^{\frac{1}{n}}$$



$$Q_g = \int_0^R 2\pi r dr \alpha u = \int_0^R 2\pi r \alpha_{CL} u_{CL} \left(\frac{y}{R}\right)^{\frac{1}{m}} \left(\frac{y}{R}\right)^{\frac{1}{n}} dr$$

$$= \int_0^R 2\pi (R - y) \alpha_{CL} u_{CL} \left(\frac{y}{R}\right)^{\frac{1}{m}} \left(\frac{y}{R}\right)^{\frac{1}{n}} dy$$

Integration by parts leads to $Q_g = \dfrac{2\pi R^2 \alpha_{CL} u_{CL} m^2 n^2}{(m+n+mn)(m+n+2mn)}$ (9)

Bankoff's Model-II

Similarly

$$Q = \int_0^R 2\pi r dr u = \int_0^R 2\pi r u_{CL} \left(\frac{y}{R}\right)^{\frac{1}{m}} dr$$

$$= \int_0^R 2\pi (R - y) u_{CL} \left(\frac{y}{R}\right)^{\frac{1}{m}} dy$$

Integration by parts leads to $Q = \dfrac{2\pi R^2 u_{CL} m^2}{(1+m)(1+2m)}$ (10)

From Eqs. (9) and (10), we get

$$\beta = \frac{\alpha_{CL}(1+m)(1+2m)n^2}{(m+n+mn)(m+n+2n)} \qquad (11)$$

Bankoff's Model-III

$$\overline{\alpha} = \frac{1}{\pi R^2} \int_0^R 2\pi r dr \alpha = \int_0^R 2\pi r \alpha_{CL} \left(\frac{y}{R}\right)^{\frac{1}{n}} dr$$

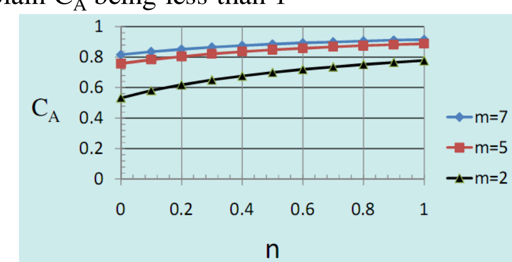
$$\overline{\alpha} = \frac{2\alpha_{CL} n^2}{(1+n)(1+2n)} \qquad (12)$$

From Eqs. (11) and (12), we get

$$\frac{\beta}{\alpha} = \frac{1}{C_A} = \frac{2(m+n+mn)(m+n+2mn)}{(1+m)(1+2m)(1+n)(1+2n)} \qquad (13)$$

Bankoff's Model-IV

- The value of C_A as a function of n for various m are given.
- It may be observed for turbulent profiles, C_A hovers between 0.8 to 0.9
- Thus variable velocity and density profile is able to explain C_A being less than 1



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Local Time Averaged Quantities-I

- From previous discussions we realized that homogeneous model overpredicts the void fraction
- This has been due to the neglect of void distribution
- To define the local phase velocities, there is a need to appreciate the time averaged local parameters

$$\tilde{\alpha} = \frac{1}{T}\int_0^T dt_g = \frac{T_g}{T} \qquad \tilde{u}_g = \frac{1}{T_g}\int_0^T u_g dt_g$$

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Local Time Averaged Quantities-II

$$\tilde{j}_g = \frac{1}{T}\int_0^T u_g dt_g = \frac{1}{T}\frac{T_g}{T_g}\int_0^T u_g dt_g = \tilde{u}_g\tilde{\alpha}$$

Similarly

$$\tilde{u}_l = \frac{1}{T_l}\int_0^T u_l dt_l \qquad \tilde{j}_l = \frac{1}{T}\int_0^T u_l dt_l = \frac{1}{T}\frac{T_l}{T_l}\int_0^T u_l dt_l = \tilde{u}_l(1-\tilde{\alpha})$$

By definition

$$\tilde{j} = \tilde{j}_l + \tilde{j}_g \qquad \tilde{u}_{gj} = \tilde{u}_g - \tilde{j} = \tilde{u}_d$$

All of the time averaged local quantities defined above can be measured.

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Zuber-Findlay Model-I

- Zuber-Findlay (xxxx) introduced the void weighted averages
- The motivation comes from the following

$$Q_g = \int_A \tilde{u}_g dA_g \quad = \int_A \tilde{u}_g\tilde{\alpha} dA \quad \Rightarrow \frac{Q_g}{A_g} = \frac{\int_A \tilde{u}_g\tilde{\alpha} dA}{\int_A \tilde{\alpha} dA}$$

- Thus the phase averaged velocity u_g is the void weighted average. This is denoted by

$$\Rightarrow \hat{u}_g = \frac{\int_A \tilde{u}_g\tilde{\alpha} dA}{\int_A \tilde{\alpha} dA}$$

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Zuber-Findlay Model-II

- Void weighted average for any parameter F can be written as

$$\Rightarrow \hat{F} = \frac{\frac{1}{A}\int_A \tilde{F}\tilde{\alpha} dA}{\frac{1}{A}\int_A \tilde{\alpha} dA} = \frac{\langle \tilde{F}\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle}$$

- Thus,

$$\Rightarrow \frac{j_g}{\alpha} = \hat{u}_g = \frac{\langle \tilde{u}_g\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle} = \frac{\langle(\tilde{j} + \tilde{u}_{gj})\,\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle} = \frac{\langle \tilde{j}\,\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle} + \frac{\langle \tilde{u}_{gj}\,\tilde{\alpha}\rangle}{\langle\tilde{\alpha}\rangle}$$

Zuber-Findlay Model-III

- Thus,

$$\Rightarrow \frac{j_g}{\alpha} = \frac{C_0 \langle \tilde{j} \rangle \langle \tilde{\alpha} \rangle}{\langle \tilde{\alpha} \rangle} + \frac{\langle \tilde{u}_{gj}\ \tilde{\alpha} \rangle}{\langle \tilde{\alpha} \rangle} = C_0 \langle \tilde{j} \rangle + \hat{u}_{gj}$$

$$\Rightarrow \frac{j_g}{\langle \tilde{j} \rangle \alpha} = \frac{\beta}{\alpha} = C_0 + \frac{\hat{u}_{gj}}{j}$$

where $C_0 = \frac{\langle \tilde{j}\, \tilde{\alpha} \rangle}{\langle \tilde{j} \rangle \langle \tilde{\alpha} \rangle}$

- Depending on different flow regimes, the value of C_0 and weighted drift velocity are empirically closed