# EN634 Nuclear Reactor Thermal Hydraulics

# Stratified Flow Modelling

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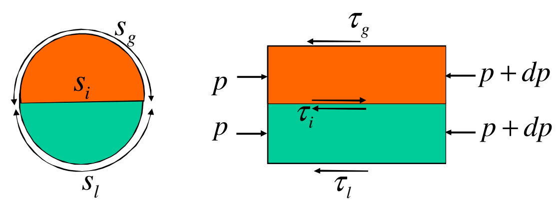
## Motivation for this lecture

- The stratified flow is a special case where mixture correlations such as Lockhart-Martinelli fail severely
- This model is also the fundamental basis for the Taitel-Dukler model that has been most rational model to delineate flow patterns for gas-liquid flow
- Stratified flow is the one flow pattern that most want to avoid, if the system is heated by chemical/nuclear means as it leads to overheating and structural failure
- We shall restrict it to fully developed flow in horizontal ducts, in which the interface will be horizontal



## Momentum Balance




- In steady state, $u_g$ and $u_l$ will be independent of position, hence momentum leads to

**For Liquid Phase**

$$A_l(-dp) - \tau_l S_l \Delta x + \tau_i S_i \Delta x = 0$$

$$\frac{dp}{dx} = \frac{-\tau_l S_l}{A_l} + \frac{\tau_i S_i}{A_l} \quad (1)$$

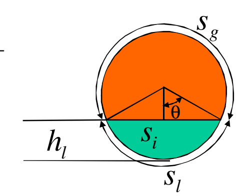
**For Gas Phase**

$$A_g(-dp) - \tau_g S_g \Delta x - \tau_i S_i \Delta x = 0$$

$$\frac{dp}{dx} = \frac{-\tau_g S_g}{A_g} - \frac{\tau_i S_i}{A_g} \quad (2)$$



## Geometrical Relations




$$Cos\theta = \frac{R-h_l}{R} = \frac{D-2h_l}{D} = 1 - \frac{2h_l}{D}$$

$$\Rightarrow \theta = Cos^{-1}\left(1 - \frac{2h_l}{D}\right)$$

$$S_l = 2R\theta = D\theta \quad S_g = \pi D - D\theta \quad S_i = 2RSin\theta = DSin\theta$$

$$A_l = R^2\theta - \frac{1}{2}2RCos\theta \; RSin\theta = \frac{D^2}{4}(\theta - Sin\theta Cos\theta)$$

$$A_g = \frac{\pi D^2}{4} - A_l = \frac{D^2}{4}(\pi - (\theta - Sin\theta Cos\theta))$$

## Non-Dimensional Parameters

- If we non-dimensionalise the variables as follows

$$\tilde{A}_l = \frac{A_l}{D^2},\ \tilde{A}_g = \frac{A_g}{D^2},\ \tilde{S}_l = \frac{S_l}{D},\ \tilde{S}_g = \frac{S_g}{D},\ \tilde{S}_i = \frac{S_i}{D},\ \tilde{h}_l = \frac{h_l}{D}$$

- From the dimensional relations in previous slide, we can write,

$$\tilde{S}_l = \theta,\ \tilde{S}_g = \pi - \theta,\ \tilde{S}_i = Sin\theta \qquad (3)$$

$$\tilde{A}_l = \frac{1}{4}(\theta - Sin\theta Cos\theta),\ \tilde{A}_g = \frac{1}{4}(\pi - \theta + Sin\theta Cos\theta), \qquad (4)$$

$$\theta = Cos^{-1}\left(1 - 2\tilde{h}_l\right) \qquad (5)$$

All non-dimensional geometric parameters are functions of $\tilde{h}_l$



## Closure Relations-I

- We have two momentum equations
- The geometric parameters $S_l$, $S_g$, $S_i$, $A_l$, $A_g$, are unique functions of $h_l$ and D
- Thus, in Eqs,. (1) and (2), we have 5 independent unknown parameters, viz., p, $\tau_l$, $\tau_g$, $\tau_i$, and $h_l$
- To close the set, we need to define, the three shear stresses
- Taitel and Dukler closed these by defining appropriate friction factors



## Closure Relations-II

- They defined.

$$\tau_l = \frac{1}{2}\rho_l u_l^2 f_l,\ \tau_g = \frac{1}{2}\rho_g u_g^2 f_g,\ \tau_i = \frac{1}{2}\rho_g u_g^2 f_i, \qquad (6)$$

Where, $f_l = C_l\, Re_l^{-n},\ f_g = C_g\, Re_g^{-n},\ f_i = f_g, \qquad (7)$

$$C_l = C_g = 16/0.046,\ n = 1/0.2$$

$$\mathrm{Re}_l = \frac{\rho_l u_l d_l}{\mu_l},\ \mathrm{Re}_g = \frac{\rho_g u_g d_g}{\mu_g} \qquad (8)$$

$$d_l = \frac{4A_l}{S_l},\ d_g = \frac{4A_g}{S_g + S_i} \qquad (9)$$



## Solution Methodology-I

- To generalise, they introduced,

$$\mathrm{Re}_{ls} = \frac{\rho_l u_{ls} D}{\mu_l},\ \mathrm{Re}_{gs} = \frac{\rho_g u_{gs} D}{\mu_g} \qquad (10)$$

- From Eqs. (8) and (10) we can write,

$$\frac{\mathrm{Re}_l}{\mathrm{Re}_{ls}} = \tilde{u}_l \tilde{d}_l,\ \frac{\mathrm{Re}_g}{\mathrm{Re}_{gs}} = \tilde{u}_g \tilde{d}_g \qquad (11)$$

Where,.

$$\tilde{u}_l = \frac{u_l}{u_{ls}} = \frac{A}{A_l} = \frac{\pi}{4\tilde{A}_l},\ \tilde{u}_g = \frac{u_g}{u_{gs}} = \frac{A}{A_g} = \frac{\pi}{4\tilde{A}_g} \qquad (12)$$

## Solution Methodology-II

Similar to Eq. (7), we can write

$$f_{ls} = C_{ls}\, Re_{ls}^{-n}, \quad f_{gs} = C_{gs}\, Re_{gs}^{-n} \qquad (13)$$

Assuming $C_{ls} = C_l$, $C_{gs} = C_g$ , we can write

$$\frac{f_l}{f_{ls}} = \left(\frac{\mathrm{Re}_l}{\mathrm{Re}_{ls}}\right)^{-n} = \left(\tilde{u}_l \tilde{d}_l\right)^{-n} \qquad \frac{f_g}{f_{gs}} = \left(\frac{\mathrm{Re}_g}{\mathrm{Re}_{gs}}\right)^{-n} = \left(\tilde{u}_g \tilde{d}_g\right)^{-n} \qquad (14)$$

- We know that $\left.\dfrac{dp}{dx}\right|_{ls} = \dfrac{4\rho_l u_{ls}^2}{2D} f_{ls}$, $\left.\dfrac{dp}{dx}\right|_{gs} = \dfrac{4\rho_g u_{gs}^2}{2D} f_{gs}$

$$\Rightarrow \chi^2 = \left.\frac{dp}{dx}\right|_{ls} \bigg/ \left.\frac{dp}{dx}\right|_{gs} = \frac{\rho_l u_{ls}^2}{\rho_g u_{gs}^2}\frac{f_{ls}}{f_{gs}} \qquad (15)$$

- Now having closed the set by defining shear stresses, we will now attempt to solve for $h_l$ and dp/dx



## Solution Methodology-III

- Equating the right hand sides of Eqs. (1) and (2), and rearrangement leads to

$$\frac{-\tau_l S_l}{A_l} + \frac{\tau_i S_i}{A_l} = \frac{-\tau_g S_g}{A_g} - \frac{\tau_i S_i}{A_g} \qquad \Rightarrow \frac{\tau_g S_g}{A_g} \frac{-\tau_l S_l}{A_l} + \tau_i S_i\left(\frac{1}{A_l} + \frac{1}{A_g}\right) = 0$$

$$\Rightarrow \frac{0.5\rho_g u_g^2 f_g S_g}{A_g} \frac{-0.5\rho_l u_l^2 f_l S_l}{A_l} + 0.5\rho_g u_g^2 f_i S_i\left(\frac{1}{A_l} + \frac{1}{A_g}\right) = 0$$

$$\Rightarrow \frac{0.5\rho_g \tilde{u}_g^2 u_{gs}^2 f_{gs}\left(\tilde{u}_g \tilde{d}_g\right)^{-n} \tilde{S}_g D}{\tilde{A}_g D^2} \frac{-0.5\rho_l \tilde{u}_l^2 u_{ls}^2 f_{ls}\left(\tilde{u}_l \tilde{d}_l\right)^{-n} \tilde{S}_l D}{\tilde{A}_l D^2}$$

$$+0.5\rho_g \tilde{u}_g^2 u_{gs}^2 f_{gs}\left(\tilde{u}_g \tilde{d}_g\right)^{-n} \tilde{S}_i D \frac{1}{D^2}\left(\frac{1}{\tilde{A}_l} + \frac{1}{\tilde{A}_g}\right) = 0$$



## Solution Methodology-IV

- Cancelling D/D$^2$ and Dividing by $0.5u_{gs}^2 f_{gs}$

$$\frac{\tilde{u}_g^2\left(\tilde{u}_g \tilde{d}_g\right)^{-n} \tilde{S}_g}{\tilde{A}_g} \frac{-\rho_l u_{ls}^2 \tilde{u}_l^2 f_{ls}\left(\tilde{u}_l \tilde{d}_l\right)^{-n} \tilde{S}_l}{\rho_g u_{gs}^2 f_{gs} \tilde{A}_l} + \tilde{u}_g^2\left(\tilde{u}_g \tilde{d}_g\right)^{-n} \tilde{S}_i\left(\frac{1}{\tilde{A}_l} + \frac{1}{\tilde{A}_g}\right) = 0$$

- Employing Eq. (15), we get,

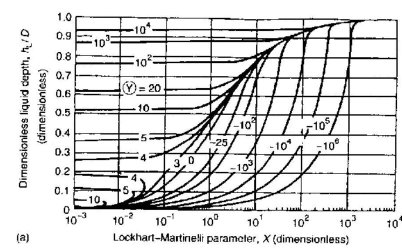
$$\frac{\tilde{u}_g^2\left(\tilde{u}_g \tilde{d}_g\right)^{-n} \tilde{S}_g}{\tilde{A}_g} \frac{-\chi^2 \tilde{u}_l^2\left(\tilde{u}_l \tilde{d}_l\right)^{-n} \tilde{S}_l}{\tilde{A}_l} + \tilde{u}_g^2\left(\tilde{u}_g \tilde{d}_g\right)^{-n} \tilde{S}_i\left(\frac{1}{\tilde{A}_l} + \frac{1}{\tilde{A}_g}\right) = 0$$

$$\Rightarrow \chi^2 = \frac{\tilde{u}_g^2\left(\tilde{u}_g \tilde{d}_g\right)^{-n}\left(\dfrac{\tilde{S}_i}{\tilde{A}_l} + \dfrac{\tilde{S}_i}{\tilde{A}_g} + \dfrac{\tilde{S}_g}{\tilde{A}_g}\right)}{\dfrac{\tilde{u}_l^2\left(\tilde{u}_l \tilde{d}_l\right)^{-n} \tilde{S}_l}{\tilde{A}_l}} \qquad (16)$$



## Solution Methodology-V

- The above equation indicates that $\tilde{h}_l$ is a unique function of $\chi^2$, which is similar to previous observation that α is a function of $\chi^2$
- From Eq. (16), we can generate the non-dimensional variation of $\tilde{h}_l$ for given $\chi$ numerically as a root finding problem



## Solution Methodology-VI

- It is fairly straight forward to show that $\phi_l^2$ or $\phi_g^2$ is a unique function of $\chi^2$. The two phase multiplier are defined as follows

$$\phi_l^2 = \frac{dp}{dx} \Bigg/ \frac{dp}{dx}\bigg|_{ls} \qquad \phi_g^2 = \frac{dp}{dx} \Bigg/ \frac{dp}{dx}\bigg|_{gs}$$

- The solution for $\phi_l^2$ or $\phi_g^2$ can similarly be obtained