EN634 Nuclear Reactor Thermal Hydraulics
Flow Patterns

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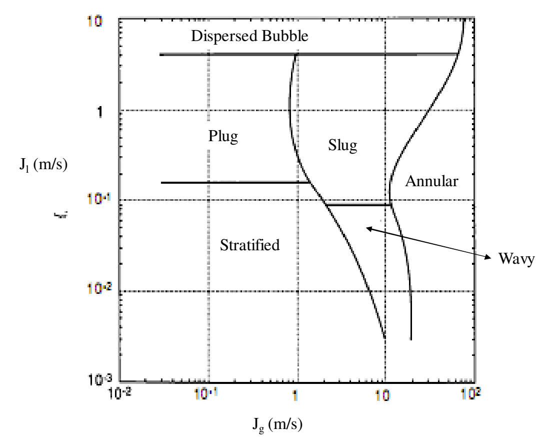
Motivation for this lecture

- We had understood that during two-phase flow, several patterns can exist
- We also understood that it may not be possible to have correlations for pressure drop that will be valid for all flow patterns
- However, owing to similarity of flow structure, the correlations may be generalised for the same flow pattern
- We shall look at how flow pattern transitions can be predicted
- Hence there is incentive to evolve methods for predicting flow patterns

Horizontal Pipe Flow-Patterns



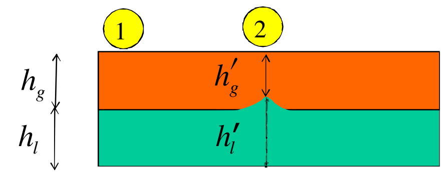
Taitel-Dukler's Model

- The fundamental pattern is assumed to be stratified flow
- Existence of other flow patterns are considered to arise out of instability of stratified flow
- Hence it is necessary to appreciate the fundamental instability called the Kelvin-Helmholtz instability
- There are several approaches to understand this.
- We shall choose the simplest route followed by Taitel and Dukler

Simplified Treatment of K-H Instability

- The assumptions made are:
  - One dimensional analysis adequate
  - Inviscid flow
  - Incompressible flow
  - Bernoulli's equation is applicable during the initial stage of this instability (quasi-steady)

Momentum Balance



- In this treatment we consider liquid to be stationary
- Applying Bernoulli's equation on the gas side

$$p_g - p_g' = \left(\frac{\rho_g u_g'^2}{2} - \frac{\rho_g u_g^2}{2}\right) + \rho_g g\left(h_l' - h_l\right) \quad (1)$$

Non-Dimensional Parameters

- For the stationary liquid, we can write

$$p_l' = p_l - \rho_l g\left(h_l' - h_l\right) \quad (2)$$

- For the wave to grow

$$p_l' > p_g' \quad (3)$$

$$\Rightarrow \cancel{p_l} - \rho_l g\left(h_l' - h_l\right) > \cancel{p_g} - \rho_g\left(\frac{u_g'^2}{2} - \frac{u_g^2}{2}\right) - \rho_g g\left(h_l' - h_l\right)$$

$$\Rightarrow g\left(h_l' - h_l\right)\left(\rho_g - \rho_l\right) > -\rho_g\left(\frac{u_g'^2}{2} - \frac{u_g^2}{2}\right)$$

$$\Rightarrow (\rho_l - \rho_g)\, g\left(h_g - h_g'\right) < \rho_g\left(\frac{u_g'^2}{2} - \frac{u_g^2}{2}\right)$$

Non-Dimensional Parameters

- Since from continuity $u_g' A_g' = u_g A_g$

$$\Rightarrow (\rho_l - \rho_g)\, g\left(h_g - h_g'\right) < \left(\frac{\rho_g u_g^2}{2}\left(\left(A_g / A_g'\right)^2 - 1\right)\right) \quad (4)$$

- Since $A_g' = A_g + \frac{dA_g}{dh_l}\left(h_l' - h_l\right)$, we can write

$$\frac{A_g' - A_g}{\frac{dA_g}{dh_l}} = \left(h_l' - h_l\right) = \left(h_g - h_g'\right) \quad (5)$$

- From Eqs. (4) and (5) we can write

$$\Rightarrow (\rho_l - \rho_g)\, g\,\frac{\left(A_g - A_g'\right)}{\frac{dA_g}{dh_l}} < \left(\frac{\rho_g u_g^2}{2}\left(\left(A_g / A_g'\right)^2 - 1\right)\right)$$

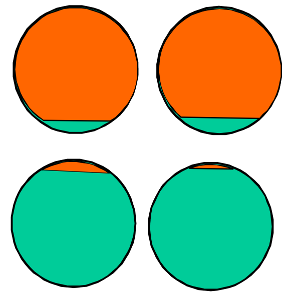
$$\Rightarrow 2\frac{(\rho_l - \rho_g)g}{\rho_g}\frac{\left(A_g - A_g'\right)}{\left(\left(A_g/A_g'\right)^2 - 1\right)\frac{dA_g}{dh_l}} < u_g^{\,2}$$

$$\Rightarrow 2\frac{\Delta\rho g}{\rho_g}\frac{A_g'\left(A_g/A_g' - 1\right)}{\left(\left(A_g/A_g'\right)^2 - 1\right)\frac{dA_g}{dh_l}} < u_g^{\,2} \qquad \Rightarrow 2\frac{\Delta\rho g}{\rho_g}\frac{A_g'}{\left(\left(A_g/A_g'\right) + 1\right)\frac{dA_g}{dh_l}} < u_g^{\,2}$$

$$\Rightarrow 2\frac{\Delta\rho g}{\rho_g}\frac{A_g'^{\,2}}{\left(A_g + A_g'\right)\frac{dA_g}{dh_l}} < u_g^{\,2} \qquad \Rightarrow 2\frac{\Delta\rho g}{\rho_g}\frac{A_g'^{\,2}}{A_g^{\,2}}\frac{A_g^{\,2}}{\left(A_g + A_g'\right)\frac{dA_g}{dh_l}} < u_g^{\,2}$$

$$\Rightarrow 2\frac{\Delta\rho g A_g}{\rho_g}\frac{A_g'^{\,2}}{A_g^{\,2}}\frac{A_g}{\left(A_g + A_g'\right)\frac{dA_g}{dh_l}} < u_g^{\,2} \qquad \Rightarrow 2\frac{\Delta\rho g A_g}{\rho_g}\frac{A_g'^{\,2}}{A_g^{\,2}}\frac{1}{2\frac{dA_g}{dh_l}} < u_g^{\,2}$$

- Taitel modelled A_g'/A_g as $1-(h_l/d)$.



- As $A_g'/A_g \rightarrow 1$, $1-(h_l/d) \rightarrow 1$,
- As $A_g'/A_g \rightarrow 0$, $1-(h_l/d) \rightarrow 0$,
- Thus for the wave to grow, $u_g^{\,2} > \frac{\Delta\rho g A_g}{\rho_g} C^2 \frac{1}{S_i}$

  where $\frac{dA_l}{dh_l} = S_i$ and $C = 1 - \frac{h_l}{d}$

Stratified-Non-stratified Transition

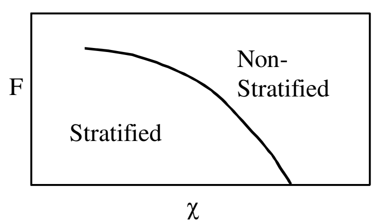
- Non-dimensionalising as done earlier with d and d^2, and u_{gs} as, length, area and velocity scales, we get

$$\tilde{u}_g^{\,2} u_{gs}^{\,2} > \frac{\Delta\rho g \tilde{A}_g D^2}{\rho_g} C^2 \frac{1}{\tilde{S}_i D} \qquad \Rightarrow \frac{\tilde{u}_g^{\,2}\tilde{S}_i}{\tilde{A}_g C^2}\frac{u_{gs}^{\,2}}{gD\frac{\Delta\rho}{\rho_g}} > 1$$

$$\Rightarrow \frac{\tilde{u}_g^{\,2}\tilde{S}_i}{\tilde{A}_g C^2} F^2 > 1$$

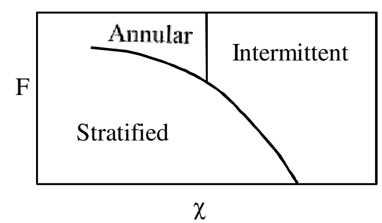
where $F^2 = \frac{u_{gs}^{\,2}}{gD\frac{\Delta\rho}{\rho_g}}$

For K-H instability to occur $F^2 > \frac{\tilde{A}_g C^2}{\tilde{u}_g^{\,2}\tilde{S}_i}$



Intermittent-Annular Transition

- If the parameter of flow are such that the stratified flow is unstable, then either annular flow or intermittent flow sets in.
- Taitel and Dukler postulated that if $\tilde{h}_l > 0.5$, then instability shall trigger intermittent flow
- Otherwise the would be annular



Slug-Dispersed Bubble Transition-I

- If the liquid shear is high, it will shear larger bubbles into small ones making the flow dispersed
- However, if the buoyancy is high then it will favour sustenance of large bubbles.
- Taitel and Dukler postulated that if buoyancy force per unit length is greater than shear force per unit length, then intermittent flow will exist. Otherwise the flow shall be dispersed bubble
- Buoyancy force per unit length $= g\Delta\rho A_g$
- Mean turbulence shear force per unit length $= \frac{\tau_w}{2} S_i$

Slug-Dispersed Bubble Transition-II

- The shear stress can be expressed as.

$$\tau_w = \frac{1}{2}\rho_l u_l^2 f_l$$

- For dispersed bubbly flow to sustain

$$\frac{1}{2}\rho_l u_l^2 f_l \frac{S_i}{2} > g\Delta\rho A_g$$

- Non-dimensionalising as done before, we get

$$\frac{1}{2}\rho_l \tilde{u}_l^2 u_{ls}^2 f_l \frac{\tilde{S}_i d}{2} > g\Delta\rho\tilde{A}_g d^2$$

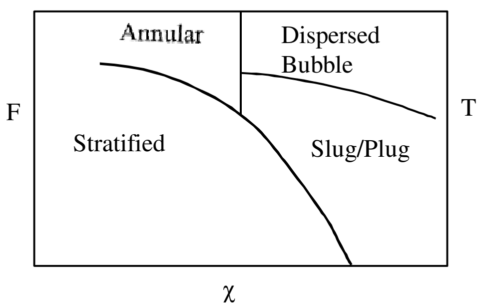
$$\frac{1}{2}\rho_l \tilde{u}_l^2 u_{ls}^2 f_{ls}(\tilde{u}_l\tilde{d}_l)^{-n} \frac{\tilde{S}_i d}{2} > g\Delta\rho\tilde{A}_g d^2$$

$$\frac{1}{2}\rho_l u_{ls}^2 \frac{4 f_{ls}}{4d}\tilde{u}_l^2(\tilde{u}_l\tilde{d}_l)^{-n}\frac{\tilde{S}_i}{2} > g\Delta\rho\tilde{A}_g$$

Slug-Dispersed Bubble Transition-III

$$\left.\frac{dp}{dx}\right|_{ls}\frac{1}{g\Delta\rho} > \frac{8\tilde{A}_g}{\tilde{S}_i\tilde{u}_l^2(\tilde{u}_l\tilde{d}_l)^{-n}} \qquad T^2 > \frac{8\tilde{A}_g}{\tilde{S}_i\tilde{u}_l^2(\tilde{u}_l\tilde{d}_l)^{-n}}$$

where $T = \left.\frac{dp}{dx}\right|_{ls}\frac{1}{g\Delta\rho}$



Stratified-Wavy Stratified Transition-I

- It is noticed that smaller stable waves appear before the K-H instability sets in
- Jefferey has analysed this aspect has come out with a criterion for the onset of waves.

$$\frac{(u_g - c)^2}{g\frac{\Delta\rho}{\rho_g}\frac{\nu_l}{c}} > \frac{4}{s}$$

In this expression c is the wave speed, s is the sheltering coefficient recommended to be between 0.01-0.03

- Taitel and Dukler argued that $c \sim u_l$, $u_g >> u_l$ and recommended s = 0.01 for satisfactory prediction
- Thus, for wave formation $u_g^2 > \frac{4}{s} g \frac{\Delta\rho}{\rho_g}\frac{\nu_l}{u_l}$

- Non-dimensionalising as done earlier, we get

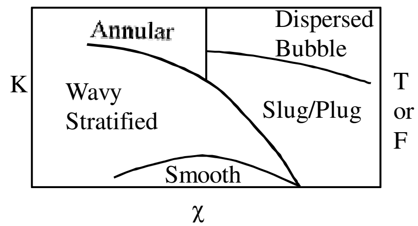
$$\frac{u_g^{2}\tilde{u}_g^{2}}{g\,\frac{\Delta\rho}{\rho_g}\,\frac{\nu_l}{u_l\tilde{u}_l}} > \frac{4}{s}$$

- Rearranging, we get

$$\frac{u_{gs}^{2}\tilde{u}_g^{2}}{g\,\frac{\Delta\rho}{\rho_g}\,d\,\frac{\nu_l}{du_{ls}\tilde{u}_l}} > \frac{4}{s}$$

$$F^2\,Re_{ls} > \frac{4}{s}\,\frac{1}{\tilde{u}_l\tilde{u}_g^{2}}$$

$$K^2 > \frac{4}{s}\,\frac{1}{\tilde{u}_g^{2}\tilde{u}_l}$$ where $K^2 = F^2\,Re_{ls}$



Flow Pattern Logic

- For given flow rates, geometry and fluid properties,
  1. Compute χ
  2. Find $\tilde{h}_l$ using iterative procedure
  3. Compute F, T, K, $\tilde{A}_g, \tilde{u}_g, \tilde{u}_l, \tilde{S}_i$