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#### Taitel-Dukler's Model-I

- $\square$  In horizontal systems we had put the map in the nondimensional planes with  $\chi$  as the fundamental parameter and with F, T and K as the deciding parameters
- ☐ In vertical systems such a treatment is absent
- ☐ All arguments are dimensional
- $\Box$  Usually it is plotted in  $j_g$ - $j_l$  plane
- □ We can assume the properties that may decide the flow patterns are j<sub>g</sub>, j<sub>l</sub>, D, ρ<sub>l</sub>, ρ<sub>g</sub>, μ<sub>l</sub>, μ<sub>g</sub>, σ and g.
- $\square$  With 9 variables, we can get 6  $\pi$  groups. Thus, ideally it will be a complex map.

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#### Taitel-Dukler's Model-II

- ☐ However, the state of the art is far from satisfactory
- ☐ There have been many papers
- ☐ Taitel-Dukler's model seems rational (AIChE J, <u>26</u>, 345, 1980)
- ☐ Discussions are fairly complete in Kazimi and Todreas
- ☐ We shall briefly look at the arguments.

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### **Bubbly-Slug Transition-I**

- ☐ In fully developed flow, there is a constant slip.
- ☐ In stationary liquid systems the final velocity reached by the gas bubbles is called the bubble rise velocity.
- ☐ From large number of studies, it has been established that the bubble rise velocity can be expressed as

$$V_{\infty} = 1.53 \left( \frac{g \Delta \rho \sigma}{\rho_{i}^{2}} \right)^{\frac{1}{4}}$$



☐ When liquid velocity is superimposed, we can write

$$V_{\sigma}$$
 -  $V_{I}$  =  $V_{\infty}$ 



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### **Bubbly-Slug Transition-II**

☐ From basic relations, we can write

$$V_g = \frac{V_{gs}}{\alpha}, \ V_l = \frac{V_{ls}}{1-\alpha}$$



 $\Box$  From equations 1, 2 and 3, we can write

$$\frac{V_{ls}}{1-\alpha} = \frac{V_{gs}}{\alpha} - 1.53 \left(\frac{g\Delta\rho\sigma}{\rho_1^2}\right)^{1/4}$$



- ☐ They argued that,
  - $\square$  Bubble collision frequency  $\rightarrow \infty$  as  $\alpha \rightarrow 0.3$
  - $\Box$  Bubbles wobble and coalesce at  $\alpha = 0.25$

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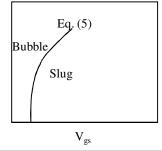
#### **Bubbly-Slug Transition-III**

 $\square$  Substituting  $\alpha = 0.25$  in Eq. (4) and rearranging, we get,

$$\frac{V_{ls}}{V_{gs}} = 3 - \frac{1.15}{V_{gs}} \left( \frac{g\Delta\rho\sigma}{\rho_1^2} \right)^{1/4}$$



Although the equation is linear, in a log-log plot it is non-linear



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- ☐ It has been observed that in very small tubes, bubbly flow is not seen.
- ☐ Taitel and Dukler rationalised as follows
  - ☐ It is knon that Taylor bubble velocity can be expressed as  $V = 0.35 \text{ (gD)}^{0.5}$
  - ☐ If the bubble rise velocity is larger than the Taylor bubble velocity, then the bubble will rise and coalesce into a Taylor bubble
  - ☐ Hence the condition for bubbly flow not to exist is to equate the bubble rise velocity to Taylor bubble velocity

$$0.35\sqrt{gD} \le 1.53 \left(\frac{g\Delta\rho\sigma}{\rho_l^2}\right)^{\frac{1}{4}} \implies \left(\frac{\rho_l^2 gD^2}{\Delta\rho\sigma}\right)^{\frac{1}{4}} \le 4.36$$



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### Bubble-Dispersed Bubble Transition-I

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- ☐ As discussed in horizontal systems, if there is high liquid shear it will tear larger bubbles into smaller ones.
- ☐ However, gravity will have no role in this case.
- ☐ Taitel and Dukler took the study of Hinze (1955) on bubble development in agitated flows. Hinze showed that maximum diameter of the bubble to be

$$d_{\text{max}} = k \left(\frac{\sigma}{\rho_l}\right)^{\frac{3}{5}} \varepsilon^{-\frac{2}{5}}$$

where,  $\epsilon$  is energy dissipated per unit mass,

$$k = 0.725$$

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## Bubble-Dispersed Bubble Transition-II

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☐ The dissipation rate can be treated as follows

$$\frac{power}{mass} = \frac{\Delta pQ}{A\rho\Delta x} = \frac{dp}{dx} \frac{V_m}{\rho_m}$$

$$\frac{dp}{dx} = \frac{4f}{2D} \rho_m V_m^2 \quad \text{with} \quad f = 0.046 \left(\frac{V_m D}{V_l}\right)^{-0.2}$$

☐ Further, when the bubbles are too small, they do not coalesce. Brodkey (1967) had shown that the critical diameter whove which coalescence happens can be given as  $(0.4\sigma)^{0.5}$ 

$$d_{crit} = \left(\frac{0.4\sigma}{\Delta \rho g}\right)^{0.5}$$

#### Bubble-Dispersed Bubble

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- Transition-III
  ☐ Taitel argued that for isolated bubbles to exist,  $d_{\text{max}} > d_{\text{crit}}$
- ☐ Substituting the expressions arrived for the dmax and d crit and equating the gives

and d crit and equating the gives
$$\left(\frac{\left(\rho_{l}/\rho_{l}\right)^{0.5}V_{l}^{0.08}}{\left(\sigma/\rho_{l}\right)^{0.1}D^{0.48}}\right)(V_{ls}+V_{gs})^{1.12}=3$$



- ☐ Kazimi and Todreas gets RHS as 4.72
- ☐ Barnea has modified it as

RHS - = 3 + 17 
$$\left(\frac{V_{gs}}{(V_{ls} + V_{gs})}\right)^{0.5}$$



# Transition-IV Dispersed Bubble Eq. (7) Bubble Slug

**Bubble-Dispersed Bubble** 

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### Dispersed Bubble-Slug

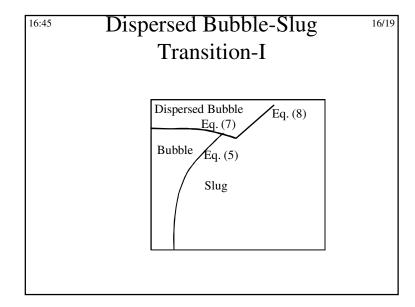
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#### Transition-I

- ☐ If we have spheres in a cubic lattice, the maximum packing one can obtain is 0.52. This implies that it is physically impossible to have bubbly flow beyond  $\alpha = 0.52$
- ☐ Further, in dispersed bubble flow, one can expect homogeneous model to be valid. Hence, we can write

$$\alpha = \beta = \frac{V_{gs}}{(V_{ls} + V_{gs})} = 0.52$$





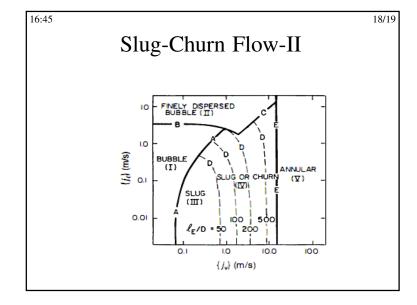
Slug-Churn Flow-I

- ☐ Taitel and Dukler argued that Churn flow occurs before stabilization of the slug
- ☐ Thus churn flow was looked at as an entrance phenomenon. If the entrance length is large, then the entire pipe will have churn flow.
- ☐ Using several empirical arguments they came out with an expression for the entrance length as

$$\frac{l_e}{D} = 42.6 \left( \frac{(V_{ls} + V_{gs})}{\sqrt{gD}} + 0.29 \right)$$

☐ Thus, the transition line will depend on I/D of the pipe. First it will be churn followed by slug

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#### Transition to Annular Flow

- ☐ Taitel and Dukler proposed that annular flow will result if the drag overcomes the weight of a fragmented drop
- ☐ They used the data of Hinze on the fragmented drop size, used the value of coefficient of drag as 0.44 and finally assuming that  $V_g \sim V_{gs}$ , they arrived at the transition criterion as

$$\frac{V_{gs}\rho_g^{0.5}}{(\sigma g\Delta\rho)^{0.25}} = 3.1$$

- $\square$  Note that this will be a vertical line in  $V_{oc}$ - $V_{ls}$  plot
- ☐ Refer to figure in previous slide