# Introduction to Choking

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## Choked Flow-I

- We have looked into concepts to compute pressure drop and void fraction in two-phase flows, given the geometry and flow conditions and have seen their application
- In boilers and nuclear reactors, when the high pressure fluid inside the system is exposed to ambient conditions, either by design or by accident, the pressurised fluid will rush out of the system
- To characterize the transient behaviour in the system, it is necessary to predict the mass flow rate of the fluid issuing out of the system



## Choked Flow-II

- The issuing fluid exits typically at a speed equal to the local sonic speed. Such a state is called critical flow.
- We shall begin with single phase flows, which is covered in the first course in fluid mechanics and logically extend it to two-phase flows.



## Choked Flow-III

- Let us begin with the energy equation that we derived for single phase flow

$$\frac{\partial(\rho A e)}{\partial t}+\frac{\partial(\rho A u e)}{\partial s}=A\frac{\partial p}{\partial t}+q''P-\tau_w P u-W'$$

- For steady adiabatic frictionless flow with no shaft work, the above equation simplifies to

$$\frac{\partial(\rho A u e)}{\partial s}=0$$

- Since at steady state, the mass flow rate along the duct is constant, it reduces to

$$\frac{\partial e}{\partial s}=0 \quad \text{or} \quad e=h+u^2/2+gz=\text{cons tan t}$$

## Choked Flow-IV

$h_o$ } h, u

- We will define stagnation condition as that when the velocity is zero or very small

$$\Rightarrow h_o = h + u^2/2 \quad (1)$$

- Assuming the fluid to behave like ideal gas with constant specific heat,

$$h = c_p T, \; h_o = c_p T_o \quad (2)$$

- From the above two equations, we can say that

$$u^2 = 2(h_o - h) = 2c_p(T_o - T) \quad (3)$$



## Choked Flow-V

- As we are dealing with adiabatic flow with no friction, viz., isentropic flow

$$p\rho^{-\gamma} = constant \quad \Rightarrow p^{1-\gamma}T^{\gamma} = constant \quad (4)$$

$$\Rightarrow T = constant\,(p)^{\frac{\gamma-1}{\gamma}}$$

- Eq. (3) implies

$$u^2 = 2c_pT_o\left(1 - \frac{T}{T_o}\right)$$

$$\Rightarrow u = \left[2c_pT_o\left(1 - \left(\frac{p}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]^{0.5} \quad (5)$$



## Choked Flow-VI

- The mass flow rate through the duct can be written as

$$\dot{m} = \rho A u = A\rho_o\left(\frac{p}{p_o}\right)^{\frac{1}{\gamma}}\left[2c_pT_o\left(1 - \left(\frac{p}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]^{0.5}$$

$$\Rightarrow \dot{m} = A\rho_o\left[2c_pT_o\left(\left(\frac{p}{p_o}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_o}\right)^{\frac{\gamma+1}{\gamma}}\right)\right]^{0.5} \quad (6)$$

- For a given $p_o$ and $T_o$, mass flow rate is a function of $p/p_o$



## Choked Flow-VII

- For maximum flow rate,

$$\frac{d\dot{m}}{d(p/p_o)} = 0$$

$$\Rightarrow \frac{p}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad (7)$$

- Substituting Eq. (7) in Eq. (5) we get,

$$\Rightarrow u = \left[2c_pT_o\left(1 - \frac{2}{\gamma+1}\right)\right]^{0.5}$$

## Choked Flow-VIII

- Expressing for $T_o$ in terms of T, we get,

$$\Rightarrow u = \left[ 2c_p T \left( \frac{p_o}{p} \right)^{\frac{\gamma-1}{\gamma}} \left( \frac{\gamma-1}{\gamma+1} \right) \right]^{0.5} \qquad (8)$$

- Substituting for $p_o/p$ from Eq. (7) in the above equation and simplifying, we get

$$\Rightarrow u = [\gamma RT]^{0.5} = a$$

$$\Rightarrow M = u/a = 1 \qquad (9)$$



## Choked Flow-IX

- From undergraduate course, we also know that if stagnation pressure inside is high then pressure at the exit will be given by Eq. (7), viz.,

$$p = p_o \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

- For air $\gamma = 1.4$ is satisfactory. For steam, $\gamma = 1.3$ is satisfactory



## Choked Flow-X

- When Pressurised fluid that is sub-cooled or a mixture of steam and water is present under stagnation conditions, approach similar to that for single-phase can be employed
- The concepts we used in single-phase are,
  - Energy equation under adiabatic-frictionless conditions
  - Equation of State (Ideal gas law)
  - Isentropic Condition
- Let us look at its implementation



## Homogeneous Equilibrium Model-I

- Let us begin with homogeneous equilibrium model
- From energy equation

$$h_0 = xh_g + (1-x)h_f + \frac{u_H{}^2}{2}$$

$$\Rightarrow u_H = \left[ 2(h_0 - (xh_g + (1-x)h_f)) \right]^{0.5}$$

- Now mass flow rate can be expressed as

$$\dot{m} = \rho_H A u_H = \frac{A\left[ 2(h_0 - (xh_g + (1-x)h_f)) \right]^{0.5}}{(xv_g + (1-x)v_f)} \qquad (10)$$

## Homogeneous Equilibrium Model-II

- Invoking isentropic flow, x can be expressed as

$$x = \frac{s_0 - s_f}{s_{fg}} \qquad (11)$$

- For given stagnation conditions, the mass flow rate at any part of the duct is now a function of local p as $h_f$, $h_g$, $s_f$, $s_g$, $v_f$ and $v_g$ are all functions of p
- Since equation of state for water is complex, analytical expression is not possible.
- However, as properties are available in the form of computerised steam tables, the maximisation of mass flow rate can be done numerically



## Homogeneous Equilibrium Model-III

- The procedure for determination of mass flow rate is as follows.
  1. For a given $p_o$ and $T_o$ determine $s_o$ from steam routine
  2. Assume a p at choking plane, and get all the relevant saturation properties from steam routine
  3. Get x using Eq. (11) and $\dot{m}$ using Eq. (10)
  4. Repeat steps 2, 3 for pressure equal to p +$\Delta$p and compute $\dot{m}$ and $d\dot{m}/dp$
  5. Repeat steps 1-4 for another near by pressure
  6. Shoot for $d\dot{m}/dp = 0$



## Homogeneous Equilibrium Model-IV

- Experiments indicate that for L/D > 40, the leak rates given by HEM are very satisfactory and is the preferred model
- However, when L/D is less, velocity equilibrium does not set in and mass flow rate is found to be larger
- This calls for slip effects to be introduced
- We shall look at a popular model called Moody's model
- In further analysis, let us do the analysis per unit area of the duct and obtain the mass flux G



## Moody's Model-I

- The mass flow rate per unit area can be expressed as,

$$G = \frac{\dot{m}}{A} = \frac{\rho_g A_g u_g + \rho_l A_l u_l}{A} = \rho_g \alpha u_g + \rho_l (1-\alpha) u_l$$

$$= \rho_g u_g \left( \alpha + \frac{\rho_l}{\rho_g} (1-\alpha) \frac{1}{s} \right) \qquad (12)$$

- Using the expression for $\alpha$ (x,s)

$$\alpha = \frac{1}{1 + \frac{\rho_g}{\rho_l} \frac{(1-x)}{x} s}$$

- Eliminating $\alpha$ in Eq. (12) using the above expression and simplification results in

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# Moody's Model-II

$$G = \rho_g u_g \left( \frac{1}{x + (1-x)s \dfrac{\rho_g}{\rho_l}} \right) \quad (13)$$

- Now from energy equation, stagnation enthalpy for this case shall be

$$h_0 = x\left(h_g + \frac{u_g^{\,2}}{2}\right) + (1-x)\left(h_f + \frac{u_f^{\,2}}{2}\right)$$

$$= xh_g + (1-x)h_f + \frac{u_g^{\,2}}{2}\left(x + \frac{(1-x)}{s^2}\right)$$

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# Moody's Model-III

- The above equation can be rewritten to get an expression for $u_g^{\,2}$,

$$u_g^{\,2} = \frac{(h_o - (xh_g + (1-x)h_f))2}{\left(x + \dfrac{(1-x)}{s^2}\right)}$$

- Squaring both sides of Eq. (13) and substitution of the square of the velocity from the above equation, we get

$$G^2 = \rho_g^{\,2} \frac{2(h_o - (xh_g + (1-x)h_f)}{\left(x + (1-x)s\dfrac{\rho_g}{\rho_l}\right)^2 \left(x + \dfrac{1-x}{s^2}\right)} \quad (14)$$

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# Moody's Model-IV

- The above equation indicates G = G(x,p,s,)
- Invoking isentropic flow,

$$x = \frac{s_0 - s_f}{s_{fg}} \quad (15)$$

- Thus G = G(p,s), for a given $s_0$
- Moody used,

$$\frac{\partial G}{\partial p} = 0; \quad \frac{\partial G}{\partial s} = 0$$

- Note that numerator of Eq. (14) is independent of s and hence it is enough to differentiate the denominator with respect to s and equate it to zero. This leads to

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# Moody's Model-V

$$s = \left(\frac{\rho_f}{\rho_g}\right)^{1/3} \quad (16)$$

- Substituting the expression for s from Eq. (16) in Eq. (14), we get

$$G^2 = \rho_g^{\,2} \frac{2(h_o - (xh_g + (1-x)h_f)}{\left(x + (1-x)\left(\dfrac{\rho_g}{\rho_l}\right)^{2/3}\right)^3} \quad (17)$$

# Moody's Model-VI

- The procedure for determination of mass flow rate is as follows.
  1. For a given $p_o$ and $T_o$ determine $s_o$ from steam routine
  2. Assume a p at choking plane, and get all the relevant saturation properties from steam routine
  3. Get x using Eq. (15), s using Eq. (16) and G using Eq.(17)
  4. Repeat steps 2, 3 for pressure equal to p +Δp and compute $\dot{m}$ and $d\dot{m}/dp$
  5. Repeat steps 1-4 for another near by pressure
  6. Shoot for $d\dot{m}/dp = 0$



# Moody's Model-VII

- It is interesting to note that Eq. (16) can be directly obtained by minimising the specific kinetic energy (kinetic energy per unit mass flow rate) with slip

$$\frac{\partial}{\partial s}\left(\frac{x u_g^{\,2} + (1-x) u_f^{\,2}}{2}\right) = 0$$

- Fauske used specific momentum (momentum per unit mass flow rate) minimisation,

$$\frac{\partial}{\partial s}\left(x u_g + (1-x) u_f\right) = 0$$

And showed that,

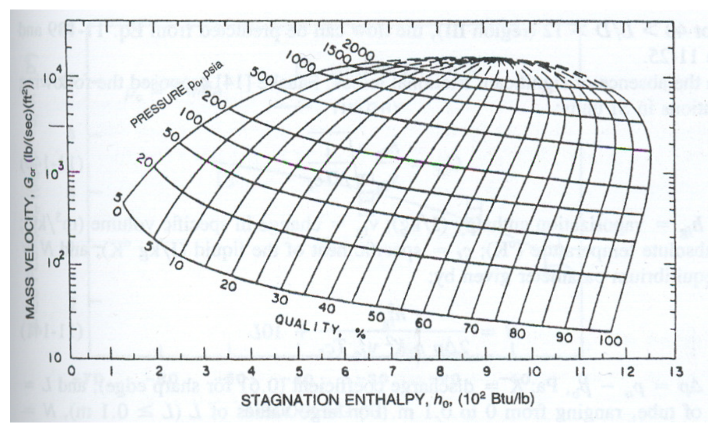
$$s = \left(\frac{\rho_f}{\rho_g}\right)^{1/2} \qquad (18)$$



# Fauske's Model

- The procedure for computation of Mass flux using Fauske's model is very similar to what has been presented for Moody's model with Eq. (18) for s instead of Eq. (16)
- It has been seen that Moody's model predicts the highest mass flux and Homogeneous model predicts the least with Fauske's model lying in between
- As pointed above, homogeneous model predicts well for L/d >40. However, as L/d decreases, the mass flux increases and Fauske's and Moody's model progressively do better
- Though Moody's model overpredicts leak rates, it is still used as a conservative model for safety analysis





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# Non-Equilibrium Model

- Fauske conducted experiments and proposed simple empirical equations for the prediction of Mass fluxes
- When L/d = 0, the leak rates were found to be predicted by the orifice equation

$$G = 0.61\sqrt{2\rho_f(p_o - p_b)} \qquad (19)$$

- For 0 < L/d < 40, the leak rates were found to be predicted by the equation

$$G = 0.61\sqrt{2\rho_f(p_o - p_{cr})} \qquad (20)$$

where $p_{cr}$ is read from the figure is the next slide

- For L/d > 40, the leak rates were found to be satisfactorily predicted by the homogeneous model

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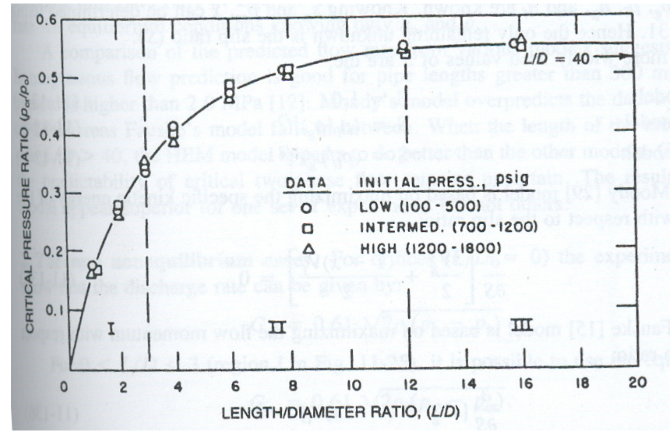


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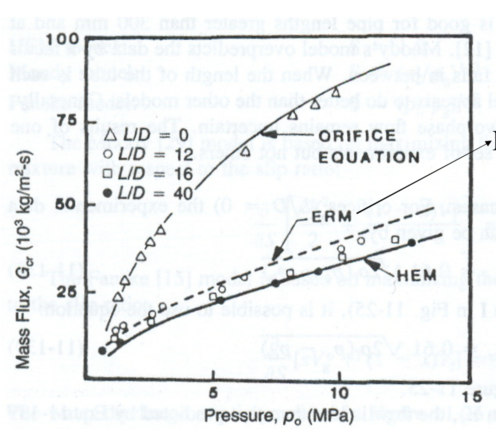


Eq. (20)