Introduction to Condensation Heat Transfer

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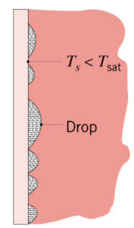
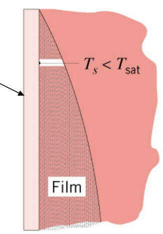
Condensation Heat Transfer-I

- Condensation implies transformation of vapor back to liquid
- There are basically two mechanisms for condensation
  - ➢ Film Condensation
  - ➢ Drop Condensation

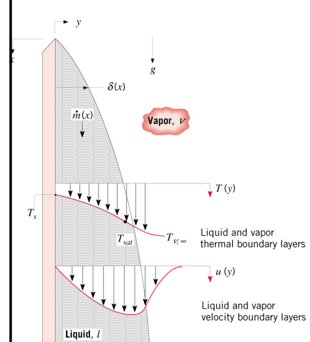




Condensation Heat Transfer-II

- Film Condensation
  - ➢ Entire surface is covered by the condensate, which flows continuously from the surface and provides a resistance to heat transfer between the vapor and the surface.
  - ➢ Thermal resistance is reduced through use of short vertical surfaces and horizontal cylinders.
- Dropwise Condensation
  - ➢ Surface is covered by drops ranging from a few micrometers to agglomerations visible to the naked eye.
  - ➢ Thermal resistance is greatly reduced due to absence of a continuous film.
  - ➢ Surface coatings may be applied to inhibit *wetting* and stimulate dropwise condensation.

Analysis of Film Condensation-I



Nusselt Analysis for Laminar Flow

Assumptions:

- ➢ A pure vapor at T_{sat}
- ➢ Creeping boundary layer flow
- ➢ Negligible shear stress at liquid/vapor interface.

We shall derive it from the basic equations

Analysis of Film Condensation-II

- Momentum Equation

$$\rho_l\left(\cancel{u\frac{\partial(u)}{\partial x}}+\cancel{v\frac{\partial(u)}{\partial y}}\right)=-\frac{\partial p}{\partial x}+\mu_l\left(\frac{\partial^2 u}{\partial y^2}\right)+\rho_f g$$

Negligible $\rho_v g$

$$\Rightarrow (\rho_f-\rho_v)g=-\mu_l\left(\frac{d^2u}{dy^2}\right)\quad\Rightarrow\left(\frac{d^2u}{dy^2}\right)=-\frac{(\rho_f-\rho_v)g}{\mu_l}$$

On Integration we get

$$u=-\frac{(\rho_f-\rho_v)g}{\mu_l}\frac{y^2}{2}+c_1y+c_2$$

Boundary Conditions $u=0,\ y=0$ $\Rightarrow c_2=0$

Analysis of Film Condensation-III

$\frac{\partial u}{\partial y}=0,\ y=\delta$ $\quad\Rightarrow c_1=\frac{\rho_f-\rho_v}{\mu_l}g\delta$

$$\Rightarrow u=\frac{(\rho_f-\rho_v)g\delta^2}{\mu_l}\left(\frac{y}{\delta}-\frac{y^2}{2\delta^2}\right)$$

- Energy Equation

$$\left(\cancel{u\frac{\partial(T)}{\partial x}}+\cancel{v\frac{\partial(T)}{\partial y}}\right)=\alpha\left(\frac{\partial^2T}{\partial y^2}\right)\qquad\Rightarrow\left(\frac{\partial^2T}{\partial y^2}\right)=0$$

Negligible (Advection neglected)

$$\Rightarrow T=c_1y+c_2$$

Analysis of Film Condensation-IV

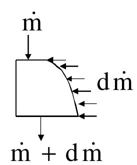
Boundary Conditions $T=T_W,\ y=0$ $\quad T=T_{Sat},\ y=\delta$

$$\Rightarrow T=T_w+(T_{sat}-T_w)\frac{y}{\delta}$$

Note that the boundary layer thickness is still unknown

This is obtained in the following manner

$$\dot m=\int_0^\delta\rho_l u\,dy$$ For unit width



$$=\int_0^\delta\rho_l\frac{(\rho_l-\rho_v)g\delta^2}{\mu_l}\left(\frac{y}{\delta}-\frac{y^2}{2\delta^2}\right)dy$$

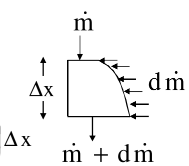
$$=\rho_l\frac{(\rho_l-\rho_v)g\delta^3}{3\mu_l}$$

Analysis of Film Condensation-V

$$d\dot m=\rho_l\frac{(\rho_l-\rho_v)g3\delta^2}{3\mu_l}d\delta\quad(1)$$

From Energy Balance per unit width

Advection neglected implies that heat travels only in y direction



$$h_{fg}d\dot m=\left(k_l\frac{\partial T}{\partial y}\bigg|_{y=\delta}\right)\Delta x=k_l\left(\frac{T_{sat}-T_w}{\delta}\right)\Delta x$$

$$d\dot m=\frac{k_l}{h_{fg}}\left(\frac{T_{sat}-T_w}{\delta}\right)\Delta x\quad(2)$$

Equating 1 and 2 we get $\quad\frac{\rho_l(\rho_l-\rho_v)g\delta^2}{\mu_l}d\delta=\frac{k_l}{h_{fg}}\left(\frac{T_{sat}-T_w}{\delta}\right)\Delta x$

$$\Rightarrow\delta^3\frac{d\delta}{dx}=\frac{k_l\mu_l(T_{sat}-T_w)}{\rho_l(\rho_l-\rho_v)h_{fg}g}\qquad\Rightarrow\frac{\delta^4}{4}=\frac{k_l\mu_l(T_{sat}-T_w)}{\rho_l(\rho_l-\rho_v)h_{fg}g}x+c_1$$

Analysis of Film Condensation-VI

Using the Boundary Condition $\delta = 0 \text{ at } x = 0$ $\Rightarrow c_1 = 0$

$$\Rightarrow \delta = \left(\frac{4 k_l \mu_l (T_{sat} - T_w) x}{\rho_l (\rho_l - \rho_v) h_{fg}\, g} \right)^{1/4}$$

To account for sub-cooling the following correction is made

$$h'_{fg} = h_{fg} (1 + 0.68\, Ja)$$

$$\text{Where, } Ja = \frac{c_p (T_{sat} - T_w)}{h_{fg}}$$ Jacob Number

Finally,

$$h = \frac{q''}{T_{sat} - T_w} = \frac{1}{T_{sat} - T_w} k_l \frac{\partial T}{\partial y} = \frac{k_l}{T_{sat} - T_w} \frac{(T_{sat} - T_w)}{\delta} = \frac{k_l}{\delta}$$

Analysis of Film Condensation-VII

$$\Rightarrow Nu_x = \frac{hx}{k_l} = \frac{k_l}{\delta} \frac{x}{k_l} = \frac{x}{\delta} \quad \Rightarrow Nu_x = \left(\frac{\rho_l (\rho_l - \rho_v) h'_{fg}\, g x^3}{k_l \mu_l (T_{sat} - T_w)} \right)^{1/4}$$

$$Nu_x = 0.707 \left(\frac{\rho_l (\rho_l - \rho_v) h'_{fg}\, g x^3}{k_l \mu_l (T_{sat} - T_w)} \right)^{1/4}$$

By integration one can show that

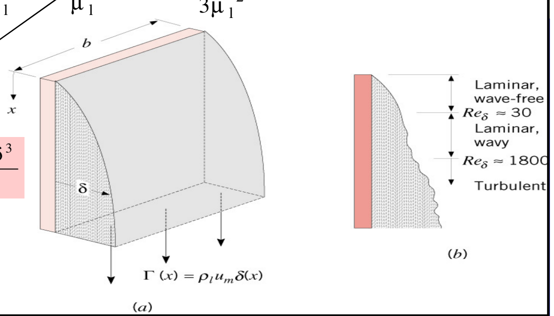
$$\overline{Nu}_L = \frac{4}{3} Nu_L = 0.943 \left(\frac{\rho_l (\rho_l - \rho_v) h'_{fg}\, g L^3}{k_l \mu_l (T_{sat} - T_w)} \right)^{1/4}$$

Analysis of Film Condensation-VIII

- Effects of Turbulence
  - Three flow regimes may be identified and delineated in terms of a Reynolds number defined as

$$\Rightarrow Re_\delta = \frac{4 \rho_l \bar{u} \delta}{\mu_l} = \frac{4 \dot{m}}{\mu_l} = \frac{4 g \rho_l (\rho_l - \rho_v) \delta^3}{3 \mu_l^2}$$

$$\because \dot{m} = \rho_l \frac{(\rho_l - \rho_v) g \delta^3}{3 \mu_l}$$



Analysis of Film Condensation-IX

Laminar Region $Re_\delta < 30$

$$\frac{\overline{h}_L (\nu_l^2 / g)^{1/3}}{k_l} = 1.47 (Re_\delta)^{-1/3}$$

Wavy Laminar Region $30 < Re_\delta < 1800$

$$\frac{\overline{h}_L (\nu_l^2 / g)^{1/3}}{k_l} = \frac{Re_\delta}{1.08 (Re_\delta)^{1.22} - 5.2}$$

Turbulent Region $Re_\delta > 1800$

$$\frac{\overline{h}_L (\nu_l^2 / g)^{1/3}}{k_l} = \frac{Re_\delta}{8750 + 58\, Pr_l^{-0.5} (Re_\delta^{0.75} - 253)}$$

We can obtain $\overline{h}_L$ provided we know Re_δ

Analysis of Film Condensation-X

- Calculation procedure:

To use the equations in the previous slide, we first use the relation for mass flow rate per unit width

$$\dot{m}_{(x=L)} = \frac{\overline{h}_L \times L\,(T_{sat} - T_w)}{h'_{fg}}$$

Since $\quad Re_\delta = \dfrac{4\dot{m}}{\mu_l} = \dfrac{4}{\mu_l}\dfrac{\overline{h}_L L\,(T_{sat} - T_w)}{h'_{fg}}$

$$\Rightarrow \overline{h}_L = \frac{Re_\delta\, h'_{fg}\, \mu_l}{4L\,(T_{sat} - T_w)}$$

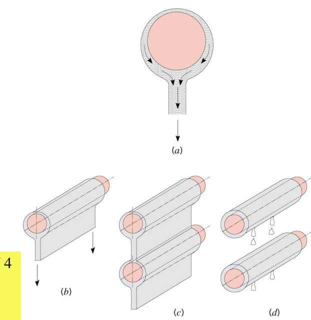
Thus, the left hand side of the equations in previous slide can also be expressed in terms of Reynolds number and known parameters. Hence we can solve for Re_δ, and hence for $\overline{h}_L$

Analysis of Film Condensation-XI

- A single tube or sphere:

$$\overline{h}_D = C\left(\frac{\rho_l(\rho_l - \rho_v)k_l^3 h'_{fg}\, g}{\mu_l(T_{sat} - T_w)D}\right)^{1/4}$$

Tube: C = 0.729 $\qquad$ Sphere: C = 0.826



- Vertical Tier of N-tubes:

$$\overline{h}_{D,N} = 0.729\left(\frac{\rho_l(\rho_l - \rho_v)k_l^3 h'_{fg}\, g}{N\mu_l(T_{sat} - T_w)D}\right)^{1/4}$$

Condensation Inside Tubes

- Shah Correlation (1979):

$$h_f(x)\frac{D}{k_f} = 0.023\, Re_e^{0.8}\, Pr_f^{0.4}\, f(x, p_R) \qquad p_R = p/p_c$$

$$f(x, p_R) = \left[(1-x)^{0.8} + \left(3.8x^{0.76}(1-x)^{0.04}\right)p_R^{-0.38}\right]$$

It is suggested that x = 0.999 to start the method properly rather than x = 1.

Applicability in the range

10.8 <G< (kg/m^2-s)

$Re_{f0} > 350$ for tubes and $Re_{f0} > 3000$ for annuli

$0.002 < p_R < 0.44$

$0<x<1$