Introduction to LOCA

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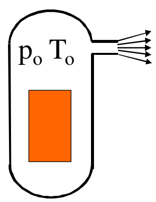
LOCA Modelling

- One of the main application of choked flows is in depressurisation of pressurised vessels due to breaks in pipes associated with the system
- In reactors these kind of accidents are called Loss of Coolant Accidents (LOCA)
- Analysis of such accidents are very complex. But for illustrative purposes, simple lumped depressurisation is discussed in this lecture
- We shall first discuss single-phase cases and then move on to two-phase cases

Single-Phase Depressurisation-I

- The total break area is A
- For single-phase case, the choked mass flow rate was obtained in the lecture on choking as



$$\Rightarrow \dot{m} = A\rho_o\left[2c_pT_o\left(\left(\frac{p}{p_o}\right)^{\frac{2}{\gamma}}-\left(\frac{p}{p_o}\right)^{\frac{\gamma+1}{\gamma}}\right)\right]^{0.5} \quad (1)$$

- Under choked conditions, it was shown that the critical pressure ratio was given by

$$\Rightarrow \frac{p}{p_o}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma-1}{\gamma}} \quad (2)$$

Single-Phase Depressurisation-II

- Plugging Eq. (2) in Eq. (1) and simplification gives

$$\Rightarrow \dot{m} = A\left[\rho_o{}^2 2c_pT_o\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{\gamma-1}{2}\right)\right]^{0.5}$$

$$= A\left[\rho_o{}^2 2c_p\frac{p_o}{\rho_oR}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{\gamma-1}{2}\right)\right]^{0.5}$$

- Expressing c_p/R in terms of γ, the above equation can be simplified as

$$= A\left[p_o\rho_oC\right]^{0.5} \quad \text{where,} \quad C=\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \quad (3)$$

Single-Phase Depressurisation-III

- Writing the mass balance for the fluid in the vessel

Mass Balance

$$\frac{d(V\rho)}{dt} = -\dot{m}$$

$$\Rightarrow V\frac{d(\rho)}{dt} = -\dot{m} \quad \Rightarrow V\frac{d}{dt}\left(\frac{p}{RT}\right) = -\dot{m}$$

- Treating the instantaneous pressure and density in the vessel as the stagnation pressure, and using Eq. (3) for mass flow rate, we can write

$$\Rightarrow \frac{V}{R}\frac{d}{dt}\left(\frac{p}{T}\right) = -A\sqrt{p\rho C} \qquad (4)$$

Single-Phase Depressurisation-IV

- Writing the energy balance for the fluid in the vessel

Energy Balance

$$\frac{d(V\rho c_v T)}{dt} = \dot{Q} - \dot{m}h$$

$$\frac{d\left(V\frac{p}{RT}c_v T\right)}{dt} = \dot{Q} - \dot{m}c_p T$$

$$\frac{Vc_v}{R}\frac{dp}{dt} = \dot{Q} - \dot{m}c_p T \qquad (5)$$

- Eqs. (4) and (5) are two ODE's which can be solved numerically with the initial conditions,

$$p = p_o, \quad T = T_o \quad \text{at} \quad t = 0$$

Single-Phase Depressurisation-V

- We can obtain analytical solutions under some special cases
- If the heat generated is such that T = constant, then we can say $T = T_o$. In such a case energy equation will not be required

Mass Balance

$$\frac{V}{RT_o}\frac{dp}{dt} = -A\sqrt{p\frac{p}{RT_o}C}$$

$$\Rightarrow \frac{dp}{dt} = -\frac{p}{\tau} \quad \text{where, } \tau = \frac{V}{A\sqrt{CRT_o}} \qquad \Rightarrow p = p_o e^{-t/\tau}$$

Single-Phase Depressurisation-VI

- Similarly, we can also obtain an analytical solution for the adiabatic case by employing isentropic law

$$\Rightarrow T = T_o\left(\frac{p_o}{p}\right)^{\frac{1-\gamma}{\gamma}}$$

- Substituting in the mass balance equation $\quad \frac{V}{R}\frac{d}{dt}\left(\frac{p}{T}\right) = -A\sqrt{p\rho C}$

$$\frac{V}{R}\frac{d}{dt}\left\langle \frac{p}{T_o\left\langle \frac{p_o}{p}\right\rangle^{\frac{1-\gamma}{\gamma}}}\right\rangle = -A\sqrt{\frac{p^2}{RT}C} = -Ap\sqrt{\frac{C}{RT_o\left\langle \frac{p_o}{p}\right\rangle^{\frac{1-\gamma}{\gamma}}}}$$

Single-Phase Depressurisation-VII

$$\Rightarrow \frac{V}{RT_o(p_o)^{\frac{1-\gamma}{\gamma}}}\frac{d}{dt}\left(p^{1+\frac{1-\gamma}{\gamma}}\right) = -Ap^{1+\frac{1-\gamma}{2\gamma}}\sqrt{\frac{C}{RT_o(p_o)^{\frac{1-\gamma}{\gamma}}}}$$

$$\Rightarrow \frac{d}{dt}\left(p^{\frac{1}{\gamma}}\right) = -\frac{A}{V}p^{\frac{\gamma+1}{2\gamma}}\sqrt{CRT_o(p_o)^{\frac{1-\gamma}{\gamma}}}$$

$$\Rightarrow \frac{1}{\gamma}p^{\frac{1}{\gamma}-1}\frac{dp}{dt} = -\frac{A}{V}p^{\frac{\gamma+1}{2\gamma}}\sqrt{CRT_o(p_o)^{\frac{1-\gamma}{\gamma}}}$$

$$\Rightarrow p^{\frac{2-2\gamma-(\gamma+1)}{2\gamma}}\frac{dp}{dt} = -\frac{\gamma}{\tau}(p_o)^{\frac{1-\gamma}{2\gamma}} \Rightarrow p^{\frac{1-3\gamma}{2\gamma}}\frac{dp}{dt} = -\frac{\gamma}{\tau}(p_o)^{\frac{1-\gamma}{2\gamma}}$$

Single-Phase Depressurisation-VIII

- Integration of the above equation gives

$$\Rightarrow \frac{2\gamma}{1-\gamma}p^{\frac{1-\gamma}{2\gamma}}\Big|_{p_0}^{p} = -\frac{\gamma}{\tau}(p_o)^{\frac{1-\gamma}{2\gamma}}t$$

$$\Rightarrow \left(p^{\frac{1-\gamma}{2\gamma}} - p_0^{\frac{1-\gamma}{2\gamma}}\right) = (p_o)^{\frac{1-\gamma}{2\gamma}}\frac{\gamma-1}{2}\frac{t}{\tau}$$

$$\Rightarrow p^{\frac{1-\gamma}{2\gamma}} = (p_o)^{\frac{1-\gamma}{2\gamma}}\left(1+\frac{\gamma-1}{2}\right)\frac{t}{\tau}$$

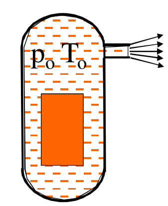
$$\Rightarrow p = p_o\left[\left(1+\frac{\gamma-1}{2}\right)\frac{t}{\tau}\right]^{-\frac{2\gamma}{\gamma-1}}$$

Two-Phase Depressurisation-I

- In water reactors, as the depressurisation is intitiated, two-phase conditions are encountered.
- The choking condition would also involve two-phase choking concepts discussed earlier
- Three models are considered for discussion:
  - Homogeneous Fluid Model
  - Separated Fluid Model
  - Bubble Rise Model

Two-Phase Depressurisation-II

- First let us consider the homogeneous model, which is valid when the depressurisation is rapid
- The fluid is treated as homogeneous



Mass Balance $\quad V\frac{d\bar{\rho}}{dt} = -\dot{m}$ 1

Energy Balance $\quad V\frac{d}{dt}\left\{\bar{\rho}\left(\bar{h}-\frac{p}{\bar{\rho}}\right)\right\} = -\dot{m}\bar{h}+\dot{Q}$ 2

$$V\left[-\frac{dp}{dt}+\bar{\rho}\frac{d\bar{h}}{dt}+\bar{h}\frac{d\bar{\rho}}{dt}\right] = -\dot{m}\bar{h}+\dot{Q}$$

Two-Phase Depressurisation-III

- From Eqs. (1) and (2), we get

$$V\left[-\frac{dp}{dt}+\bar{\rho}\frac{d\bar{h}}{dt}-\bar{h}\cancel{\frac{\dot{m}}{V}}\right]=-\cancel{\dot{m}\bar{h}}+\dot{Q}$$

$$\Rightarrow \frac{d\bar{h}}{dt}-\frac{1}{\bar{\rho}}\frac{dp}{dt}=\frac{\dot{Q}}{V\bar{\rho}} \qquad (3)$$

- Eq. (1) may be written as

$$V\frac{d(1/\bar{v})}{dt}=-\dot{m} \quad \Rightarrow \frac{1}{\bar{v}^2}\frac{d\bar{v}}{dt}=\frac{\dot{m}}{V}$$

$$\Rightarrow \frac{d(v_f+xv_{fg})}{dt}=\frac{\dot{m}}{V}\bar{v}^2$$

Two-Phase Depressurisation-IV

- As v_f and v_g are functions of p we can use chain rule to write,

$$\frac{dp}{dt}\left(\frac{dv_f}{dp}+x\frac{dv_{fg}}{dp}\right)+v_{fg}\frac{dx}{dt}=\frac{\dot{m}}{V}\bar{v}^2 \qquad (4)$$

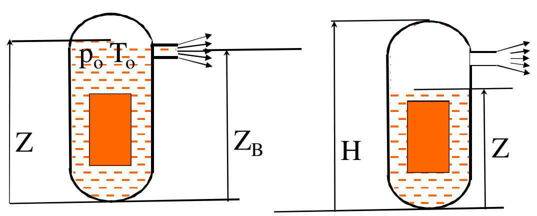
- Similarly, Eq. (3) may be written as

$$\frac{dp}{dt}\left(\frac{dh_f}{dp}+x\frac{dh_{fg}}{dp}-\bar{v}\right)+h_{fg}\frac{dx}{dt}=\frac{\dot{Q}}{V}\bar{v} \qquad (5)$$

- Eqs. (4) and (5) can be modified to get explicit expressions for dp/dt and dx/dt and these can be solved numerically by ODE solvers

Two-Phase Depressurisation-V

- Now let us consider the separated model, which is more of a conceptual model



Two-Phase Blowdown $Z > Z_B$ Single-Phase Blowdown $Z < Z_B$

Two-Phase Depressurisation-VI

- The overall lumped mass and energy balances are identical
- The difference is in the treatment of stagnation enthalpy of the fluid coming out of the break.
- This is modelled as follows

$$h_0=h_f \;\; for\; Z\geq Z_B$$
$$h_0=h_g \;\; for\; Z< Z_B \qquad (6)$$

- Thus, we need to track the interface height.

Two-Phase Depressurisation-VII

- We can continue to use the previous Eqs. (4) and (5) as the governing equations for the transient variation of pressure, p and vapour mass fraction, x
- The volume fraction α can be expressed as a function of mass fraction as

$$\alpha = \frac{1}{1+\left(\frac{1-x}{x}\right)\frac{\rho_g}{\rho_f}} \tag{7}$$

- Further for a cylindrical vessel we can write

$$1-\alpha = \frac{Z}{H}$$

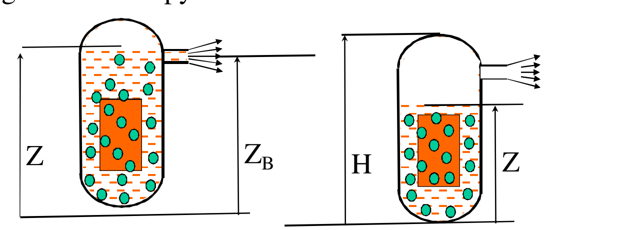
- Thus the transient variation of Z can be obtained

Two-Phase Depressurisation-VIII

- The separated model described is not that realistic as vapour is generated in the liquid and has to be transported to the top space
- Hence there will always be some bubbles in the liquid.
- Now we will discuss a more realistic model called the bubble rise model
- It is fairly similar to the previous model but needs to distribute the steam in fluid appropriately
- In this model, we consider two regions
  - Steam Dome
  - Frothing Liquid with uniformly distributed bubbles

Two-Phase Depressurisation-IX

- The bubbles are assumed to be of one diameter and are assumed to rise with a uniform velocity
- The blowdown modelling is similar and expressions for stagnation enthalpy are discussed later



Two-Phase Blowdown $Z > Z_B$ Single-Phase Blowdown $Z < Z_B$

Two-Phase Depressurisation-X

- Let us define
  Mass of liquid - M_l
  Mass of vapour in bubbles - M_{gb}
  Mass of vapour is steam dome - M_{sd}
  Mass of steam in vessel – M_s
  Average void fraction in the vessel - α
- The overall mass and energy balance continues to be the same as outlined in Eqs. (4) and (5). But tracking of interface height needs more elaborate book keeping.
- This is modelled as follows.

Two-Phase Depressurisation-XI

$$M_s = A_V H\alpha\rho_g = M_{gb} + M_{sd} \quad (8)$$

$$M_{sd} = A_V\left(H - Z\right)\rho_g \quad (9)$$

$$M_{gb} = A_V H\alpha\rho_g - A_V\left(H - Z\right)\rho_g$$

$$\Rightarrow M_{gb} = A_V\rho_g\left(Z - (1-\alpha)H\right) \quad (10)$$

- Further for the steam dome we can write

$$\frac{dM_{sd}}{dt} = \dot{m}_{gain} - \dot{m}_{loss} \quad (11)$$

Two-Phase Depressurisation-XII

- The loss term in the previous equation can be modelled as

$$\dot{m}_{loss} = 0 \;\; for\ Z \geq Z_B$$
$$\dot{m}_{loss} = \dot{m}_{break} \;\; for\ Z < Z_B \quad (12)$$

- To model the steam added in the dome due to interface movement, consider a time Δt, during which interface moves by ΔZ

$$mass\ gain\ in\ dome = A_V\alpha_{gb}(V_b + \frac{dz}{dt})\Delta t\rho_g$$

$$\Rightarrow \dot{m}_{gain} = A_V\alpha_{gb}(V_b + \frac{dz}{dt})\rho_g \quad (13)$$

Two-Phase Depressurisation-XIII

$$M_{gb} = \alpha_{gb}A_V Z\rho_g$$

$$\Rightarrow \dot{m}_{gain} = \frac{M_{gb}}{Z}(V_b + \frac{dz}{dt}) \quad (14)$$

- Substituting the expression for M_{gb} from Eq. (10) and neglecting dz/dt in comparison with V_b, we get

$$\dot{m}_{gain} = \frac{A_V\rho_g\left(Z-(1-\alpha)H\right)}{Z}V_b \quad (15)$$

- Substituting the expression for $\dot{m}_{gain}$ from Eq. (17) in Eq. (11), we get

$$\frac{d\left(A_V\left(H-Z\right)\rho_g\right)}{dt} = A_V\rho_g\left(1-(1-\alpha)\frac{H}{Z}\right)V_b - \dot{m}_{loss}$$

Two-Phase Depressurisation-XIV

$$\Rightarrow A_V\left(H-Z\right)\frac{d\rho_g}{dt} - A_V\rho_g\frac{(dZ)}{dt} = A_V\rho_g\left(1-(1-\alpha)\frac{H}{Z}\right)V_b - \dot{m}_{loss}$$

$$\Rightarrow \frac{dZ}{dt} = \frac{(H-Z)}{\rho_g}\frac{d\rho_g}{dp}\frac{dp}{dt} - \left(1-(1-\alpha)\frac{H}{Z}\right)V_b + \frac{\dot{m}_{loss}}{\rho_g A_V} \quad (16)$$

- Thus, the overall solution is obtained as follows
  - Eqs.(4) and (5) are solved to obtain p, x.
  - The overall α is obtained using is Eq. (7)
  - Suitable choking model is used. The definition of stagnation enthalpy is as follows

$$h_0 = \frac{M_l h_f + M_{gb} h_g}{M_l + M_{gb}} \quad for\ Z \geq Z_B$$

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$$h_0 = h_g \quad for\ Z < Z_B$$

- M_{gb} is obtained using Eq. (10)
- M_l can be obtained as

$$M_l = A_V H(1-\alpha)\rho_f$$

- Finally, Z can be obtained by solving Eq. (16)
- Thus the entire solution can be obtained numerically