Heat Generation in Nuclear Reactor

- Heat generation in nuclear reactor is not uniform
- Heat generated per unit volume depends on the number of fissions occurring per unit volume
- This can be shown to be proportional to the variation of neutron density in the reactor
- This information is provided to a thermal hydraulic engineer by the reactor physicist
- While the exact evaluation is fairly complex, some simplifying approximations shall be made in the next slide

Heat Distribution - I

- Heat generation in nuclear reactor is distributed amongst
  1. Kinetic energy of fission fragments (KE-FF)
  2. Kinetic Energy of neutrons (KE-N
  3. Betas, Gammas and neutrinos
- Further not all energy is liberated instantly.
- We can typically approximate them as follows

Receiver	Instantaneous fraction (~90%)	Delayed fraction (~10%)
Fuel Element	KE- FF - 83%	Betas – 4%
Dispersed in Fuel, Mod., structure	KE-N 2.5%, Prompt Gamma 4%	Capture Gamma 3% Fission Product Gamma 3.5%

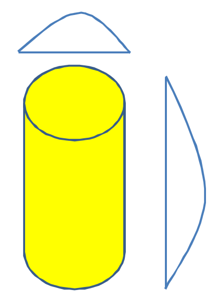
- ~ 90% in fuel and ~10% elsewhere

Thermal Constraints

- The objective of a thermal hydraulic analysis is to satisfy the following Constraints:
  - The maximum fuel temperature has to be within the allowable limit set by the materials engineer
  - Typically this is metal softening temperature.
  - Similarly, the Clad temperature shall be within its applicable safety limit.
  - This is dictated by the metal water reaction temperature.
  - In addition, the heat flux should be below the Critical Heat Flux (CHF limit)
  - We shall discuss this during boiling heat transfer

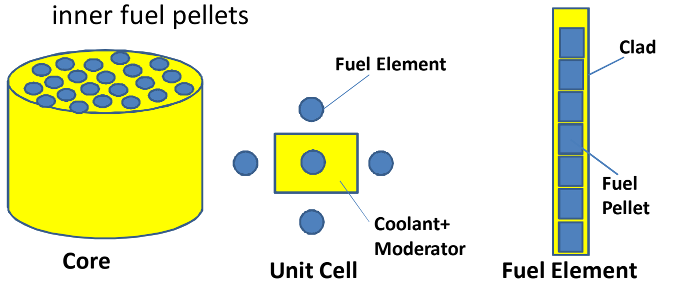
Heat Distribution - II

- The spatial variation for bare cylindrical reactors can be shown to be Cosine axially and Bessel function of zeroth order in radial direction
- For complex systems, these are complicated
- Shape supplied by Physicist
- We shall keep the discussion general so as to grasp the fundamentals



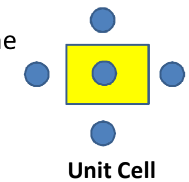
Definitions - I

- The reactor core by definition is the assembly of the fuel material along with moderator and coolant material
- It can be assumed to be an array of rods distributed in a uniform manner
- As discussed each rod has the outer clad and the inner fuel pellets



Definitions-II

- The discussion is presently illustrated using a square pitch (p). Should be fairly straight forward to extend it to any type of unit cell
- For a unit cell, the volume of the core can be viewed as the total volume of the unit cell
- Power density (Q''') is defined as the thermal power generated per unit core volume
- Linear heat rate (q') is defined as the thermal power generated per unit length of the fuel rod
- Heat flux (q'')is defined as the thermal power generated per unit external surface area of the fuel rod



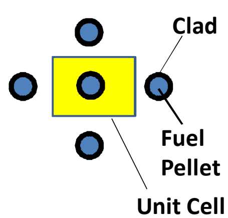
Definitions-III

- Volumetric heat generation (q''') rate is defined as the thermal power generated in the core per unit fuel volume
- Consider an infinitesimal volume (dV) of unit cell and let the thermal power generated be (dQ). The power density can be written as

$$Q''' = \frac{dQ}{p^2 dl}$$

- Similarly, the linear heat rate can be written for the control volume of length dl as

$$q' = \frac{dQ}{dl}$$



Definitions-IV

- Similarly, the heat flux for the control volume can be written as

$$q'' = \frac{dQ}{\pi d_{rod} dl}$$

- Finally, the volumetric heat generation rate can be written as

$$q''' = \frac{dQ}{\frac{\pi}{4} d_{pellet}^2 dl}$$

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Definitions-V

- Since dQ is same in all the above equations, we note that

$$Q'''p^2 = q' = q''\pi d_{rod} = q'''\frac{\pi}{4}d_{pellet}^2$$

- Thus, the parameters defined differ only by a geometric factor. Hence if any one of them is known, the others may be computed from the geometry
- Further, since dQ varies as the neuron density, all the parameters would vary accordingly

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Thermal Analysis-I

- To keep the analysis general for any shape distribution, we shall introduce the following analysis
- For the purpose of analysis, the case of the cylindrical reactor is analysed. This is so as most power reactors are cylindrical. The argument can be extended for any other case similarly
- For a cylindrical bare core, the variation of neutron density varies as

$$n = n_{max} J_0\left(2.405\frac{r}{R}\right)Cos\left(\pi\frac{z}{H}\right)$$

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Thermal Analysis-II

- This would imply that

$$Q''' = Q'''_{max} J_0\left(2.405\frac{r}{R}\right)Cos\left(\pi\frac{z}{H}\right)$$

$$\Rightarrow Q = \int_{Vol} Q'''\ dV$$

$$= \int_0^R \int_{-H/2}^{H/2} Q'''_{max} J_0\left(2.405\frac{r}{R}\right)Cos\left(\pi\frac{z}{H}\right)2\pi r\ drdz$$

$$\Rightarrow \overline{Q'''} = \frac{Q'''_{max}}{\pi R^2}\int_0^R J_0\left(2.405\frac{r}{R}\right)2\pi r\ dr\ \frac{1}{H}\int_{-H/2}^{H/2} Cos\left(\pi\frac{z}{H}\right)dz$$

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Thermal Analysis-III

$$\Rightarrow \overline{Q'''} = Q'''_{max}\ P(r)\ P(z)$$

- where

$$P(r) = \frac{1}{\pi R^2}\int_0^R f(r)2\pi r\ dr \quad \text{and} \quad P(z) = \frac{1}{H}\int_{-H/2}^{H/2} f(z)dz$$

- P(r) and P(z) are called radial and axial power factors
- Thus the maximum value can be found from the average value if P(r) and P(z) are known
- The values of maximum heat flux, linear heat rate and volumetric heat generation rates can be computed using the geometric factors

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Steady State Analysis in Single Channel

- Single channel Hot channel Analysis is generally carried out to assess the overall safety of a nuclear reactor

The steps are:

- Identify the hot channel, which is usually the central fuel channel
- Compute the maximum linear heat rate in central channel by dividing the global average linear heat rate by the product of radial and axial power factor.
- Compute the mass flow rate associated with the channel by dividing the average mass flow rate per channel by any radial factor (introduced by orifices).
- The analysis illustrated for a single phase constant property flow to begin with.

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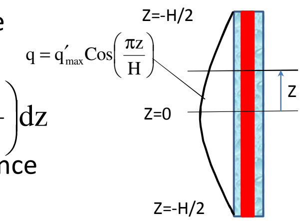
Fluid Temperature-I

- Energy Balance for an infinitesimal slice at a distance z can be written as



$$\dot{m}c_p dT_B = q'\,dz = q'_{max}\mathrm{Cos}\left(\frac{\pi z}{H}\right)dz$$

- On integration from the entrance to any general point z, we get

$$\int_{T_{B-in}}^{T_B} dT_B = \frac{q'_{max}}{\dot{m}c_p}\int_{-H/2}^{z}\mathrm{Cos}\left(\frac{\pi z}{H}\right)dz$$

$$T_B = T_{B-in} + \frac{q'_{max}}{\dot{m}c_p}\frac{H}{\pi}\left[\mathrm{Sin}\left(\frac{\pi z}{H}\right)\right]_{-H/2}^{z} = T_{B-in} + \frac{q'_{max}}{\dot{m}c_p}\frac{H}{\pi}\left[\mathrm{Sin}\left(\frac{\pi z}{H}\right)+1\right]$$

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Fluid Temperature-II

- Having found the temperature distribution for the fluid, we turn our attention to the outer clad temperature
- From the definition of convective heat transfer coefficient, we can write

$$q'' = h(T_{CO} - T_B)$$

$$\Rightarrow T_{CO}(z) = \frac{q''(z)}{h} + T_B(z)$$

- To find the clad temperature distribution we need to perform conduction analysis in clad

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Clad Temperature-I

- The governing equation for conduction is

$$\rho_c c_c \cancel{\frac{\partial T_c}{\partial t}} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk_c\frac{\partial T_c}{\partial r}\right) + \cancel{q'''}$$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(rk_c\frac{dT_c}{dr}\right) = 0 \qquad \Rightarrow rk_c\frac{dT_c}{dr} = C = R_{CO}k_c\left.\frac{dT_c}{dr}\right|_{R_{CO}}$$

- By definition

$$\Rightarrow -k_c\left.\frac{dT_c}{dr}\right|_{R_{CO}} = q'' \qquad \Rightarrow C = -q''R_{CO}$$

$$\Rightarrow -k_c\frac{dT_c}{dr} = q''\frac{R_{CO}}{r} \qquad \Rightarrow -\int_{T_C}^{T_{CO}} dT_C = \int_{r}^{R_{CO}} \frac{q''}{k_C}R_{CO}\frac{dr}{r}$$

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Clad Temperature-II

- Integration with k_c assumed to be constant gives

$$\Rightarrow T_C - T_{CO} = \frac{q''}{k_C} R_{CO} \ln \frac{R_{CO}}{r}$$

- Taking the other limit to the clad inner radius

$$\Rightarrow T_{CI} - T_{CO} = \frac{q''}{k_C} R_{CO} \ln \frac{R_{CO}}{R_{CI}}$$

- This would be the highest clad temperature and if this is safe, then the design is OK
- Now we shall perform conduction analysis in fuel pellet

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Fuel Temperature-I

- The governing equation for conduction is

$$\cancel{\rho_c c_c \frac{\partial T_f}{\partial t}} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk_f \frac{\partial T_f}{\partial r}\right) + q''' \quad \Rightarrow \frac{1}{r}\frac{d}{dr}\left(rk_f \frac{dT_f}{dr}\right) = -q'''$$

- Transposing and performing the integration we get,

$$\Rightarrow \int_{\left(rk_f \frac{dT_f}{dr}\right)_0}^{\left(rk_f \frac{dT_f}{dr}\right)_r} d\left(rk_f \frac{dT_f}{dr}\right) = -\int_0^r q''' r dr$$

- Using the boundary condition that $dT_f/dr = 0$ at centre

$$\Rightarrow rk_f \frac{dT_f}{dr} = -q''' \frac{r^2}{2}$$

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Fuel Temperature-II

- Transposing and performing integration once again

$$\Rightarrow -\int_{T_f(R_{CI})}^{T_f(r)} k_f dT_f = \int_{R_{CI}}^{r} q''' \frac{r}{2} dr$$

- If k_f is assumed to be constant, then

$$\Rightarrow k_f\left(T_f(r) - T_f(R_{CI})\right) = \frac{q'''}{4}\left(R_{CI}^2 - r^2\right)$$

- Taking the other limit of r = 0, we get

$$\Rightarrow T_f(0) = T_f(R_{CI}) + \frac{q''' R_{CI}^2}{4k_f}$$

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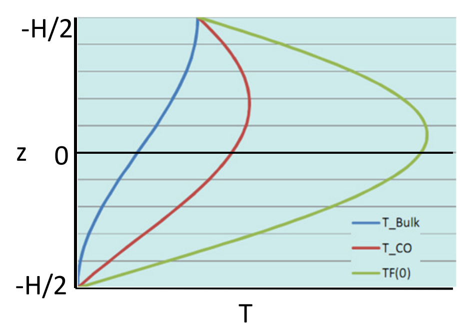
Summary

- T_{Bulk} increases monotonously
- T_{clad} has a maxima after the centreline
- $T_F(0)$ also has a maxima after the centreline.



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More Refined Analysis

- In the previous analysis, we had assumed properties of the material as constant
- However, the temperature drop in the fuel is fairly large and hence the variation of thermal conductivity with temperature has to be taken into account
- Further, we assumed that the clad inner diameter and the fuel outer temperature are identical.
- However, due to the presence of gap, there is a temperature drop that is induced.
- Over and above these two, some more considerations on oxidation and corrosion have to be considered and these are done empirically

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Heat Distribution - I

- It is to be noted that the volumetric heat generated in fuel pellet is not uniform radially
- The volumetric heat generated at the centre is lower due to self shielding effect. Often, diffusion approximation is used. The variation of q''' can be expressed as

$$q''' = AI_0(\kappa r)$$

- The above or a similar equation can be written as

$$q'''(r) = \overline{q'''} g(r)$$

where g(r) is defined as $g(r) = \dfrac{q'''(r)}{\overline{q'''}}$. This function will be given by the physicist

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Fuel Temperature-I

- The governing equation for conduction is now written as

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(rk_f \frac{dT_f}{dr} \right) = -\overline{q'''} g(r)$$

- Transposing and performing the integration we get,

$$\Rightarrow \int_{\left(rk_f \frac{dT_f}{dr}\right)_0}^{\left(rk_f \frac{dT_f}{dr}\right)_r} d\left(rk_f \frac{dT_f}{dr} \right) = -\int_0^r q''' g(r) r dr$$

- Using the boundary condition that $dT_f/dr = 0$ at centre

$$\Rightarrow rk_f \frac{dT_f}{dr} = -\int_0^r q''' g(r) r dr \qquad \Rightarrow k_f \frac{dT_f}{dr} = -\frac{1}{r}\int_0^r q''' g(r) r dr$$

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Fuel Temperature-II

- Transposing and performing integration once again, now between limits r and R_f

$$\Rightarrow -\int_{T_f(r)}^{T_f(R_f)} k_f dT_f = \overline{q'''} \int_r^{R_f} \frac{1}{r}\left(\int_0^r g(r) r dr \right) dr$$

- Previously we had understood that $T_f(R_f)$ is obtained from fluid temperature and moving inward by integration for the clad and accounting for gap conductance that will be discussed shortly
- For a given $T_f(R_f)$, LHS is a function of $T_f(r)$, as k_f as a function of temperature and would be known. On the other hand, RHS is a constant as g(r) and $\overline{q'''}$ are specified. Thus, $T_f(r)$ is obtained by a non-linear solver

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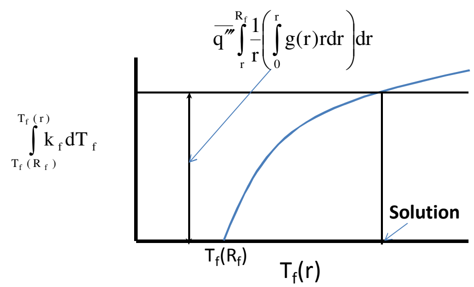
Fuel Temperature-III

- The solution can be visualized graphically in the following manner
- For known $T_f(R_f)$, we can plot LHS as shown
- The RHS is a constant as shown by the horizontal line



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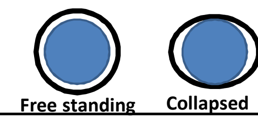
Clad Gap Temperature-I

- Earlier we had assumed that there was no gap between clad and the fuel
- In reality, a gap is provided to accommodate the thermal expansion of the fuel as otherwise the clad will rupture due to thermal stress.
- This gap can offer a large thermal resistance.
- To reduce thermal resistance, the gap is at times filled with He
- From the design point of view the clad can be free standing or can be collapsed



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Clad Gap Temperature-II

- The gap conductance will depend on the gas pressure and contact pressure
- The gas pressure varies with time due to accumulation of fission gases
- The complete treatment can be very complex and is full of uncertainty
- For simple and conservative calculations, gap conductance is taken around 5500 W/m^2-K
- The temperature drop in the gap can be estimated as

$$\Rightarrow \Delta T_{Gap} = \frac{q''}{h_{Gap}}$$

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Clad and Other Resistances-I

- The treatment of clad is identical as it is reasonable to assume that the clad thermal conductivity is constant. Thus,

$$\Rightarrow T_{CI} - T_{CO} = \frac{q''}{k_C} R_{CO} \ln\frac{R_{CO}}{R_{Ci}} = \frac{q'}{2\pi k_C} \ln\frac{R_{CO}}{R_{Ci}}$$

- Usually the clad gets oxidised in the first few days and is subsequently covered by corrosion products. An allowance is made for the presence of these.
- These are estimated empirically by using suitable thicknesses and their thermal conductivities.

Clad and Other Resistances-II

- The temperature drop in the oxide can be estimated as

$$\Delta T_{Ox} = \frac{q'}{2\pi k_{ox}} \ln \frac{R_{ox-out}}{R_{CO}}$$

- Since the thickness to radius ratio is very small, t/R<<1, we can treat the layer as a slab and can write a simpler formula

$$k\frac{\Delta T_{Ox}}{t} = q'' \qquad \Rightarrow \Delta T_{Oxide} = \frac{q''t}{k_{Ox}}$$

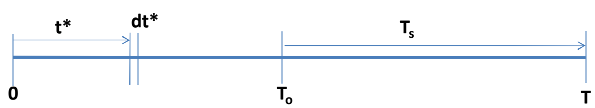
- Similarly, the temperature drop in corrosion product can be estimated as

$$\Delta T_{Oxide} = \frac{q''t}{k_{Ox}}$$

Decay Heat-I

- We have understood the behaviour of temperature in steadily operating reactors
- Unlike thermal power plants where the reaction ceases when fuel supply is cut off, nuclear reactor continues to produce heat even after shut down due to radioactive decay of fission products.
- Estimation of the power produced after shut down is very complex. However, the simple model of Way and Wigner is useful in understanding the trends.

Decay Heat-II



- Referring to the figure shown above, the coordinate t^* starts when the reactor started producing power, where as coordinate T_s starts from the time the reactor is shut down.
- During operation, it is assumed that the power of the reactor is constant
- Since there are several radioactive species present, there is no specific half time that can be assigned.

Decay Heat-III

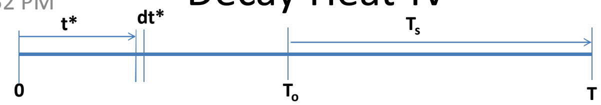
- Way and Wigner fitted a curve for the decrease in energy release with time as a power law
- The energy release rate in the form of β and γ at time t after occurrence of a fission was expressed as

$$\dot{E}_{\beta} = 1.4t^{-1.2} \text{ MeV}/(\text{fission} - \text{s})$$

$$\dot{E}_{\gamma} = 1.26t^{-1.2} \text{ MeV}/(\text{fission} - \text{s})$$

- In the above expressions, t is time expressed in seconds
- Our aim is to express the decay power as a fraction of the original full power at any given time T_s after the reactor is shut down. It is assumed that the reactor was operating for a time T_o at a uniform power

Decay Heat-IV



- Let us consider the fissions that occurred between t^* and dt^*. Also, let q''' be the volumetric heat generation rate. The number of fissions during this time shall be,

$$No.\ of\ \ fissions = q'''(W/cc)\ 3.16x10^{10}(fissions/J)\ dt^*(s)$$

$$dP'''_\beta = q'''\ 3.16x10^{10}\ 1.4(T-t^*)^{-1.2}dt^*(MeV/cc-s)$$

$$= q'''\ 4.34x10^{10}(T-t^*)^{-1.2}dt^* x1.602x10^{-13}(J/cc-s)$$

$$= q'''\ 6.95x10^{-3}(T-t^*)^{-1.2}dt^*(J/cc-s)$$

$$\Rightarrow P'''_\beta = \int_0^{T_o} q'''\ 6.95x10^{-3}(T-t^*)^{-1.2}dt^*(J/cc-s)$$

Decay Heat-V

$$\Rightarrow \frac{P'''_\beta}{P'''_0} = 6.95x10^{-3}\left.\frac{(T-t^*)^{-0.2}}{-0.2}\right|_0^{T_o}$$

$$\Rightarrow \frac{P'''_\beta}{P'''_0} = 3.48x10^{-2}[(T-T_o)^{-0.2} - T^{-0.2})]$$

$$\Rightarrow \frac{P'''_\beta}{P'''_0} = 3.48x10^{-2}[T_s^{-0.2} - T^{-0.2}]$$

Similarly $$\frac{P'''_\gamma}{P'''_0} = 3.12\,x10^{-2}[T_s^{-0.2} - T^{-0.2}]$$

Decay Heat-VI

$$\frac{P'''}{P'''_0} = 0.066[T_s^{-0.2} - T^{-0.2}]$$

- American Nuclear Society (ANS-1971) had suggested a curve that was fitted to the following expression

$$\frac{P}{P_0} = 0.005\ a[T_s^{-b} - T^{-b}]$$

where, a and b are given in the following table

T_S(s)	a	b
0.1-1	12.05	0.0639
10-150	15.31	0.1807
150-8x10^8	27.43	0.2962