10:14 **EN 634 Nuclear Reactor Thermal Hydraulics and Safety** 1/18

Governing Equations for Single-phase Flows

**Kannan Iyer**
**Kiyer@iitb.ac.in**

**Department of Mechanical Engineering**
**Indian Institute of Technology, Bombay**

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10:14 2/18

## ONE DIMENSIONAL ANALYSIS

- In reactor thermal hydraulics one dimensional area averaged approach is most popular.
- While detailed averages can be taken from the three dimensional Navier-Stokes equations, the same can be better understood through the plug flow model approach

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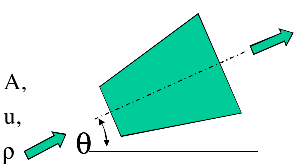
10:14 3/18

## Conservation of Mass - I

- A – Area ($m^2$)
- u – Velocity (m/s)
- ρ – Density ($kg/m^3$)
- $\dot{m}$ – Mass flow rate (kg/s)
- s – Coordinate along pipe (m)
- $\Gamma'$ – Mass source/sink (kg/s-m)




Rate of accumulation of mass in CV = Mass flow rate into CV − Mass flow rate out of CV +/- Rate of Mass generated / destructed in CV

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10:14 4/18

## Conservation of Mass - II

$$\frac{\partial(\rho A\Delta s)}{\partial t}=\cancel{\dot{m}}-\left(\cancel{\dot{m}}+\frac{\partial(\dot{m})}{\partial s}\Delta s\right)\pm\Gamma'\Delta s$$

$$\boxed{\frac{\partial(\rho A)}{\partial t}+\left(\frac{\partial(\dot{m})}{\partial s}\right)=\pm\Gamma'} \qquad (1)$$

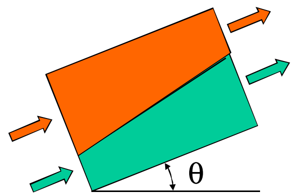
The control volume need not extend over the entire cross section. An example is given in next slide

10:14 5/18

# Conservation of Mass - III

Gas $\dfrac{\partial(\rho_g A_g)}{\partial t} + \dfrac{\partial(\dot{m}_g)}{\partial s} = \Gamma_g'$

liquid $\dfrac{\partial(\rho_l A_l)}{\partial t} + \dfrac{\partial(\dot{m}_l)}{\partial s} = \Gamma_l'$




Note: In boiling flows $\Gamma_g' = -\Gamma_l'$

10:14 6/18

# Conservation of Momentum - I

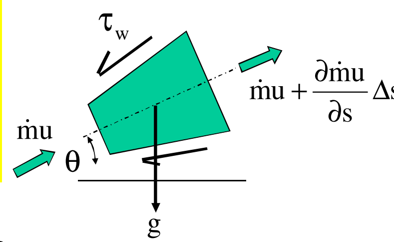
$\tau_w$ – Wall shear stress (N/m$^2$)

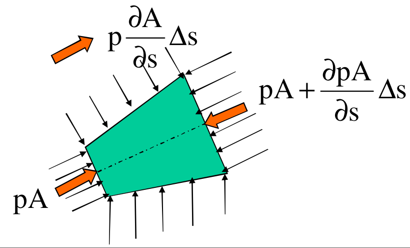
g – Gravitational acceleration (m$^2$/s)

p – Pressure (N/m$^2$)

P – Perimeter (m)

H – Elevation (m)




10:14 7/18

# Conservation of Momentum - II

| Rate of accumulation of momentum in CV | = | Momentum rate into CV | – | Momentum rate out of CV | + | Sum of all forces in positive direction acting on CV |
|---|---|---|---|---|---|---|
| 1 | | 2 | | 3 | | 4 |

Term-1 $\dfrac{\partial(\rho A \Delta s u)}{\partial t}$ Term-2 $\dot{m}u$ Term-3 $\dot{m}u + \dfrac{\partial \dot{m}u}{\partial s}\Delta s$

10:14 8/18

# Conservation of Momentum - III

Term-4 = Pressure force + Shear force + Gravity force

Pressure force $= pA - \left(pA + \dfrac{\partial pA}{\partial s}\Delta s\right) + p\dfrac{\partial A}{\partial s}\Delta s$

$= -A\dfrac{\partial p}{\partial s}\Delta s$

Shear force $= -\tau_w P \Delta s$

Gravity force $= -\rho A \Delta s g \mathrm{Sin}(\theta)$

Substitution of all the terms leads to

10:14 9/18

## Conservation of Momentum - IV

$$\frac{\partial(\rho A u \Delta s)}{\partial t} = \cancel{\dot{m}u} - \left(\cancel{\dot{m}u} + \frac{\partial(\dot{m}u)}{\partial s}\right) - A\frac{\partial p}{\partial s}\Delta s - \tau_w P \Delta s - \rho A \Delta s g \operatorname{Sin}\theta$$

$$\Rightarrow \quad \frac{\partial(\rho A u)}{\partial t} + \frac{\partial(\rho A u^2)}{\partial s} = -A\frac{\partial p}{\partial s} - \tau_w P - \rho A g \operatorname{Sin}\theta \qquad (2)$$

$$\Rightarrow \quad \frac{\partial(\dot{m})}{\partial t} = -\frac{\partial(\dot{m}u)}{\partial s} - A\frac{\partial p}{\partial s} - \tau_w P - \rho A g \operatorname{Sin}\theta \qquad (3)$$

Eqs. (2) and (3) are the conservative forms of the momentum equation

10:14 10/18

## Conservation of Momentum - V

- We can get non-conservative form by expanding the LHS and using mass conservation equation

$$\rho A\frac{\partial u}{\partial t} + u\cancel{\frac{\partial \rho A}{\partial t}} + \rho A u\frac{\partial u}{\partial s} + u\cancel{\frac{\partial \rho A u}{\partial s}} = -A\frac{\partial p}{\partial s} - \tau_w P - \rho A g \operatorname{Sin}\theta$$

$$\Rightarrow \quad \rho A\frac{\partial u}{\partial t} + \rho A u\frac{\partial u}{\partial s} = -A\frac{\partial p}{\partial s} - \tau_w P - \rho A g \operatorname{Sin}\theta \qquad (4)$$

The above expression assumes source and sink of mass is zero.

10:14 11/18

## Mechanical Energy Equation - I

- Multiplication of momentum equation with velocity will give mechanical energy equation

$$\rho A u\frac{\partial u}{\partial t} + \rho A u^2\frac{\partial u}{\partial s} = -uA\frac{\partial p}{\partial s} - u\tau_w P - u\rho A g \operatorname{Sin}\theta \qquad (5)$$

$$\rho A\frac{\partial u^2/2}{\partial t} + \rho A u\frac{\partial u^2/2}{\partial s} = -uA\frac{\partial p}{\partial s} - u\tau_w P - u\rho A g\frac{\partial H}{\partial s} \qquad (6)$$

For steady, inviscid and incompressible flow, we get

$$Au\frac{\partial \rho u^2/2}{\partial s} + Au\frac{\partial p}{\partial s} + uA\frac{\partial(\rho g H)}{\partial s} = 0$$

10:14 12/18

## Mechanical Energy Equation - I

$$\Rightarrow \quad \frac{\partial\left(p + \rho u^2/2 + \rho g H\right)}{\partial s} = 0$$

$$\Rightarrow \quad p + \rho u^2/2 + \rho g H = \text{Constant}$$

Bernoulli's Equation

10:14 13/18

# I-Law of Thermodynamics

Definitions

h - Specific enthalpy (J/kg)

Specific Flow Energy

$$e = h + {u^2}/{2} + gH$$

Specific Energy

$$i = \underbrace{h - {p}/{\rho}}_{\text{Internal energy}} + {u^2}/{2} + gZ$$

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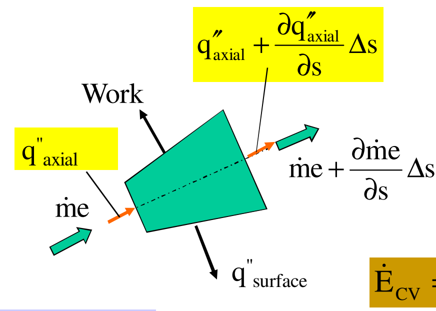
10:14 14/18

# I-Law - II




$$\dot{E}_{CV} = \dot{Q} - \dot{W} + \dot{m}\left(e_{inlet} - e_{exit}\right)$$

$$q''_{axial} = -k\frac{\partial T}{\partial s}$$

$$q''_{surface} = U(T - T_\infty)$$

k – Thermal Conductivity (W/m-K)

U – Overall heat transfer coefficient (W/m²-K)

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10:14 15/18

# I-Law - III

$$\dot{E}_{CV} = \frac{\partial(A\Delta s\rho i)}{\partial t} = A\Delta s\frac{\partial\left(\rho\left[e - {p}/{\rho}\right]\right)}{\partial t}$$

- Using Taylor series, we can write

$$\dot{m}_{inlet}e_{inlet} - \dot{m}_{inlet}e_{exit} = \dot{m}e - \left(\dot{m}e + \frac{\partial(\dot{m}e)}{\partial s}\Delta s\right)$$

$$= -\frac{\partial(\dot{m}e)}{\partial s}\Delta s$$

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10:14 16/18

# I-Law - IV

- We can write

$$\dot{W} = \tau_w P\Delta s u_H + W'_{CV}\Delta s$$

$$\dot{Q} = q''_{surface}P\Delta s + q''_{axial}A - \left(q''_{axial}A + \frac{\partial(q''_{axial}A)}{\partial s}\Delta s\right)$$

$$= q''_{surface}P\Delta s - \frac{\partial(q''_{axial}A)}{\partial s}\Delta s$$

Substitution of all the terms lead to

$$\frac{\partial(\rho Ae)}{\partial t} + \frac{\partial(\rho Aue)}{\partial s} = A\frac{\partial p}{\partial t} + q''_{surface}P - \tau_w Pu - \frac{\partial(q''_{axial}A)}{\partial s} - W'_{CV}$$

Conservative form

7

# I-Law - V



- Using mass conservation we can write

$$\rho A \frac{\partial e}{\partial t} + \rho A u \frac{\partial e}{\partial s} = A \frac{\partial p}{\partial t} + q'' P - \tau_w P u - \frac{\partial (q''_{axial} A)}{\partial s} - W'_{CV} \quad (8)$$

- Substituting the expression for e, we get

$$\rho A \frac{\partial (h + u^2/2 + gH)}{\partial t} + \rho A u \frac{\partial (h + u^2/2 + gH)}{\partial s} = \quad (9)$$

$$A \frac{\partial p}{\partial t} + q''_{surace} P - \tau_w P u - \frac{\partial (q''_{axial} A)}{\partial s} - W'_{CV}$$

$$\rho A \frac{\partial u^2/2}{\partial t} + \rho A u \frac{\partial u^2/2}{\partial s} = -uA \frac{\partial p}{\partial s} - u \tau_w P - u \rho A g \frac{\partial H}{\partial s} \quad (6)$$





# I-Law - VI

- Subtracting both sides of mechanical energy equation Eq. (6) from I–Law Eq. (9) , we can write the thermal energy equation:

$$\rho A \frac{\partial (h)}{\partial t} + \rho A u \frac{\partial (h)}{\partial s} = uA \frac{\partial p}{\partial s} + A \frac{\partial p}{\partial t} + q''_{surface} P - \frac{\partial (q''_{axial} A)}{\partial s} - W' \quad (10)$$

- Here we have assumed that shear work is lost out to the surroundings. For insulated systems this gets ploughed back and hence we have to add $\tau_w Pu$ on RHS. If there is energy split, this has to be suitably handled