# Introduction to Pool Boiling Heat Transfer

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## Boiling Heat Transfer-I

- Boiling is associated with transformation of liquid to vapor by heating
- It differs from vaporization in the sense that it is associated with the formation of bubbles
- The formation of bubbles stir the fluid and breaks the boundary layers thereby increasing the heat transfer coefficient
- The onset of bubble formation is called nucleation
- In equilibrium thermodynamics, boiling is assumed to occur when water is heated to its saturated temperature

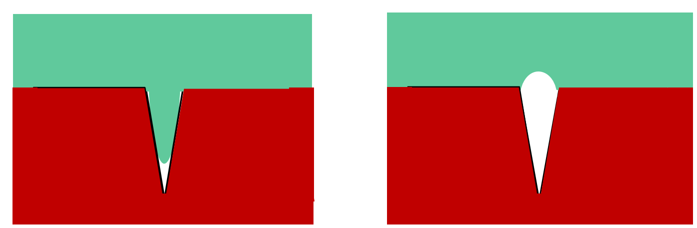


## Homogeneous Nucleation

- The bubbles are normally formed on the surface scratches. The bubbles do not appear till the wall is heated in excess of the saturation temperature, called wall superheat.
- If the surface has mirror finish the onset of boiling is considerably suppressed
- It has been seen that under clean surface conditions, no boiling is seen up to 321 $^{o}$C (~0.92 TC). According to Blander and Katz (AIChE J, 21, 833-849, 1975) the similar behaviour is seen in organic fluids (~0.89 TC).
- In industrial equipment that has sufficient scratches, this homogeneous nucleation phenomenon is not of much relevance



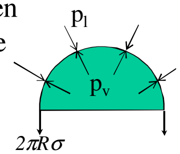
## Hetrogeneous Nucleation-I



- Bubbles generally originate from pits
- Air trapped in cavities help in nucleation
- Experiments indicate that removal air results in suppression of nucleation

# Hetrogeneous Nucleation-II

- If we assume the bubble to be of hemispherical shape and perform a force balance between surface tension and pressure, we can write




$$(p_v - p_l)\pi R^2 = 2\pi R\sigma$$

- For the bubble to grow LHS > RHS

$$(p_v - p_l) > \frac{2\sigma}{R} \qquad (1)$$

- From thermodynamics, Clausius and Clayperon equation is given as

$$\left.\frac{dp}{dT}\right|_{sat} = \frac{h_{fg}}{T_{sat} v_{fg}} \qquad (2)$$



# Hetrogeneous Nucleation-II

- From Taylor series expansion, we can write

$$T_W = T_{sat}(p_{amb}) + \left.\frac{dT}{dp}\right|_{sat} (p_{sat}(T_W) - p_{amb})$$

- Substituting for *(dT/dp)sat* we get,

$$T_W = T_{sat}(p_{amb}) + \frac{T_{sat} v_{fg}}{h_{fg}} (p_{sat}(T_W) - p_{amb})$$

- For bubble to grow, the condition for critical radius can be invoked to eliminate Δp and we can write

$$T_W - T_{sat}(p_{amb}) = \frac{T_{sat} v_{fg}}{h_{fg}} \frac{2\sigma}{R} \qquad (3)$$

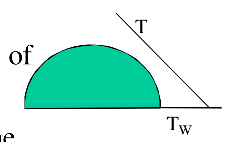


# Hetrogeneous Nucleation-II

- For water at 1 bar, and for a 1 micron cavity, we can compute the wall superheat by substituting, Tsat= 373 K, ; σ= 0.059 N/m ; hfg= 2.256 x $10^6$J/kg ; vg= 1.672 $m^3$/kg; R=$10^{-6}$ m to get $T_W$-$T_{sat}$ = 32 K
- Nucleation is significantly affected if there is a temperature gradient normal to the wall
- In flow boiling, due to boundary layer presence, there is a considerable variation
- Let us start with constant wall flux case and extend it to the general case



- The fluid temperature decreases as we move away from the wall, the pressure excess must correspond to the lowest temperature, which is at the vertical tip of the bubble



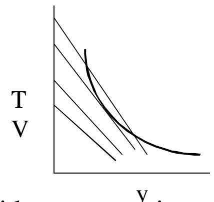

- This will ensure that at no surface of the bubble there will be condensation
- Further, as the critical size of bubbles are of the order 10 microns, the temperature profile in this scale can be assumed to be linear
- Labelling the vertical coordinate as y, we can write,

$$q'' = -k_l \frac{dT_l}{dy} = k_l \left( \frac{T_W - T_l}{y} \right) \qquad (4)$$

- For hemispherical bubble, y = $R_{crit}$ and we can write,

$$T_V - T_{sat}(p_{amb}) = \frac{T_{sat} v_{fg}}{h_{fg}} \frac{2\sigma}{y} \quad (5)$$

$$\Rightarrow \frac{dT_V}{dy} = -\frac{T_{sat} v_{fg}}{h_{fg}} \frac{2\sigma}{y^2} \quad (6)$$




- For linear temperature profile in liquid, we can write,

$$\frac{dT_l}{dy} = -\frac{q''}{k_l} \quad (7)$$

- The slope will increase as heat flux is increased. Davis and Anderson postulated tangency of the liquid profile as the condition for nucleation



- Equating the slope at the point of nucleation, we get

$$\Rightarrow y^2 = \frac{T_{sat} v_{fg}}{h_{fg}} \frac{2\sigma k_l}{q''} \quad (8)$$

- At the point of tangency, the values of temperatures of vapour and liquid also have to be same. Eqs. (4) and (5) can be rewritten as,

$$T_l = T_W - \frac{q''y}{k_l} \qquad T_V = T_{sat}(p_{amb}) + \frac{T_{sat} v_{fg}}{h_{fg}} \frac{2\sigma}{y}$$

$$T_W - T_{sat}(p_{amb}) = \frac{q''y}{k_l} + \frac{T_{sat} v_{fg}}{h_{fg}} \frac{2\sigma}{y}$$

$$T_W - T_{sat}(p_{amb}) = \frac{1}{y}\left(\frac{q''y^2}{k_l} + \frac{T_{sat} v_{fg} 2\sigma}{h_{fg}}\right)$$



- Substituting for y from Eq. (8), we get

$$T_W - T_{sat}(p_{amb}) = \frac{1}{\left(\frac{T_{sat} v_{fg}}{h_{fg}} \frac{2\sigma k_l}{q''}\right)^{0.5}} \left(\frac{2\sigma T_{sat} v_{fg}}{h_{fg}} + \frac{T_{sat} v_{fg} 2\sigma}{h_{fg}}\right)$$

$$(T_W - T_{sat}(p_{amb}))^2 = \frac{q''}{k_l} \frac{1}{\left(\frac{T_{sat} v_{fg} 2\sigma}{h_{fg}}\right)} \left(\frac{4\sigma T_{sat} v_{fg}}{h_{fg}}\right)^2 = \left(\frac{8\sigma T_{sat} v_{fg}}{h_{fg}} \frac{q''}{k_l}\right)$$

$$(T_W - T_{sat}(p_{amb}))^2 = \left(\frac{8\sigma T_{sat} v_{fg}}{h_{fg}} \frac{q''}{k_l}\right) \quad (9)$$

$$\frac{(T_W - T_{sat}(p_{amb}))^2}{T_W - T_{bulk}} = \left(\frac{8\sigma T_{sat} v_{fg}}{h_{fg}} \frac{h}{k_l}\right) \quad (10)$$



- Eq. (9) is used for the case of heat flux specified cases, whereas Eq.(10) can be used for wall temperature specified cases
- Frost and Dzakowic (ASME 67-HT-61(Ht. Tr. Conf.) showed that

$$(T_W - T_{sat}(p_{amb}))^2 = \left(\frac{8\sigma T_{sat} v_{fg}}{h_{fg}} \frac{q''}{k_l}\right)\left(\frac{1}{pr_l}\right)^2$$

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## Introduction-I

- We have understood that boiling starts with nucleation of bubbles in crevices
- We have also established criterion for the onset of bubble both in the presence of temperature gradient as well is in its absence.
- We shall now look at the growth of these bubbles as they have a say in predicting the heat transfer during nucleate boiling.
- The exact solution demands solution of three dimensional Navier-Stokes equations with complex boundary conditions as the interfaces have to be tracked.

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## Introduction-II

- We shall look at crude modelling by making several assumptions to get some analytical solutions that are illustrative
- The two extremes that are normally addressed in bubble growth are:
  - Inertia controlled growth in the initial stages
  - Heat transfer controlled growth in the latter parts of the growth
- We shall look at them now

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## Inertia Controlled regime

- Rayleigh derived this equation by using the first law of thermodynamics
- The assumptions are:
  - The pressure inside the bubble is constant at $p_{sat}(T_W)$
  - The control volume is the infinite shell of water with bubble interface on one side (p = constant)
  - One-dimensional spherically symmetric incompressible flow conditions exist
- Continuity equation implies

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V_r\right)=0$$

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$$\Rightarrow \left(r^2 V_r\right)=C$$

At $r = R_B$, $V_r = dR_B/dt = \dot{R}_B$

$$\Rightarrow V_r = \frac{R_B^2 \dot{R}_B}{r^2}$$

- The kinetic energy of the liquid from $r=R_B$ to $\infty$ can be written as

$$\int_{R_B}^{\infty} 4\pi r^2 dr \rho_f \frac{V_r^2}{2} = \int_{R_B}^{\infty} 4\pi r^2 dr \rho_f \frac{1}{2}\left(\frac{R_B^2 \dot{R}_B}{r^2}\right)^2$$

$$= 2\pi\rho_f R_B^4 \dot{R}_B^2 \int_{R_B}^{\infty} \frac{1}{r^2}\, dr = 2\pi\rho_f R_B^4 \dot{R}_B^2 \frac{1}{R_B} = 2\pi\rho_f R_B^3 \dot{R}_B^2$$

- Invoking first law, where the change of KE should be equal to work done (under no heat transfer), we can write,

$$2\pi\rho_f R_B^3 \dot{R}_B^2 = \frac{4}{3}\pi\left(R_B^3 - R_0^3\right)\left(p_{sat}(T_W) - p_\infty\right)$$

- Differentiating the above expression, we get

$$2\pi\rho_f\left(R_B^3 2\dot{R}_B\ddot{R}_B + \dot{R}_B^2 3R_B^2\dot{R}_B\right) = 4\pi R_B^2\dot{R}_B\left(p_{sat}(T_W) - p_\infty\right)$$

- Simplifying, we get,

$$R_B\ddot{R}_B + \frac{3}{2}\dot{R}_B^2 = \frac{p_{sat}(T_W) - p_\infty}{\rho_f} \qquad \boxed{1}$$



- Using Clayperon's Equation,

$$\frac{dp}{dT} = \frac{h_{fg}}{T_{sat} v_g}$$

- Using linearized expansion, we can write

$$\frac{p_{sat}(T_W) - p_\infty}{\left(T_W - T_{sat}(p_\infty)\right)} = \frac{h_{fg}}{T_{sat} v_g}$$

$$\Rightarrow p_v - p_\infty = \frac{h_{fg}}{T_{sat} v_g}\left(T_W - T_{sat}(p_\infty)\right) \qquad \boxed{2}$$



- Eliminating the pressure difference from Eqs.(1) and (2), we can write

$$\left(R_B\ddot{R}_B + \frac{3}{2}\dot{R}_B^2\right) = \frac{h_{fg} v_f}{T_{sat} v_g}\left(T_W - T_{sat}(p_\infty)\right)$$

- At very low bubble radius, the first term is negligible and hence we can write,

$$\frac{3}{2}\dot{R}_B^2 = \frac{h_{fg} v_f}{T_{sat} v_g}\left(T_W - T_{sat}(p_\infty)\right)$$

$$\dot{R}_B = \sqrt{\frac{2}{3}\frac{h_{fg} v_f}{T_{sat} v_g}\left(T_W - T_{sat}(p_\infty)\right)}$$

- Since RHS is a constant, bubble grows linearly with time



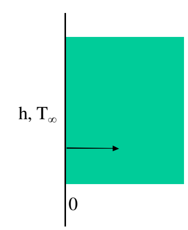
## Heat transfer Control

- Before we look at heat transfer control, let us look at heat transfer in semi-infinite plate
- We will link it to bubble dynamics a bit later
- The governing equation

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \quad 0 \le x \le \infty;\ 0 \le t$$

- Boundary conditions

$T(0,x) = T_i$ ; $T(t,0) = T_s$ ; $T(t,x\rightarrow\infty) = T_i$



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## 1-D Transient in a Semi-Infinite Plate-II

- Mathematically, problems that have boundary at infinity are often solved by a method called similarity solution
- In this method, a new variable, called similarity variable is introduced
- This variable is chosen such that T becomes a function of only this variable
- Thus the governing equation will be transformed into an ODE from PDE
- There are systematic ways by which this can be derived, but often involves some qualitative arguments

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## 1-D Transient in a Semi-Infinite Plate-III

- In this course we shall give you the form of the variable. Note that it will be a combination of x and t

$$\eta = \frac{x}{(4\alpha t)^{0.5}} \Rightarrow \frac{\partial \eta}{\partial x} = \frac{1}{(4\alpha t)^{0.5}}, \frac{\partial \eta}{\partial t} = \frac{x}{(4\alpha)^{0.5}}\frac{-0.5}{t^{1.5}}$$

- Using chain rule $\frac{\partial T}{\partial x} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial x} = \frac{dT}{d\eta}\frac{1}{(4\alpha t)^{0.5}}$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\frac{dT}{d\eta}\frac{1}{(4\alpha t)^{0.5}} = \frac{d^2T}{d\eta^2}\frac{\partial \eta}{\partial x}\frac{1}{(4\alpha t)^{0.5}}$$

$$= \frac{d^2T}{d\eta^2}\frac{1}{(4\alpha t)}$$

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial t} = \frac{dT}{d\eta}\frac{x}{(4\alpha)^{0.5}}\frac{-0.5}{(t)^{1.5}} = \frac{dT}{d\eta}\frac{-x}{2t(4\alpha t)^{0.5}} = \frac{dT}{d\eta}\frac{-\eta}{2t}$$

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## 1-D Transient in a Semi-Infinite Plate-IV

- Substituting for the partial derivatives in the heat equation, we get

$$\frac{1}{\alpha}\frac{dT}{d\eta}\frac{-\eta}{2t} = \frac{d^2T}{d\eta^2}\frac{1}{(4\alpha t)}$$

$$\Rightarrow \frac{d^2T}{d\eta^2} = -\frac{dT}{d\eta}2\eta$$

- Thus we get an ODE in η $\frac{d^2T}{d\eta^2} = -2\eta\frac{dT}{d\eta}$
- The boundary conditions

$T(t,0) = T_s \Rightarrow T(\eta = 0) = T_s$

$T(0,x) = T_i$ ; $T(t,x\to\infty) = T_i \Rightarrow T(\eta = \infty) = T_i$

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## 1-D Transient in a Semi-Infinite Plate-V

- The governing equation is $\frac{d^2T}{d\eta^2} = -2\eta\frac{dT}{d\eta}$
- To get the solution, we make the transformation

$$\frac{dT}{d\eta} = T^+ \Rightarrow \frac{d^2T}{d\eta^2} = \frac{dT^+}{d\eta}$$

- The equation in the new variables can be written as

$$\frac{dT^+}{d\eta} = -2\eta T^+ \qquad \Rightarrow \frac{dT^+}{T^+} = -2\eta d\eta$$

u is a dummy variable

- Integration gives $ln(T^+) = -\eta^2 + C \quad \Rightarrow T^+ = C_1 e^{-\eta^2}$

$$\Rightarrow \frac{dT}{d\eta} = C_1 e^{-\eta^2} \Rightarrow \int_{T_S}^{T} dT = T - T_S = C_1 \int_0^{\eta} e^{-\eta^2} d\eta = C_1 \int_0^{\eta} e^{-u^2} du$$

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## Error Function

- The integral $\int_0^{\eta} e^{-u^2} du$ occurs very frequently in physics
- Though not integrable in explicit form, it has been integrated with series expansion and tables have been constructed under what is called Error Function

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^{x} e^{-u^2} du$$

**erf(x) is tabulated in in many books**

**x=3 is as good as ∞**

**Erf(3) = 0.99998**

- It turns out that $\int_0^{\infty} e^{-u^2} du = \frac{\sqrt{\pi}}{2}$
- Hence erf(∞)=1, erf(0) = 0
- A complimentary error function is also defined as

  erfc(x) = 1 – erf(x)

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## Integration

Consider $I = \int_{\infty}^{\infty} \int_{\infty}^{\infty} e^{-(x^2+y^2)} dxdy = \int_0^{\infty} \int_0^{2\pi} e^{-(r^2)} r dr d\theta$

Put $r^2 = t$ $\Rightarrow I = \int_0^{\infty} \int_0^{2\pi} e^{-t} \frac{dt}{2} d\theta = \frac{2\pi}{2} \left( -e^{-t} \Big|_0^{\infty} \right) = \pi$

Note that $I = \int_{\infty}^{\infty} e^{-x^2} dx \int_{\infty}^{\infty} e^{-y^2} dy = 4 I_1^2, \; where \; I_1 = \int_0^{\infty} e^{-x^2} dx$

From above $4 I_1^2 = \pi \; or \; I_1 = \frac{\sqrt{\pi}}{2}$

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## 1-D Transient in a Semi-Infinite Plate-VI

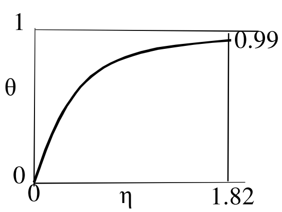
- The solution $T = C_1 \int_0^{\eta} e^{-u^2} du + T_S = C_1 \frac{\sqrt{\pi}}{2} erf(\eta) + T_S$
  - The Boundary condition, $T(\eta=\infty) = T_i$ implies

$$T_i = C_1 \frac{\sqrt{\pi}}{2} erf(\infty) + T_s \qquad \Rightarrow T_i = C_1 \frac{\sqrt{\pi}}{2} + T_s$$

$$\Rightarrow C_1 = (T_i - T_s) \frac{2}{\sqrt{\pi}}$$

$$\therefore T = (T_i - T_s) erf(\eta) + T_s$$

$$\Rightarrow \frac{T - T_s}{(T_i - T_s)} = erf(\eta) = \theta$$

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## 1-D Transient in a Semi-Infinite Plate-VII

- For η = 1.82, θ=0.99; This implies that η = 1.82 is for all practical purposes is ∞

$$\eta = 1.82 \Rightarrow \frac{x}{(4\alpha t)^{0.5}} = 1.82 \qquad \Rightarrow x = 3.64(\alpha t)^{0.5}$$

- The above numbers can be interpreted in the following manner
- $x > 3.64\ (\alpha t)^{0.5}$ can be considered as infinitely thick
- Similarly for $t < x^2/(13.25\ \alpha)$, the plate can be considered infinite
- Now we will turn our attention to heat transferred

# 1-D Transient in a Semi-Infinite Plate-VIII

$\frac{\partial T}{\partial x} = \frac{dT}{d\eta}\frac{1}{(4\alpha t)^{0.5}}$

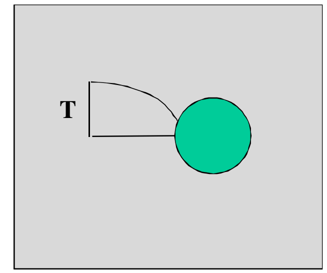
$$q'' = -k\left.\frac{\partial T}{\partial x}\right|_{x=0} = -k\left.\frac{dT}{d\eta}\right|_{\eta=0}\frac{1}{(4\alpha t)^{0.5}}$$

- We had shown that $\frac{dT}{d\eta} = C_1 e^{-\eta^2} \Rightarrow \left.\frac{dT}{d\eta}\right|_{\eta=0} = C_1 = (T_i - T_s)\frac{2}{\sqrt{\pi}}$

$$\therefore q'' = -k\frac{(T_i - T_s)}{(4\alpha t)^{0.5}}\frac{2}{\sqrt{\pi}} = k\frac{(T_s - T_i)}{(\pi\alpha t)^{0.5}}$$



- The heat transferred into the bubble can be viewed similar to semi-infinite plate as shown
- The infinite pool can be viewed as the semi-infinite region
- Rate of vaporization can be estimated by using the energy balance as




$$\frac{q'' 4\pi R_B^2}{h_{fg}} = \frac{dm_g}{dt} = \frac{d}{dt}\frac{4}{3}\rho_g \pi r R_B^3 = 4\pi\rho_g R_B^2\frac{dR_B}{dt}$$



$$\frac{dR_B}{dt} = \frac{q''}{h_{fg}\rho_g} = k_l\frac{(T_s - T_i)}{(\pi\alpha t)^{0.5}h_{fg}\rho_g}$$

$$\frac{dR_B}{dt} = \frac{k_l}{\rho_l c_{pl}}\frac{\rho_l c_{pl}(T_s - T_i)}{(\pi\alpha_l t)^{0.5}h_{fg}\rho_g} = Ja\sqrt{\frac{\alpha_l}{\pi t}} \qquad \boxed{1}$$

Where $Ja = \frac{\rho_l c_{pl}(T_s - T_i)}{h_{fg}\rho_g}$

Integration of Eq. (1) gives

$$R_B = 2Ja\sqrt{\frac{\alpha_l t}{\pi}} + c$$



- Using BC of $R_B = 0$ at $t = 0$, implies $c = 0$

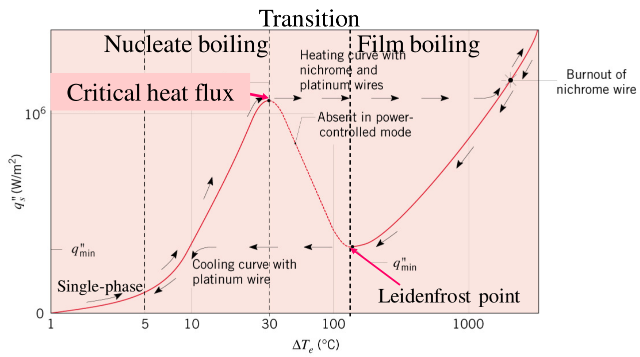
$$R_B = 2Ja\sqrt{\frac{\alpha_l t}{\pi}} \qquad \Rightarrow \dot{R}_B = Ja\sqrt{\frac{\alpha_l}{\pi t}}$$

- More complex analysis gives,

$$\Rightarrow R_B = 2\sqrt{3}Ja\sqrt{\frac{\alpha_l t}{\pi}} \qquad \Rightarrow \dot{R}_B = \sqrt{3}Ja\sqrt{\frac{\alpha_l}{\pi t}}$$

- Bubble grows as square root of time in the later half,
- Though $R_B$ and its time derivative are functions of time, its product does not depend on time. This was exploited by Forster and Zuber to arrive at a correlation for heat transfer

# Pool Boiling Heat Transfer-I




Boiling curve at 1 atm



# Pool Boiling Heat Transfer-II

- Free convection region $\Delta T_{Sat} < 5$ °C (single phase)
- Vapor formed at the free surface
- Onset of nucleation $\Delta T_{sat} \sim 5$ °C
- Bubbles nucleate, grow and detach from the surface
- Increase of wall superheat leads to more vigorous nucleation and rapid increase in heat transfer
- As the superheat is increased, the vapor formation become vigorous, it blankets the surface and the heat transfer decreases. This turn around point is called the Critical Heat Flux or Boiling crisis



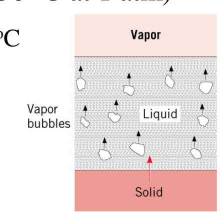
# Pool Boiling Heat Transfer-III

- As the superheat is increased, more blanketing causes the heat transfer to drop, till the entire heated surface is blanketed
- Now the radiation heat transfer also starts playing a role and eventually, the heat transfer starts increasing due to increase convection and radiation heat transfer
- The second turnaround point is called Leidenfrost point or rewetting point.
- The heat transfer beyond this point is called film boiling
- We shall briefly look at some details

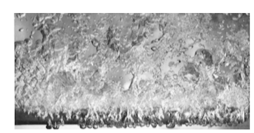


# Pool Boiling Heat Transfer-IV

- Nucleate Boiling (Wall super heat < 30 °C at 1 atm)
  - ➢ Isolated bubbles Region $5 < \Delta T_{Sat} < 10$ °C
    - Liquid motion is strongly influenced by nucleation of bubbles at the surface.
    - 'h' rapidly increases with wall superheat
    - Heat transfer is principally due to contact of liquid with the surface (single-phase convection) and not due to vaporization.

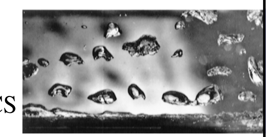



  - ➢ Jets and Columns Region ($10 < \Delta T_{Sat} < 30$ °C at 1 atm)
    - Increasing nucleation density causes bubbles to coalesce to form jets and slugs
    - Liquid wetting impaired
    - 'h' starts decreasing with increase in superheat



# Pool Boiling Heat Transfer-V

- Critical Heat Flux (Wall super heat ~30 $^oC$ at 1 atm)
  - Typically 1MW/m$^2$ at 1 atm and increases with pressure
  - If the wall pumps heat flux, there is a potential for the wall to melt as the heat transfer coefficient is very low here due to vapor blanketing.
- Film Boiling (Wall super heat >120 $^oC$ at 1 atm)
  - Heat transfer by conduction and radiation across vapor blanket
  - Usually not a preferred mode of cooling but can occur during the ECCS injection in an uncovered core





# Pool Boiling Heat Transfer-VI

- Transition Boiling ( 30 $^oC$ < $\Delta$Tsat < 120 $^oC$ at 1 atm)
  - Called Unstable film boiling
  - Surface conditions oscillate between nucleate and film boiling.
- Boiling Heat Transfer Correlations
  - Different models exist and there is no single view on this aspect.
  - We shall just list some correlations for application purposes



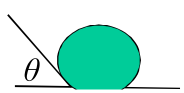
# Models for Nucleate Boiling-I

- Taking cue from turbulent convection heat transfer promoted by the movement of eddies, nucleate boiling heat transfer can also be represented by the classic Dittus-Boelter type equation

$$\Rightarrow Nu = A\,Re^n\ Pr^m$$

- To properly account for bubble scales, Rohsenow used departure bubble diameter as the length scale

$$\Rightarrow L_b \propto \theta \sqrt{\frac{2\sigma}{g(\rho_l - \rho_g)}}$$




**Ref: Von Carey, Liquid-Vapor Phase-Change Phenomenon**



# Models for Nucleate Boiling-II

- If one performed an energy balance on the heated surface, one can write a bubble velocity scale as

$$G h_{fg} = q'' \qquad \Rightarrow V_b \rho_g h_{fg} = q''$$

$$\Rightarrow V_b = \frac{q''}{\rho_g h_{fg}}$$

- Experiments suggest that subcooling had little influence on the heat transfer coefficient. Hence the $T_{sat}$ rather than $T_B$ is used as the temperature scale

$$\Rightarrow h = \frac{q''}{T_W - T_{sat}}$$

# Models for Nucleate Boiling-III

- As the Reynolds number has to be that of liquid that cools the surface

$$\Rightarrow Re_l = \frac{GL_b}{\mu_l}$$

- From the previous slide,

$$Gh_{fg} = q''$$

- As this mass flux can be equally viewed from either the liquid or the vapour standpoint, $Re_l$ is defined as

$$\Rightarrow Re_l = \frac{\rho_g V_b L_b}{\mu_l}$$



# Models for Nucleate Boiling-IV

- Defining Nu to be $Nu = \frac{hL_b}{k_l}$ Rohsenow used

$$Nu = A\,Re^{1-r}\,Pr^{1-s}.$$

- Plugging in the expressions for each non-dimensional number as stated, he arrived at the corrrelation

$$\frac{q''}{\mu_l h_{fg}} = \frac{1}{(C_{sf})^{1/r}} \left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{0.5} Pr_l^{-s/r} \left[\frac{c_{pl}\Delta T_{sat}}{h_{fg}}\right]^{1/r}$$

- In the above equation,

$$C_{sf} = \frac{C\theta}{A}, \quad r = 0.33, \quad s = 1.7$$



# Models for Nucleate Boiling-V

- $C_{sf}$ is an empirical constant found by experiment and is tabulated in handbooks. Typically its value is 0.013. Some have recommended value of s=1
- An alternate route has been followed by Forster and Zuber
- They used scales derived from bubble dynamics

$$D_{scale} = R_c \left[\frac{\rho_f \dot{R}_B^{\,2}}{2\sigma / R_c}\frac{R_B^{\,2}}{R_c^{\,2}}\right]^{0.25} \qquad Nu = \frac{q''}{(T_W - T_l)}\frac{D_{scale}}{k_l}$$

- Similarly, Re was scaled as

$$Re = \frac{\rho_f \dot{R}_B R_B}{\mu_l}$$



# Models for Nucleate Boiling-VI

- By substituting for the bubble Radius and Bubble velocity as derived earlier, we can write

$$Re = \pi\ Ja^2\, Pr_l^{-1}$$

- $R_C$ from our previous derivation can be written as

$$R_c = \frac{2\rho}{(p_{sat}(T_W) - p_{sat}(T_l))}$$

- Using the above definitions, Forster and Zuber fitted experimental data into the form

$$\Rightarrow Nu = A\,Re^n\,Pr^m$$

A= 0.0015, n=0.62, m = 0.33

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## Models for Nucleate Boiling-VII

- The above equation when substituted with appropriate scales reduce to the following equation

$$\frac{q''}{T_W - T_l} = 0.00122 \frac{\Delta T_{sat}^{\,0.24} \Delta p_{sat}^{\,0.75} c_{pl}^{\,0.45} \rho_l^{\,0.49} k_l^{\,0.79}}{\sigma^{0.5} h_{fg}^{\,0.24} \mu_l^{\,0.29} \rho_g^{\,0.24}}$$

- There are other correlations. At times different correlations are off by as much as 100% and one needs to be careful when sizing surface area for HX.

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## Models for CHF-I

- Critical heat flux is the limiting factor in the operation of heat flux driven systems (nuclear)
- Kuteteladze, based on his Russian work gave an empirical relation

$$q'' = 0.131 \rho_g h_{fg} \left( g(\rho_f - \rho_g)\sigma \right)^{0.25}$$

- Based on the work of Zuber, Lienhard, we can construct the following physics
- CHF is governed by Rayleigh-Taylor as well as Kelvin-Helmholtz Instabilities,

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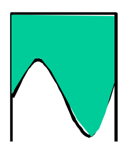
## Models for CHF-II

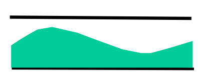
- Based on inviscid flow assumption, one can show that Taylor wave length to be

$$\lambda_T = 2\pi \sqrt{\frac{3\sigma}{\left( g(\rho_f - \rho_g) \right)}}$$



- Similarly, one can derive the Helmholtz wave length to be under the assumption that $u_g >> u_l$, $\rho_l >> \rho_g$

$$\lambda_H = \frac{2\pi\sigma}{\rho_g u_g^{\,2}}$$



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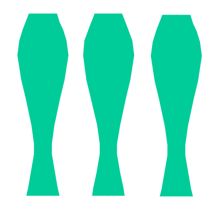
## Models for CHF-III

- Performing energy balance, we get

$$\lambda_T^{\,2} q'' = \dot{m}_g h_{fg}$$

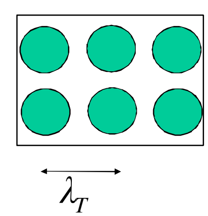
$$\dot{m}_g = \rho_g u_g \frac{\pi}{4} d^2$$



- Assuming $d = \lambda_T/2$, and from the above two equations, we can write

$$u_g = \frac{16}{\pi} \frac{q''}{\rho_g h_{fg}}$$

- Also, $u_g = \sqrt{\dfrac{2\pi\sigma}{\rho_g \lambda_H}}$



# Models for CHF-IV

- Substituting for $u_g$ from the last equation into the one above, assuming $\lambda_H = \lambda_T$, and substituting the expression for $\lambda_T$, Lienhard derived

$$q'' = 0.149 \rho_g h_{fg} \left( g( \rho_f - \rho_g )\sigma \right)^{0.25}$$

- Earlier using similar arguments and using p/d $= \sqrt{6}$ Zuber derived,

$$q'' = 0.131 \rho_g h_{fg} \left( g( \rho_f - \rho_g )\sigma \right)^{0.25}$$

- This is same as Kuteteladze's experimental fit.
- Lienhard's model is assumed to be a better fit than Zuber's. Many more corrections are available



# Models for CHF-V

- For the minimum critical heat flux at the Leidenfrost's point, correlations are available

$$q'' = 0.09 \rho_g h_{fg} \left[ \frac{g( \rho_f - \rho_g )\sigma}{( \rho_f - \rho_g )^2} \right]^{0.25}$$

**Berenson, 1961**

- Several others are available. See Collier and Thome, Carey, Tong and Tang, etc.

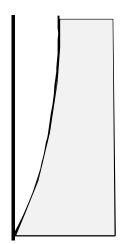


# Film Boiling-I

- Similar to Nusselt Condensation, we can derive

$$Nu_x = 0.707 \left[ \frac{\rho_g ( \rho_l - \rho_v ) h_{fg} g x^3}{k_g \mu_g ( T_w - T_{sat} )} \right]^{1/4}$$

$$h_x = 0.707 \left[ \frac{\rho_g ( \rho_l - \rho_v ) h_{fg} g k_g^3}{\mu_g ( T_w - T_{sat} ) x} \right]^{1/4}$$



- We can derive the average h as done before
- During film boiling radiation heat transfer also plays an important role



# Film Boiling-II

$$h_{rad} = \frac{\sigma \rho_g ( T_w^2 + T_{sat}^2 )( T_w + T_{sat} )}{\frac{1}{\varepsilon_{wall}} + \frac{1}{\varepsilon_{interface}} \quad 1} (\ )^{1/4}$$

- Since radiation and convection is present simultaneously, they get coupled.
- Explicit for of effective heat transfer coefficient can be obtained by using the following relations

$$h_{overall} = h_{convection} + 0.75\, h_{radiation}$$

**For $h_{rad} < h_{conv}$**

# Film Boiling-III

**For $0 < h_{rad} / h_{conv} < 1$**

$$h_{overall} = h_{conv} + h_{rad} \quad 0.75 + 0.25 \frac{h_{rad}}{h_{conv}} \quad \frac{1}{2.62 + \frac{h_{rad}}{h_{conv}}}$$

- Convection has to be modified for turbulent conditions and relations are available