EN644 Two-Phase Flow and Heat Transfer
Assignment -4

1. Consider a simple problem to illustrate the development of empirical relations for a hypothetical One dimensional model.. The problem is a bit lengthy but very illustrative (I guess). Consider flow of air-liquid between two horizontal parallel plates separated by a distance D, with air flowing on top of the liquid (stratified flow). The flow may be assumed to be fully developed and laminar. In such a case, the exact axial momentum equation for liquid and gas can be simplified to give $\frac{dp_g}{dx} = \mu_g \frac{d^2 u_g}{dy^2}, \frac{dp_l}{dx} = \mu_l \frac{d^2 u_l}{dy^2}$. In the above formulation, u_g and u_l are functions of y alone and p is only a function of x. Note that x is along the plates and y is normal to the plates. Since dp/dx is only a function of x and $\mu_i \frac{d^2 u_i}{dy^2}$ is only a function of y, each of them will be a constant (denoted as dp/dx). Thus u_g and u_l can be shown to be of the form $u_i = (dp/dx)\, y^2/2 + a_{1i}\, y + a_{2i}$. The values of the four constants are obtained using four conditions, viz., the velocities at wall shall be 0 and the velocity and the shear stress at the interface ($y = y_{interface}$) shall be continuous. Thus we have found the velocity profiles in terms of two unknowns, viz., dp/dx and $y_{interface.}$ These can be evaluated by integrating the velocity profiles and equating them to the respective phase volumetric flow rates per unit width, viz., Q_g and Q_l. This completes the formulation. Now having carried out the algebra, try to show that dp/dx is only a function of χ. Note that for only gas are liquid flowing, the profile will be parabolic and dp/dx can be obtained accordingly as a function of Q_g or Q_l. If possible, obtain the exact function. Try to express β / α in the drift flux form.
2. Consider an experimental loop in which saturated liquid at 5 bar is pumped into the test section. The hydraulic diameter and the flow cross sectional area of the test section is 6 mm and 22 cm^2. If the test section, which is oriented vertically, is 6 m long and is heated with a linear heat rate of 10 kW/m, estimate the accelerational, frictional and gravitational pressure gradient using (a) homogeneous model (b) slip flow model with CISE (Premoli) correlation for void closure and Friedel correlation for frictional closure. (c) X_{tt} method for void closure and B Chisholm coefficient method done in the class for friction closure. The required correlations are given at the end of this assignment
3. Starting from the definition of χ^2, show that

$$\chi^2 = \left(\frac{1-x}{x}\right)^{2-n} \left(\frac{\mu_l}{\mu_g}\right)^n \frac{\rho_g}{\rho_l}$$

Notice that square root of this expression is used in previous problem for X_{tt} with n = 0.2

Further, using the definitions of ϕ_{lo}^2 and ϕ_{ls}^2, show that

$$\phi_{lo}^2 = \phi_{ls}^2 (1-x)^{2-n}$$

4. Numerically obtain the variation of $\tilde{h}_l$ with χ and compare it with the Taitel's result at a few points for turbulent gas and turbulent liquid. The figure is given at the end of the assignment

Premoli

The equations of Premoli et al. [1971] are given in terms of the slip ratio S to be applied to Equation 12.

$$S = 1 + F_1 \left[\frac{y}{1 + yF_2} - yF_2\right]^{1/2} ,$$

where

$$F_1 = 1.578\ \mathrm{Re}_L^{-0.19}(\rho_f/\rho_g)^{0.22} ,$$
$$F_2 = 0.0273\ We_L \mathrm{Re}_L^{-0.51}(\rho_f/\rho_g)^{-0.08} ,$$
$$y = \frac{\beta}{1 - \beta} ,$$

and

Re_L = liquid Reynolds number, $\dfrac{GD_i}{\mu_f}$,

We_L = liquid Weber number, $\dfrac{G^2 D_i}{\sigma \rho_f g_c}$,

σ = surface tension,

g_c = gravitational constant,

β = volumetric quality, $\dfrac{1}{1 + \left(\dfrac{1 - x}{x}\right)\left(\dfrac{\rho_g}{\rho_f}\right)}$

2.4. Friedel model

The last correlation considered is the Friedel correlation (1979), given as

$$\Phi_{\mathrm{Lo}}^2 = E + \frac{3.24FH}{Fr_{\mathrm{h}}^{0.045} We_{\mathrm{L}}^{0.035}}$$

where

$$Fr_{\mathrm{h}} = \frac{G^2}{gD_H \rho_{\mathrm{h}}^2}$$

$$E = (1 - x)^2 + x^2 \frac{\rho_{\mathrm{L}} f_{\mathrm{G}}}{\rho_{\mathrm{G}} f_{\mathrm{L}}}$$

$$F = x^{0.78}(1 - x)^{0.224}$$

$$H = \left(\frac{\rho_{\mathrm{L}}}{\rho_{\mathrm{G}}}\right)^{0.91} \left(\frac{\mu_{\mathrm{G}}}{\mu_{\mathrm{L}}}\right)^{0.19} \left(1 - \frac{\mu_{\mathrm{G}}}{\mu_{\mathrm{L}}}\right)^{0.7}$$

The liquid Weber number We_{L} is defined as

$$We_{\mathrm{L}} = \frac{G^2 D_H}{\sigma \rho_{\mathrm{h}}}$$

with the homogeneous density ρ_{h} given as

$$\frac{1}{\rho_{\mathrm{h}}} = \left(\frac{x}{\rho_{\mathrm{G}}} + \frac{1 - x}{\rho_{\mathrm{L}}}\right)$$

X_{tt}-Correlated

Another group of correlations avoids the use of a form of the homogeneous equation by employing the Lockhart-Martinelli (L-M) correlating parameter X_{tt} defined as:

$$X_{tt} = \left(\frac{1 - x}{x}\right)^{0.9} P.I._2^{0.5} \ . \tag{16}$$

Lockhart-Martinelli. The well-known early L-M pressure drop work [1949] presented void fraction data as a function of X_{tt} on two-phase/two-component adiabatic flows near atmospheric conditions. These data were approximated by equations developed by Wallis [1969] and refined by Domanski and Didion [1983] for $X_{tt} > 10$. The equations are:

$$\alpha = f(X_{tt})$$

$$= (1 + X_{tt}^{0.8})^{-0.378} \qquad \text{for } X_{tt} \leq 10 \ , \tag{17}$$

$$= 0.823 - 0.157 \ln X_{tt} \qquad \text{for } X_{tt} > 10 \ . \tag{18}$$

$$P.I._2 = \left(\frac{\mu_f}{\mu_g}\right)^{0.2} \cdot \frac{\rho_g}{\rho_f}$$