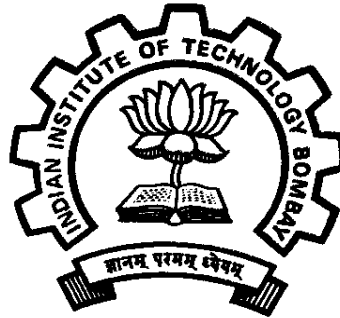


Solution Strategy for Single-Phase Governing Equations

Lecture-3

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Final Set of Equations

Mass Balance

$$\frac{\partial(A\rho)}{\partial t} + \frac{\partial(\dot{m})}{\partial s} = 0$$

Momentum Balance

$$-\frac{\partial p}{\partial s} = \frac{1}{A} \frac{\partial(\dot{m})}{\partial t} + \frac{1}{A} \frac{\partial(\rho A u^2)}{\partial s} + \tau_w \frac{P}{A} + \rho g \frac{\partial H}{\partial s}$$

Energy Balance

$$\rho A \frac{\partial(h)}{\partial t} + \rho A u \frac{\partial(h)}{\partial s} = u A \frac{\partial p}{\partial s} + A \frac{\partial p}{\partial t} + q''_{\text{surface}} P - \frac{\partial(q''_{\text{axial}} A)}{\partial s} - W'$$

Solution Strategy-I

	independent	dependent
Variables	A, H, P, t, s	$\rho, u, \dot{m}, p, \tau_w, h, q''_{\text{surface}}, q''_{\text{axial}}$
Equations	3	u, p, h chosen as independent
Closing relations	5	

$$\rho = \rho(p, h) \quad q''_{\text{surface}} = q''_{\text{surface}} \text{ (specified / } u, \text{ properties)}$$

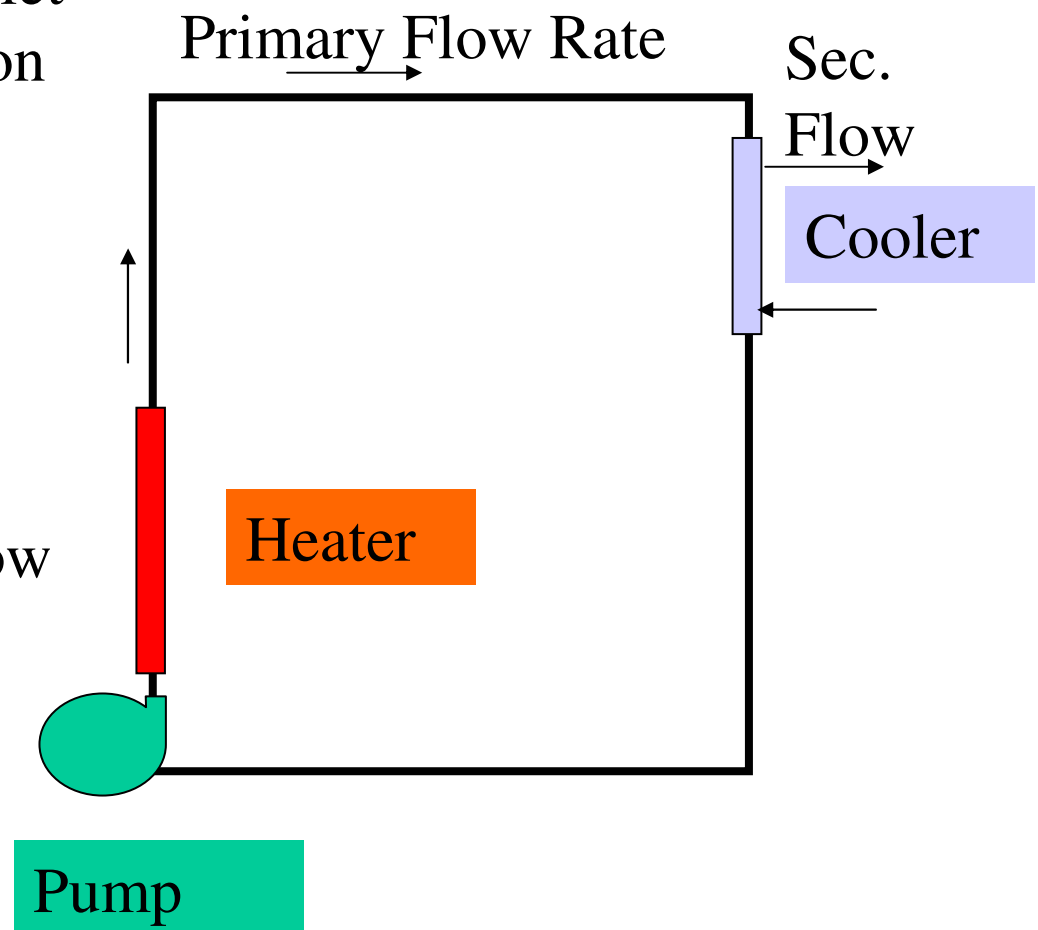
$$\dot{m} = \rho A u \quad q''_{\text{axial}} = \text{negligible}$$

$$\tau_w = \tau_w(\rho, \text{geometry}, u, \text{viscosity})$$

system is mathematically closed

Illustrative Application

- To illustrate the application let us consider forced convection loop
- For a given heater power, assuming large secondary flow in cooler, and a given pump characteristics, estimate, (a) steady mass flow rate circulating, (b) fluid temperature distribution



Application-I

Mass Balance

$$\cancel{\frac{\partial(A\rho)}{\partial t}} + \frac{\partial(\dot{m})}{\partial s} = 0$$



Mass flow rate is constant
along the duct

For incompressible fluid, even under transient,
instantaneous mass flow rate does not vary along the
length.

Application-II

Momentum Balance

Pressure term

Transient term

Acceleration term

Friction term

Gravitation term

$$-\frac{\partial p}{\partial s} = \frac{1}{A} \frac{\partial(\dot{m})}{\partial t} + \frac{1}{A} \frac{\partial(\rho A u^2)}{\partial s} + \tau_w \frac{P}{A} + \rho g \frac{\partial H}{\partial s} \quad 11$$

We shall integrate term by term over the entire loop

Pressure term

$$-\oint_{\text{loop}} \frac{\partial p}{\partial s} ds = 0$$

Transient term

$$\oint_{\text{loop}} \frac{1}{A} \frac{\partial(\dot{m})}{\partial t} ds = \frac{d\dot{m}}{dt} \oint_{\text{loop}} \frac{ds}{A} = \frac{d\dot{m}}{dt} \sum_{\text{all links}} \frac{L_i}{A_i}$$

Application-III

Acceleration
term

$$\sum_{\text{all links}} \int \frac{1}{A} \frac{\partial(\rho A u^2)}{\partial s} ds = \sum_{\text{all links}} \int \frac{\partial \left(\frac{\dot{m}^2}{\rho A^2} \right)}{\partial s} ds$$

$$= \frac{\dot{m}^2}{\rho} \sum_{\text{all links}} \left(\frac{1}{A_{i+1}^2} - \frac{1}{A_i^2} \right) = 0$$

$$\oint_{\text{loop}} \tau_w \frac{P}{A} ds = \oint_{\text{loop}} \frac{\rho u^2}{2} f \frac{4}{d_{\text{hyd}}} ds = \frac{4\dot{m}^2}{2\rho} \sum_{i=\text{all links}} \frac{1}{A_i^2} \left(\frac{f_i L_i}{d_{\text{hyd}-i}} + \frac{K_i}{4} \right)$$

$$\text{where, } f = \frac{16}{\text{Re}} \quad \forall \text{Re} < 1189.4$$

$$= 0.079(\text{Re})^{-0.25} \quad \forall \text{Re} > 1189.4$$

Friction term

Application-IV

Gravitation
term

$$\oint_{\text{loop}} \rho g \frac{\partial H}{\partial s} ds = \rho g \sum_{\text{all links}} (H_{\text{exit}-i} - H_{\text{inlet}-i}) = 0$$

For pump link

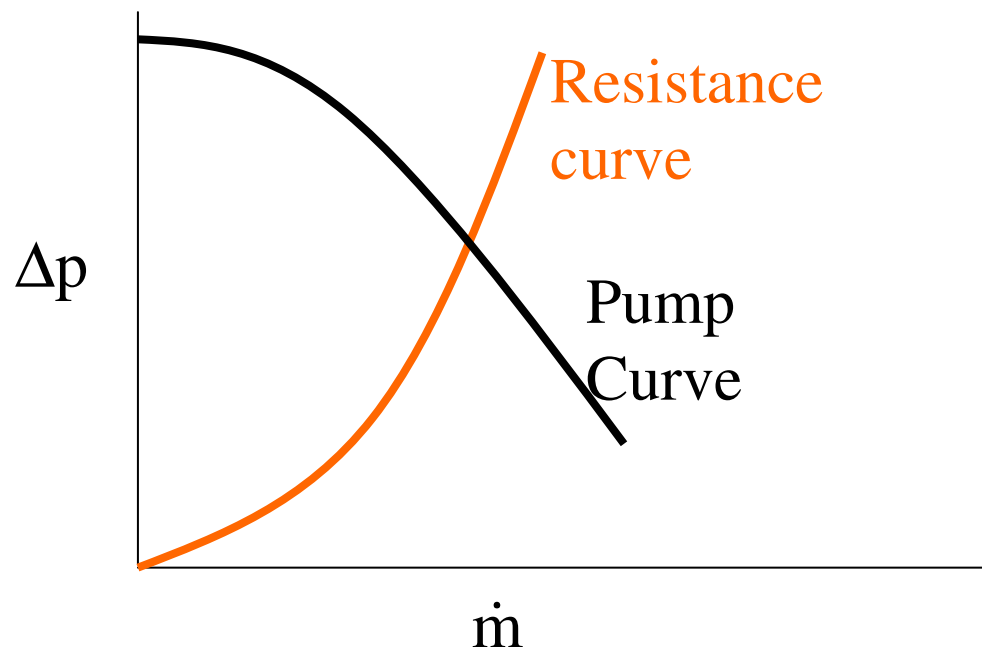
$$-\Delta p_{\text{pump}} = -\rho g H_{\text{pump}}$$

Equating pressure term to sum of all the other terms, we get

$$\frac{d\dot{m}}{dt} \sum_{\text{all links}} \frac{L_i}{A_i} + \frac{4\dot{m}^2}{2\rho} \sum_{i=\text{all links}} \frac{1}{A_i^2} \left(\frac{f_i L_i}{d_{\text{hyd}-i}} + \frac{K_i}{4} \right) - \rho g H_{\text{pump}} = 0$$

Application-V

- The integrated momentum equation has only one unknown viz., $\dot{m}$
- For a given initial condition, the solution can be marched in time. Any standard procedure for solving ODE can be employed
- For steady situations, the First term drops out and any standard procedure for solving non-linear equations can be employed



Application-VI

Energy Balance

$$\rho A \frac{\partial(h)}{\partial t} + \rho A u \frac{\partial(h)}{\partial s} = u A \frac{\partial p}{\partial s} + A \frac{\partial p}{\partial t} + q''_{\text{surface}} P - \frac{\partial(q''_{\text{axial}} A)}{\partial s} - W' + \tau_w P u$$

For steady case

$$\rho A \frac{\partial(h)}{\partial t} + \rho A u \frac{\partial(h)}{\partial s} = u A \frac{\partial p}{\partial s} + q''_{\text{surface}} P - W' + \tau_w P u$$

small

For non-pump links

$$\dot{m} c_p \frac{dT}{ds} = q''_{\text{surface}} P$$



Linear variation for constant heat flux case and exponential for constant U and T_{amb} case.

Natural Circulation-I

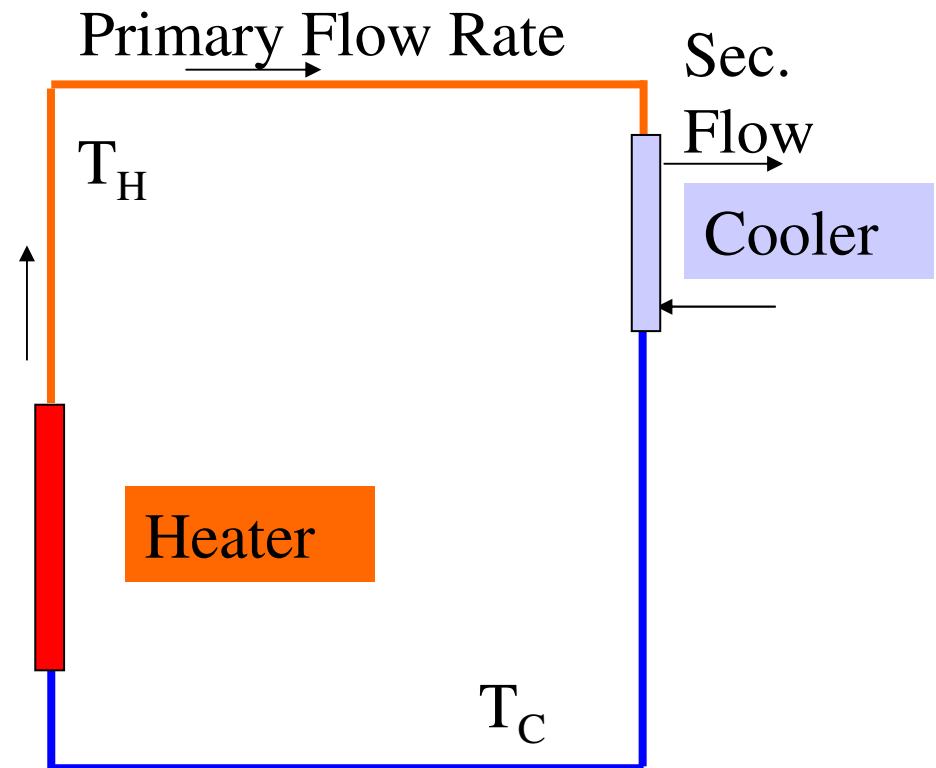
The Flow is driven by density variations and is also called Gravity Driven Flow or Thermosyphon Flow.

In these flows, momentum and energy are coupled and so need to be simultaneously solved.

Assumptions

Density only a function of Temperature. Usually assumed linear.

$$\rho = \rho_R (1 - \beta(T - T_R))$$



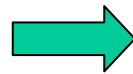
Natural Circulation-II

Boussinesq Approximation is valid. This implies that density variation with temperature need to be accounted for only in body force term of the momentum equation.

Transport properties (μ, c_p, k) are constant.

Mass Balance

$$\cancel{\frac{\partial(A\rho)}{\partial t}} + \frac{\partial(\dot{m})}{\partial s} = 0$$



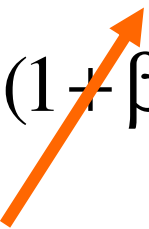
Mass flow rate is constant along the duct

Natural Circulation-III

Momentum Balance

The only difference is the integration of the gravitation term.

Gravitation
term

$$\oint_{\text{loop}} \rho g \frac{\partial H}{\partial s} ds = \oint_{\text{loop}} \rho_R (1 - \beta(T - T_R)) g dH$$
$$= \oint_{\text{loop}} \rho_R g (1 + \beta T_R) dH - \oint_{\text{loop}} \rho_R g \beta T dH$$


Thus, the integrated momentum equation leads to

Natural Circulation-IV

$$\frac{d\dot{m}}{dt} \sum_{\text{all links}} \frac{L_i}{A_i} + \frac{4\dot{m}^2}{2\rho_R} \sum_{i=\text{all links}} \frac{1}{A_i^2} \left(\frac{f_i L_i}{d_{\text{hyd}-i}} + \frac{K_i}{4} \right) - \oint_{\text{loop}} \rho_R g \beta T dH = 0$$

This equation cannot be solved unless the temperature distribution is obtained. Let us consider only steady state.

Energy Balance

$$\dot{m} c_p \frac{dT}{ds} = q''_{\text{surface}} P$$

In heater

$$T = T_C + \frac{q''_{\text{surface}} P}{\dot{m} c_p} s$$

Natural Circulation-V

Post heater

$$T = T_H$$

In cooler

$$\dot{m}c_p \frac{dT}{ds} = U(T - T_\infty)P$$

Integration leads to

$$\frac{T - T_\infty}{T_H - T_\infty} = e^{-\frac{UPs}{\dot{m}c_p}}$$

Post cooler

$$T = T_C$$

Noting that $T = T_H$ in heater when $s = L_H$ and $T = T_C$ in cooler when $s = L_c$ and elimination of T_C from the two relation leads to

Natural Circulation-VI

$$(T_H - T_\infty) = \frac{q'' P_H L_H}{\dot{m} c_p} \frac{1}{\left(1 - e^{-\frac{U P_C L_C}{\dot{m} c_p}} \right)}$$

Solution Procedure

1. Assume $\dot{m}$
2. Compute temperature profiles
3. Check if integrated momentum equation is satisfactory
4. Repeat till convergence