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Given:

$$h \text{ at } T_{ref} = 50^\circ\text{C}$$

$$= 209.42 \text{ kJ/kg (from tables)}$$

Along the constant pressure line 11*

$$dh = c_p dT + \left(v - T \left. \frac{\partial v}{\partial T} \right|_p \right) dp$$

$$\Rightarrow dh = c_p dT$$

$$\text{Thus } \int_{h_1}^{h_1^*} dh = \int_{T_1}^{T_1^*} c_{p_f} dT$$

$$h_1^* - h_1 = \int_{50}^{100} (8.41056 \times 10^{-6} T^2 - 5.81708 \times 10^{-4} T + 4.18952) dT$$

$$= \left[\frac{8.41056 \times 10^{-6}}{3} T^3 - \frac{5.81708 \times 10^{-4}}{2} T^2 + 4.18952 T \right]_{50}^{100}$$

$$= 209.75 \text{ kJ/kg}$$

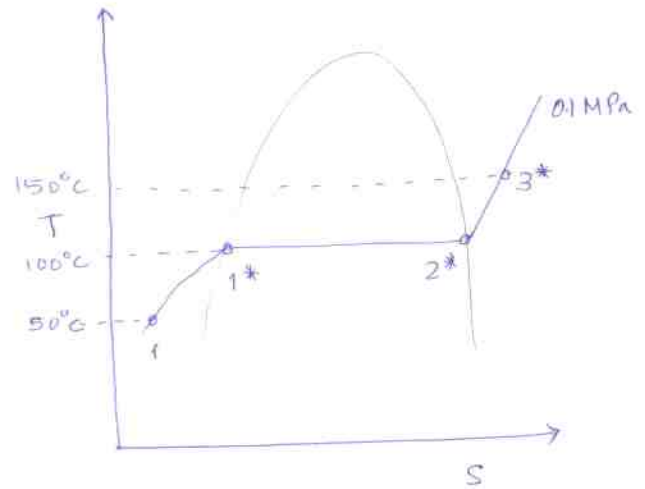
$$\Rightarrow h_1^* = 209.75 + 209.42$$

$$h_1^* = 419.17 \text{ kJ/kg}$$

Along the phase change 1*2*

Clarius Clapeyron Equation.

$$\left. \frac{dp}{dT} \right|_{T_{sat}} = \frac{h_{fg}}{T v_{fg}}$$



$$\Rightarrow \left. \frac{dp}{dT} \right|_{T_{\text{sat}}} \times T v_{fg} = h_{fg}$$

$$P = 4.99293 \times 10^{-7} T^3 - 9.69273 \times 10^{-5} T^2 + 7.99924 \times 10^{-3} T - 0.228425$$

for $(80 < T < 150^\circ\text{C})$

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$$\begin{aligned} \left. \frac{dp}{dT} \right|_{T_{\text{sat}}} &= (4.99293 \times 10^{-7} \times 3) T^2 - (9.69273 \times 10^{-5} \times 2) T + 7.99924 \times 10^{-3} \Big|_{T=100^\circ\text{C}} \\ &= 0.003593 \text{ MPa/}^\circ\text{C} \\ &= 3.593 \text{ kPa/}^\circ\text{C} \end{aligned}$$

$$\begin{aligned} \text{and } \left. v_{fg} \right|_{T_{\text{sat}}} &= -1.72712 \times 10^{-5} T^3 + 6.35816 \times 10^{-3} T^2 - 0.809280 T + 36.2854 \Big|_{T=100^\circ\text{C}} \\ &= 1.6678 \text{ m}^3/\text{kg} \end{aligned}$$

$$\text{thus. } h_{fg} = 3.593 \times 1.6678 \times 373$$

~~$$h_{fg} = 2235.17 \text{ kJ/kg}$$~~

$$h_{fg} = 2234.9 \text{ kJ/kg}$$

$$h_2^* = h_1^* + h_{fg}$$

$$h_2^* = 2654.07 \text{ kJ/kg}$$

Along the const for line 2*3*

$$dh = c_p dT$$

$$\text{and } h_3^* = h_2^* + \int_{T_2^*}^{T_3^*} c_{pg} dT$$

$$= 2654.07 + \int_{100}^{150} (-1.61558 \times 10^{-7} T^3 + 8.85428 \times 10^{-5} T^2 - 1.62498 \times 10^{-2} T + 2.97423) dT$$

$$= 2654.07 + \left[\left(\frac{-1.61558 \times 10^{-7}}{4} \right) T^4 + \left(\frac{8.85428 \times 10^{-5}}{3} \right) T^3 - \frac{1.62498 \times 10^{-2}}{2} T^2 + 2.97423 T \right]_{100}^{150}$$

$$= 2654.07 + 100.8384$$

$$h_3^* = 2754.91 \text{ kJ/kg}$$

Comparison

	calculated (kJ/kg)	Steam tables (kJ/kg)	% deviation
h_1^*	419.17	417.5	0.4%
h_2^*	2654.07	2675.5	-0.8%
h_3^*	2754.91	2776.4	-0.77%

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Along the isotherm $z^* - z^{**}$

$$dh = cp dT + \left(v - T \left. \frac{\partial v}{\partial T} \right|_p \right) dp$$

and PVT relation specified is

Redlich Kwong relation

$$P = \frac{RT}{(v-b)} - \frac{a/T^{0.5}}{v(v+b)}$$

diff. with respect to 'T' at P = const

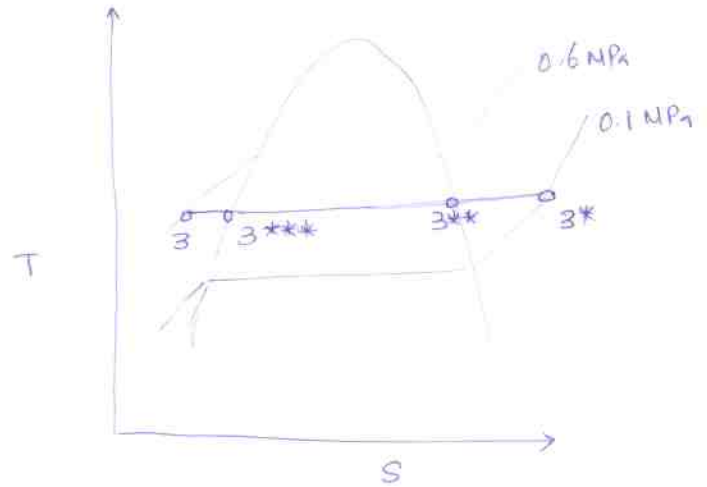
$$0 = R \left[\frac{(v-b) - T \left(\frac{\partial v}{\partial T} \right)}{(v-b)^2} \right] - a \left\{ \frac{- \left[T^{1/2} (2v+b) \frac{\partial v}{\partial T} + (v^2+bv) \frac{1}{2} T^{-1/2} \right]}{(v^2+bv)^2 T} \right\}$$

⇒

$$R \left[\frac{1}{(v-b)} - \frac{T \left(\frac{\partial v}{\partial T} \right)}{(v-b)^2} \right] = \frac{-a}{(v^2+bv)^2 T} \left\{ (2v+b) T^{1/2} \frac{\partial v}{\partial T} + (v^2+bv) \frac{1}{2} T^{-1/2} \right\}$$

$$\frac{R}{(v-b)} + \frac{a}{(v^2+bv)^2 T^{3/2}} = \frac{\partial v}{\partial T} \left[\frac{RT}{(v-b)^2} - \frac{a(2v+b)}{(v^2+bv)^2 T^{1/2}} \right]$$

$$\left. \frac{\partial v}{\partial T} \right|_p = \left[\frac{\frac{R}{(v-b)} + \frac{a}{2v(v+b)T^{3/2}}}{\frac{RT}{(v-b)^2} - \frac{a(2v+b)}{(v^2+bv)^2 T^{1/2}}} \right]$$



$$v_g \Big|_{T=150^\circ\text{C}} = -876.92 P^5 + 1468.4 P^4 - 981.15 P^3 + 333.07 P^2 - 60.25 T P + 5.4654 \quad (\text{m}^3/\text{kg}) \text{ and } P \text{ in MPa.}$$

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As the expression is complex, it is integrated numerically using Trapezoidal integration, choosing $dp = 0.1 \text{ MPa}$

which yields:

$$\int_{h_{3^*}}^{h_{3^{**}}} dh = \int_{P_{3^*}}^{P_{3^{**}}} \left(v - T \left. \frac{\partial v}{\partial T} \right|_P \right) dp.$$

$$= -6.229 \text{ kJ/kg}$$

thus

$$h_{3^{**}} = 2754.91 - 6.229$$

$$h_{3^{**}} = 2748.68 \text{ kJ/kg}$$

Along the (isothermal) phase change line $3^{**} - 3^{***}$

$$h_{fg} = \left. \frac{\partial p}{\partial T} \right|_{T_{\text{sat}}} \times T \times v_{fg}$$

$$P = 8.56688 \times 10^{-7} T^3 - 2.53198 \times 10^{-4} T^2 + 3.09515 \times 10^{-2} T - 1.36108$$

$$\left. \frac{\partial P}{\partial T} \right|_{T=150^\circ\text{C}} = (8.56688 \times 10^{-7} \times 3 \times T^2) - (2.53198 \times 10^{-4} \times 2T)$$

$$= 0.01282 \text{ MPa/deg C.}$$

$$= 12.82 \text{ kPa/}^\circ\text{C}$$

$$v_{fg} = -1.88453 \times 10^{-6} T^3 + 1.01210 \times 10^{-3} T^2 - 0.186313 T + 11.9252 \Big|_{T=150^\circ\text{C}}$$

$$= 0.3902 \text{ m}^3/\text{kg}$$

$$h_{fg} = 12.82 \times 0.3902 \times 423$$

$$h_{fg} = 2115.82 \text{ kJ/kg}$$

$$h_3^{***} = 2748.68 - 2115.82$$

$$h_3^{***} = 632.86 \text{ kJ/kg}$$

Along the isotherm 3*** - 3

as v is just a function of T .

$$h_3 \int_{h_3^{***}} dh = \left(v - T \frac{\partial v}{\partial T} \Big|_{P_3} \right) \int_{P_3^{***}}^{P_3} dp.$$

$$\frac{\partial v}{\partial T} \Big|_{T=150^\circ} = \left(3.42455 \times 10^{-9} \times 2T \right) + \left(9.67286 \times 10^{-8} \right) \Big|_{T=150^\circ\text{C}}$$
$$= 1.554 \times 10^{-05} \frac{\text{m}^3}{\text{kg } ^\circ\text{C}}$$

$$T \frac{\partial v}{\partial T} \Big| = 0.00233 \text{ m}^3/\text{kg}$$

$$v_f \Big|_{T=150^\circ\text{C}} = 0.00109 \text{ m}^3/\text{kg}$$

$$h_3 = h_{3***} + (0.00109 - 0.00233) (0.5) \times 1000$$

$$= 632.86 - 0.619$$

$$h_3 = 632.24 \text{ kJ/kg}$$

Comparison	Calculated (kJ/kg)	Steamtables (kJ/kg)	% dev.
3**	2748.68	2746.5	0.08 %
3***	632.86	632.2	0.1 %
3	632.24	632.3	-0.01 %