Assignment 11

1. The pressure ratio across a stationary normal shock wave that occurs in air is 1.25. Ahead of the shock wave, the pressure is 100 kPa and the temperature is 15°C. Find the velocity, pressure, and the temperature of the air behind the shock wave. (Ans: 319.7 m/s, 125 kPa, 307 K)
2. A normal shock wave occurs in air at a point where the velocity is 600 m/s and the stagnation temperature and pressure are 200°C and 600 kPa respectively. Find the Mach numbers pressures, and temperatures upstream and downstream of the shock wave. (Ans: T_1 = 293.8 K M_1 = 1.746, p_1 = 113.4 kPa, M_2 = 0.629, p_2 = 383 kPa, T_2 = 438.2 K)
3. Air is expanded from a large reservoir in which the pressure and temperature are 500 kPa and 35°C through a variable area duct. A normal shock occurs at point in the duct where the Mach number is 2.5. Find the pressure and temperature in the flow just downstream of the shock wave. Downstream of the shock wave, the flow is brought to rest in another large reservoir. Find the pressure and temperature in the reservoir. Assume that the flow is one-dimensional and isentropic everywhere except through the shock wave.
   (Ans: p_{02} = 249.5 kPa, T_{02} = 308 K, Note that T_0 remains the same.)
4. In the class, we had shown that the pressure ratio, density ratio, temperature ration and stagnation pressure ratio across the shock were related to M_1 and M_2, the Mach numbers at immediate the upstream and downstream of the shock respectively. Further, it was also shown that M_2 is directly related to M_1. Now using the relations referred above, express the pressure, temperature and density ratios as a function of M_1 alone.
   (Ans: $\frac{p_2}{p_2} = \frac{2\gamma M_1^2 - (\gamma-1)}{(\gamma+1)}, \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}, \frac{T_2}{T_1} = \frac{\left(2\gamma M_1^2-(\gamma-1)\right)\left(2+M_1^2(\gamma-1)\right)}{(\gamma+1)^2 M_1^2}$)
5. A perfect gas flow through a stationary normal shock. The gas velocity decreases from 480 m/s to 160 m/s through the shock. If the pressure and the density upstream of the shock are 62 kPa and 1.5 kg/m^3, find the pressure and the specific heat ratio of the gas. (Ans: γ = 1.3)
6. Consider a convergent divergent (CD) nozzle connected at the exit of a large reservoir in which the fluid can be taken to be stagnant. The inlet, throat and exit diameters are 20 mm, 8 mm and 12 mm respectively. The nozzle is operated in the ambient where pressure is 101.3 kPa. The stagnation temperature in the inlet reservoir can be taken as constant at a value of 300 K. The length of the converging (L_c) and the diverging (L_d) sections are 55 mm and 65 mm respectively. Three pressure taps are provided in the diverging section of the nozzle. These are located at the throat, 27.5 mm from the throat (d = 9.69 mm) and 55 mm from the throat (d = 11. 4 mm) respectively. (i) Assuming the flow in the nozzle to be isentropic, determine the stagnation conditions to be maintained at the inlet reservoir for Limiting Venturi Condition and Design Condition respectively. (ii) Now assuming that a normal shock is resident at at L_d /3 from the throat (d = 9.33 mm), compute the stagnation pressure at the inlet reservoir. Repeat the same if the normal shock resident at the exit of the nozzle (iii) For each of the case, compute the mass flow rate through the CD nozzle. (iv) Estimate the pressures at the three pressure taps.
7. Verify by plotting that the non-dimensional isentropic flow in a CD nozzle (in slide (33/42) agrees well with the shifted ellipse equation given in slide 34/42