## Assignment 12

1. Show that for a Van-der-Waals gas, $p_{c}=\frac{a}{27 b^{2}}, v_{c}=3 b, T_{c}=\frac{8 a}{27 R b}$, where $c$ denotes critical conditions.
2. Now show that the reduced equation of state for the Van-der-Waals gas gas is,

$$
\left(\mathrm{p}_{\mathrm{r}}+\frac{3}{\mathrm{v}_{\mathrm{r}}^{2}}\right)\left(3 \mathrm{v}_{\mathrm{r}}-1\right)=8 \mathrm{~T}_{\mathrm{r}}
$$

where, the reduced pressure, specific volume and temperature are $\mathrm{p}_{\mathrm{r}}=\mathrm{p} / \mathrm{p}_{\mathrm{c}}, \mathrm{v}_{\mathrm{r}}=\mathrm{v} / \mathrm{v}_{\mathrm{c}}, \mathrm{T}_{\mathrm{r}}=\mathrm{T} / \mathrm{T}_{\mathrm{c}}$,
3. Prove that $\left.\frac{\partial \mathrm{u}}{\partial \mathrm{p}}\right|_{\mathrm{T}}=-\left.\mathrm{T} \frac{\partial \mathrm{v}}{\partial \mathrm{T}}\right|_{\mathrm{p}}-\left.\mathrm{p} \frac{\partial \mathrm{v}}{\partial \mathrm{p}}\right|_{\mathrm{T}}$
4. Show that $\left.\frac{\partial \mathrm{p}}{\partial \mathrm{T}}\right|_{\mathrm{s}}=\frac{\mathrm{c}_{\mathrm{p}}}{\operatorname{Tv} \beta}$
5. Show that $c_{p}-c_{v}=\left.\left.T \frac{\partial v}{\partial T}\right|_{p} \frac{\partial \mathrm{p}}{\partial \mathrm{T}}\right|_{\mathrm{v}}$
6. Show that for a Van-der-Waals gas, $\mathrm{c}_{\mathrm{v}}$ is a function of T only
7. Determine $\mathrm{c}_{\mathrm{p}}-\mathrm{c}_{\mathrm{v}}$ for Van-der-Waals gas. State whether $\mathrm{c}_{\mathrm{p}}$ is a function of T only?
8. Show that Joule-Thompson's coefficient, $\mu=\left.\frac{\partial T}{\partial \mathrm{p}}\right|_{\mathrm{h}}=\frac{1}{\mathrm{c}_{\mathrm{p}}}\left(\left.\mathrm{T} \frac{\partial \mathrm{v}}{\partial \mathrm{T}}\right|_{\mathrm{p}}-\mathrm{v}\right)$ and is zero for ideal gas.
9. With reference to the figure, determine the enthalpies at the points $1^{*}, 2^{*}$ and $3^{*}$ which lie on the isobar $\mathrm{P}=0.1 \mathrm{MPa}$, using the property relations used. The temperatures at points 1 , and $3^{*}$ are $50^{\circ} \mathrm{C}$ and $150^{\circ} \mathrm{C}$ respectively. Relevant property fits at constant pressure are given below. Compare the values of enthalpy obtained with steam table values.

Property fits along the isobar ( $\mathrm{p}=0.1 \mathrm{MPa}$ )
$\mathrm{C}_{\mathrm{pf}}=8.41056 \times 10^{-06} \mathrm{~T}^{2}-5.81708 \times 10^{-04} \mathrm{~T}+4.18952(\mathrm{~kJ} / \mathrm{kgK})$
$\mathrm{C}_{\mathrm{pg}}=-1.61558 \times 10^{-07} \mathrm{~T}^{3}+8.85428 \times 10^{-05} \mathrm{~T}^{2}-1.62498 \times 10^{-02} \mathrm{~T}+2.97423(\mathrm{~kJ} / \mathrm{kgK})$
Saturation poperties
$P_{\text {sat }}=8.56688 \times 10^{-07} \mathrm{~T}^{3}-2.53198 \times 10^{-04} \mathrm{~T}^{2}+3.09515 \times 10^{-02} \mathrm{~T}-1.36108$
$\mathrm{v}_{\mathrm{fg}}=-1.72712 \times 10^{-05} \mathrm{~T}^{3}+6.35816 \times 10^{-03} \mathrm{~T}^{2}-0.809280 \mathrm{~T}+36.2854\left(\mathrm{~m}^{3} / \mathrm{kg}\right) \quad\left(80<\mathrm{T}<=120^{\circ} \mathrm{C}\right)$
$\mathrm{v}_{\mathrm{fg}}=-1.88453 \times 10^{-06} \mathrm{~T}^{3}+1.01210 \times 10^{-03} \mathrm{~T}^{2}-0.186313 \mathrm{~T}+11.9252\left(\mathrm{~m}^{3} / \mathrm{kg}\right)$
$\left(120<\mathrm{T}<=180^{\circ} \mathrm{C}\right)$
(all T are in degree celcius)
(Ans: $\mathrm{h}_{1 *}=419.168 \mathrm{~kJ} / \mathrm{kg}, \mathrm{h}_{2} *=2654.067 \mathrm{~kJ} / \mathrm{kg}, \mathrm{h}_{3} *=2754.906 \mathrm{~kJ} / \mathrm{kg}$ )

10. With value of $\mathrm{h}_{3^{*}}$ obtained in the previous problem, determine, the enthalpy at $\mathrm{P}=0.6 \mathrm{MPa}$ and $150^{\circ} \mathrm{C}$. This may be obtained by moving along the isotherm $3^{*}-3^{* *}-3 * * *$ and 3 . To calculate the enthalpy change along the isotherm $\mathrm{T}=150^{\circ} \mathrm{C}$ (in gaseous region), PVT relation is required. Redlich Kwong equation $P=\frac{R T}{(v-b)}-\frac{a / T^{0.5}}{v(v+b)}$ is to be used, where $a=0.4275 \frac{R^{2} T_{c}^{2.5}}{P_{c}}$ and $b=0.08664 \frac{R T_{c}}{P_{c}}$ where $\mathrm{P}_{\mathrm{c}}, \mathrm{T}_{\mathrm{c}}$ are the critical pressure and critical temperature ( 22.064 MPa and $373.946^{\circ} \mathrm{C}$ ). (Trapezoidal integration may be used to evaluate the integral). For the path $3^{* * *-3}$, the specific volume can be assumed to be a function of T only and is independent of P .

$$
\mathrm{v}_{\mathrm{f}}=3.42455 \times 10^{-09} \mathrm{~T}^{2}+9.67286 \times 10^{-08} \mathrm{~T}+9.99233 \times 10^{-04}
$$

(Ans: $\mathrm{h}_{3^{* *}}=2748.677 \mathrm{~kJ} / \mathrm{kg}, \mathrm{h}_{3^{* * *}}=632.857 \mathrm{~kJ} / \mathrm{kg}, \mathrm{h}_{3}=632.237 \mathrm{~kJ} / \mathrm{kg}$ )

