ME 209

Basic Thermodynamics (Lecture-12)

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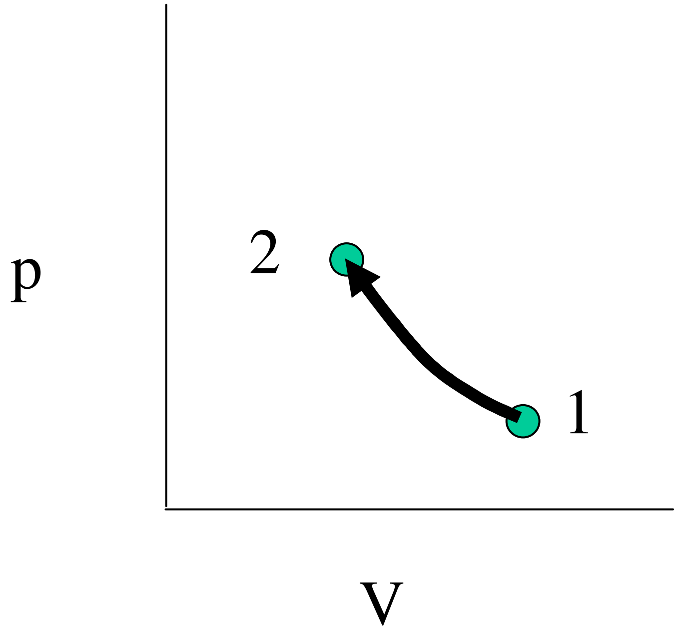
Review of Lecture 11

- Analysed a Carnot engine and showed that the Thermodynamic scale and Ideal Gas Kelvin scale are identical
- Derived the Clausius Inequality, defined entropy and entropy production
- Derived the Second law in terms of change of entropy for an infinitesinal process
- Stated the principle of increase in entropy of the universe.

Agenda for Today

- Understand property relations
- Understand h-s and T-s diagrams
- Derive the Second law for open systems

TdS Relations-I



- Consider two states 1 and 2 as shown
- The entropy change S_2-S_1 shall not depend on path
- Let us identify the reversible path 1-2
- First law for the closed system can be written as

$$dQ = dU + dW$$

$$\Rightarrow TdS = dU + pdV$$

Note that the above relation involves only properties and hence shall be independent of path

TdS Relations-II

- We can also modify the relation derived as follows

$$TdS = dU + pdV$$

$$\Rightarrow TdS = dU + d(pV) - Vdp$$

$$\Rightarrow TdS = dH - Vdp$$

- The above two property relations are the working equations for the evaluation of entropy change and are known as TdS relations

T-S Diagram-I

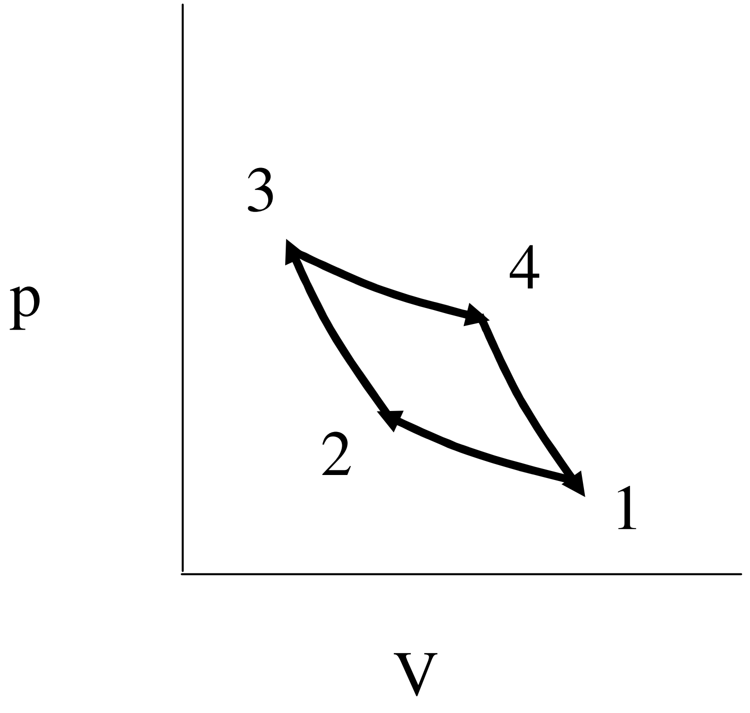
- For adiabatic and reversible process ds = 0
- Further, for a reversible process, the heat transferred per unit mass of the substance can be written as

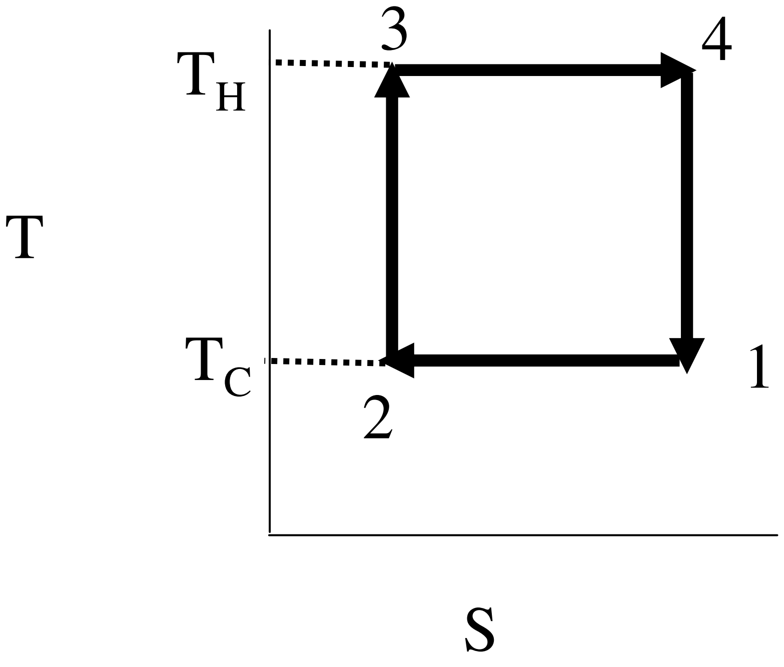
$$\Rightarrow Q_R = \int_1^2 T ds$$

- The above has motivated construction of T-s diagram. Its usefulness is established by considering analysis of Carnot Cycle

T-S Diagram-II

1. Isothermal Compression (1-2)
2. Adiabatic Compression (2-3)
3. Isothermal Expansion (3-4)
4. Adiabatic Expansion (4-1)





T-S Diagram-III

- Heat rejected in process 1-2

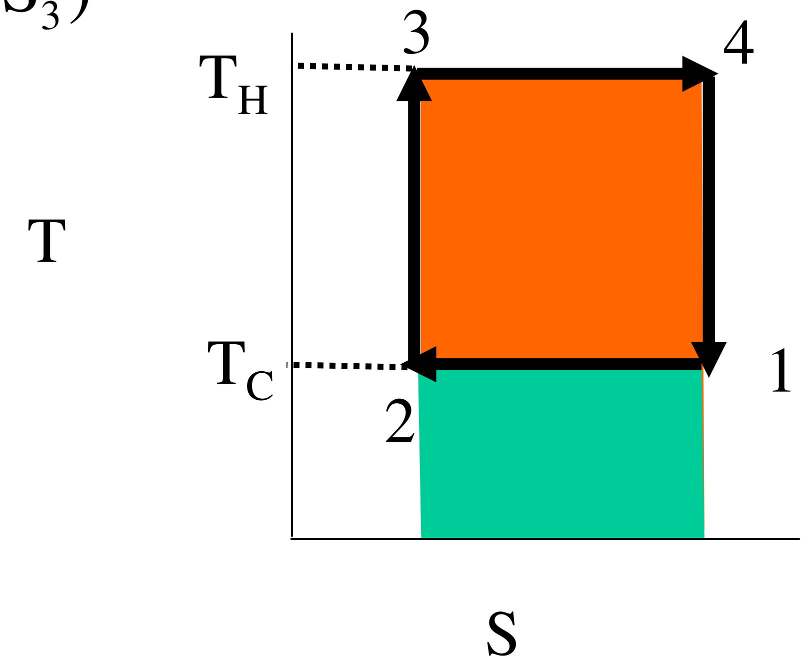
$$\Rightarrow Q_C = -Q_{12} = -\int_1^2 Tds = -T_C(S_2 - S_1)$$

- Heat added in process 1-2

$$\Rightarrow Q_H = Q_{34} = \int_3^4 Tds = T_H(S_4 - S_3)$$

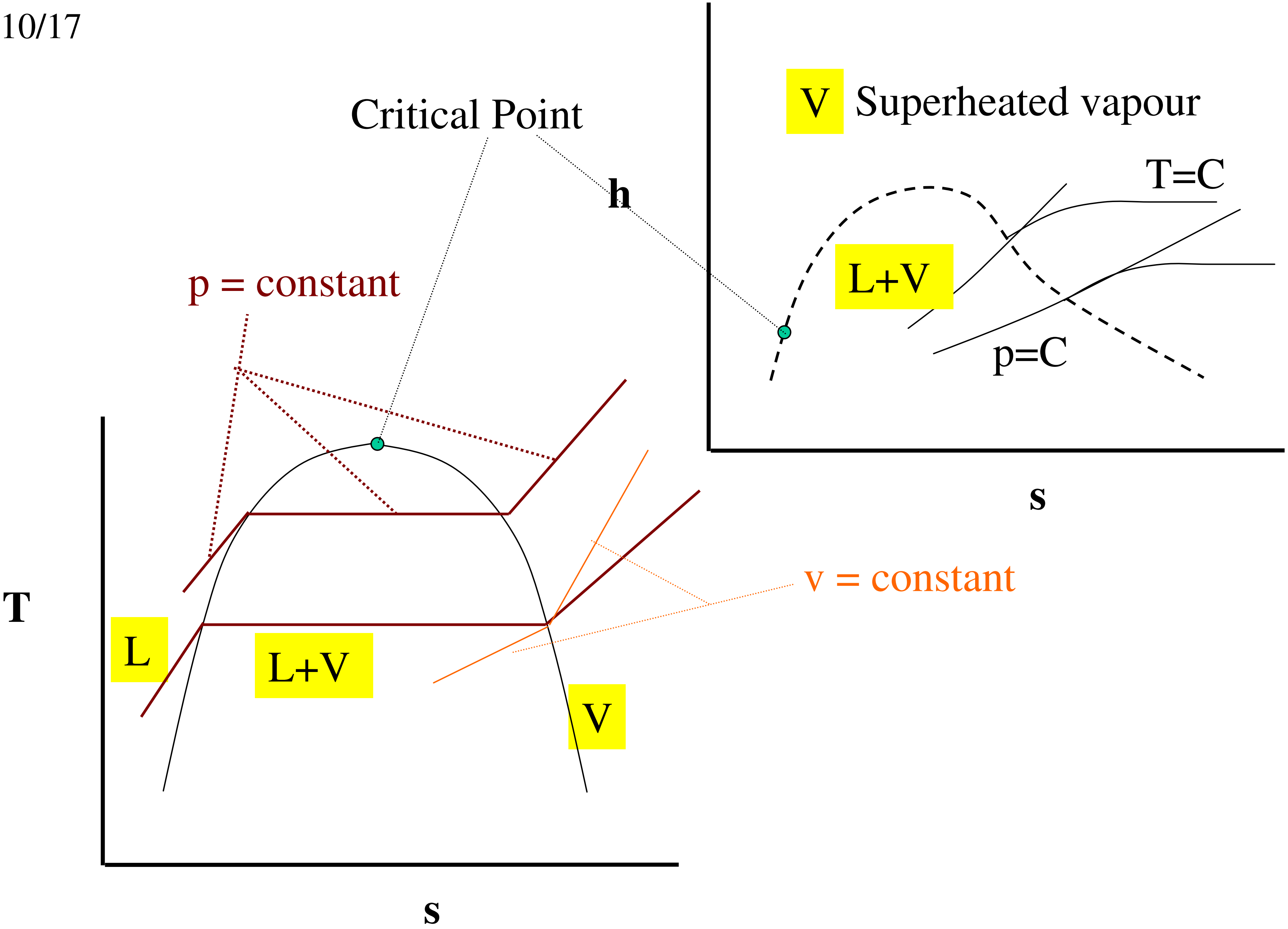
$$\Rightarrow \eta = 1 - \frac{Q_C}{Q_H}$$

$$= 1 - \frac{T_C(S_1 - S_2)}{T_H(S_4 - S_3)} = 1 - \frac{T_C}{T_H}$$



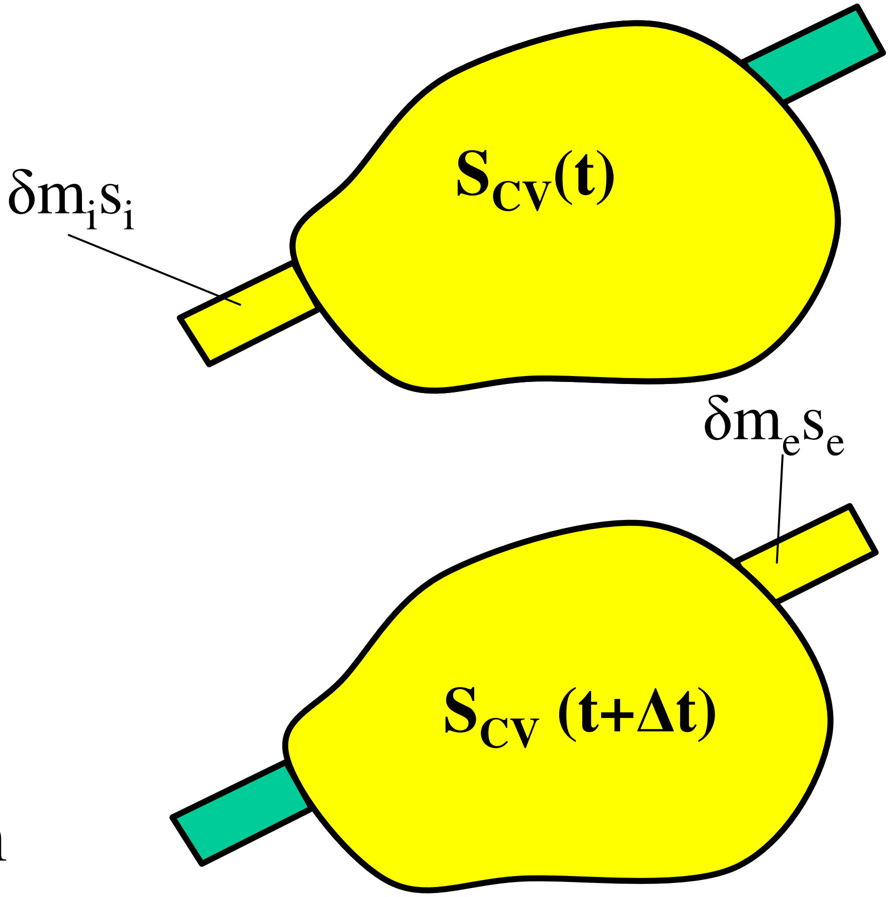
h-s diagram

- For open systems, with kinetic and potential effects neglected, we saw that for turbines, pumps/compressor and heat exchangers the heat/work per unit mass of the substance was directly related to change of enthalpy
- This motivated inventions of h-s diagrams, particularly for water substance
- These are called Mollier Diagrams


Critical Point
V
Superheated vapour
h
T=C
L+V
p=C
s
p = constant
v = constant
T
L
L+V
V
s

II Law for Open System-I

- Now let us consider the arbitrary system again
- At time, t, the total entroy of the control mass is $S_{CV}(t) + \delta m_i s_i$
- At time, t+ Δt, the total energy of the control mass is $S_{CV}(t+\Delta t) + \delta m_e s_e$
- During this Δt, the system has heat interaction of δQ. Usual Sign convention applies



II Law for Open System-II

- The second law for the control mass can be written as,

$$S_{CV}(t+\Delta t)+\delta m_e s_e - (S_{CV}(t)+\delta m_i s_i) = (\delta Q/T) + \delta S_p$$

- Rearranging and dividing both sides by Δt and shrinking the Δt to 0, we can write,

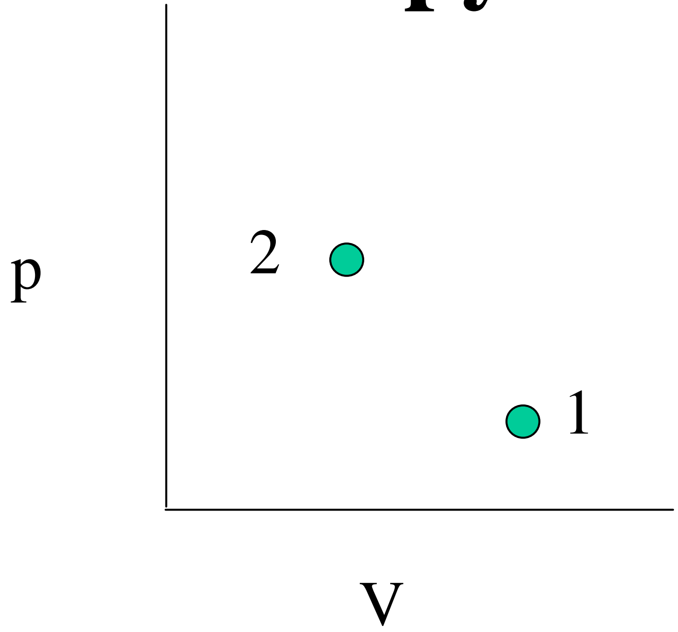
$$\left.\frac{S_{CV}(t+\Delta t)-S_{CV}(t)}{\Delta t}\right|_{\Delta t\to 0} = \left.\frac{\delta Q}{\Delta t}\frac{1}{T}\right|_{\Delta t\to 0\,\Delta t\to 0} + s_i\left.\frac{\delta m_i}{\Delta t}\right|_{\Delta t\to 0} - s_e\left.\frac{\delta m_e}{\Delta t}\right|_{\Delta t\to 0} + \left.\frac{\delta S_p}{\Delta t}\right|_{\Delta t\to 0}$$

or, $$\frac{dS_{CV}}{dt} = \frac{\dot{Q}_{CV}}{T} + \dot{m}_i s_i - \dot{m}_e s_e + \dot{S}_p$$

Evaluation of Change in Entropy-I

For the system with ideal gas

- Once the two states are known we can use the TdS relations to evaluate the entropy change



$$TdS = dU + pdV$$

$$\Rightarrow TdS = mc_v dT + \frac{mRT}{V} dV$$

$$\Rightarrow dS = mc_v \frac{dT}{T} + mR \frac{dV}{V}$$

$$\Rightarrow \int_1^2 dS = mc_v \int_1^2 \frac{dT}{T} + mR \int_1^2 \frac{dV}{V}$$

$$\Rightarrow S_2 - S_1 = mc_v \ln \frac{T_2}{T_1} + mR \ln \frac{V_2}{V_1}$$

Evaluation of Change in Entropy-II

- Proceeding on similar lines, we can use the second TdS relation

$TdS = dH - Vdp$ and show

$$\Rightarrow S_2 - S_1 = mc_p \ln\frac{T_2}{T_1} - mR\ln\frac{p_2}{p_1}$$

Evaluation of Change in Entropy-III

For the system with incompressible substance

- For incompressible substances the expansion work is absent that makes $c_p = c_v = c$

$$TdS = dU + pdV$$

$$\Rightarrow TdS = dU + \cancel{pdV}$$

$$\Rightarrow TdS = mcdT$$

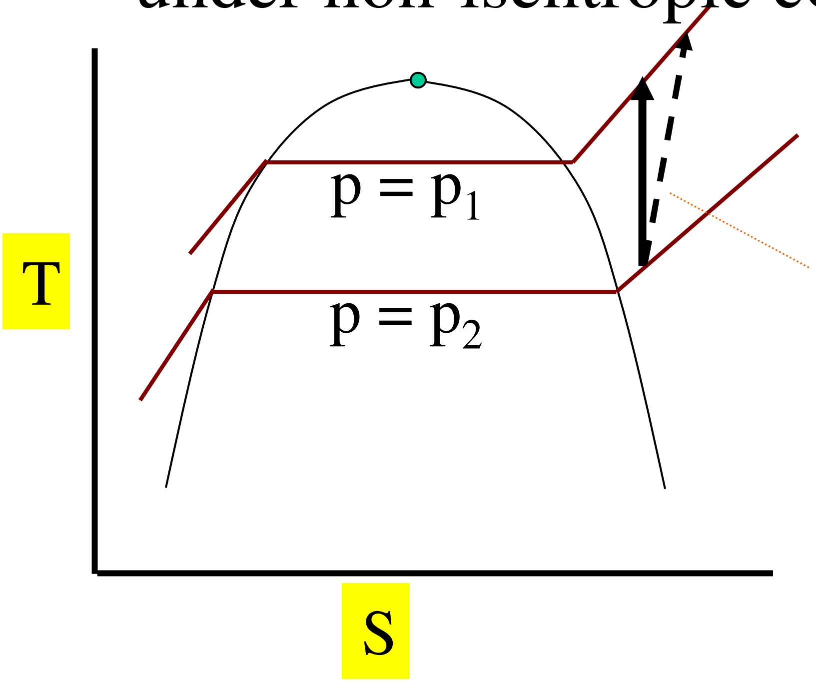
$$\Rightarrow dS = mc\frac{dT}{T}$$

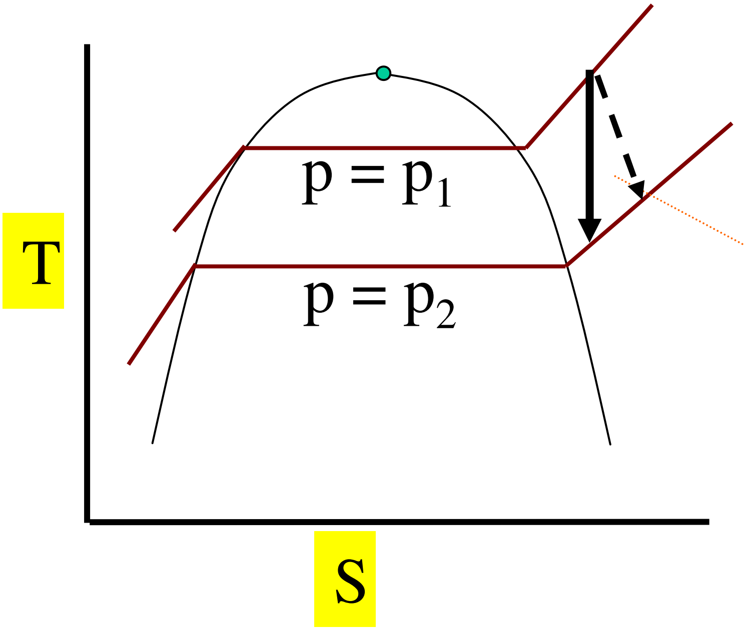
$$\Rightarrow \int_1^2 dS = mc\int_1^2 \frac{dT}{T}$$

$$\Rightarrow S_2 - S_1 = mc\ln\frac{T_2}{T_1}$$

Isentropic efficiency-I

- Turbines, compressors and nozzles, perform ideally under isentropic conditions
- Turbine output decreases, Compressor input increases and nozzle KE developed decreases, under non-isentropic conditions





Isentropic efficiency-II

- We define isentropic efficiency as

$$\eta_{s-\mathrm{Turbine/Nozzle}} = \frac{h_i - h_e}{h_i - h_{e*}}$$

$$\eta_{s-\mathrm{comp/pump}} = \frac{h_{e*} - h_i}{h_e - h_i}$$