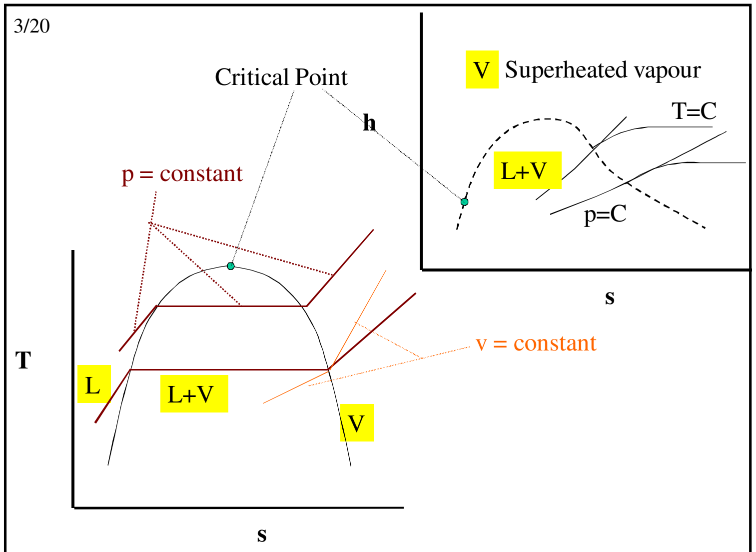
ME 209
Basic Thermodynamics (Lecture-18)
Open Systems-III

Kannan Iyer
Kiyer@me.iitb.ac.in

Department of Mechanical Engineering
Indian Institute of Technology, Bombay

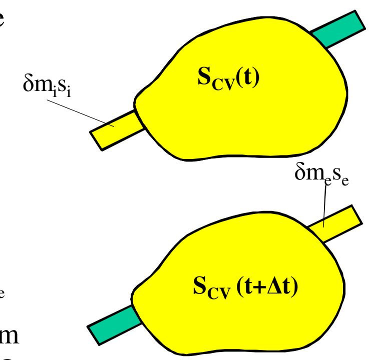
h-s diagram

- For open systems, with kinetic and potential effects neglected, we saw that for turbines, pumps/compressor and heat exchangers the heat/work per unit mass of the substance was directly related to change of enthalpy
- This motivated inventions of h-s diagrams, particularly for water substance
- These are called Mollier Diagrams



II Law for Open System-I

- Now let us consider the arbitrary system again
- At time, t, the total entroy of the control mass is $S_{CV}(t) + \delta m_i s_i$
- At time, t+ Δt, the total energy of the control mass is $S_{CV}(t+\Delta t) + \delta m_e s_e$
- During this Δt, the system has heat interaction of δQ. Usual Sign convention applies



5/20

II Law for Open System-II

- The second law for the control mass can be written as,

$$S_{CV}(t+\Delta t) + \delta m_e s_e - (S_{CV}(t) + \delta m_i s_i) = (\delta Q/T) + \delta S_p$$

- Rearranging and dividing both sides by Δt and shrinking the Δt to 0, we can write,

$$\left.\frac{S_{CV}(t+\Delta t) - S_{CV}(t)}{\Delta t}\right|_{\Delta t\to 0} = \left.\frac{\delta Q}{\Delta t}\frac{1}{T}\right|_{\Delta t\to 0\,\Delta t\to 0} + s_i\left.\frac{\delta m_i}{\Delta t}\right|_{\Delta t\to 0} - s_e\left.\frac{\delta m_e}{\Delta t}\right|_{\Delta t\to 0} + \left.\frac{\delta S_p}{\Delta t}\right|_{\Delta t\to 0}$$

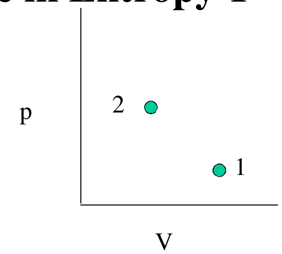
$$\text{or,}\quad \frac{dS_{CV}}{dt} = \frac{\dot{Q}_{CV}}{T} + \dot{m}_i s_i - \dot{m}_e s_e + \dot{S}_p$$

6/20

Evaluation of Change in Entropy-I

For the system with ideal gas

- Once the two states are known we can use the TdS relations to evaluate the entropy change



$$TdS = dU + pdV$$

$$\Rightarrow TdS = mc_v dT + \frac{mRT}{V}dV$$

$$\Rightarrow dS = mc_v\frac{dT}{T} + mR\frac{dV}{V}$$

$$\Rightarrow \int_1^2 dS = mc_v\int_1^2\frac{dT}{T} + mR\int_1^2\frac{dV}{V}$$

$$\Rightarrow S_2 - S_1 = mc_v\ln\frac{T_2}{T_1} + mR\ln\frac{V_2}{V_1}$$

7/20

Evaluation of Change in Entropy-II

- Proceeding on similar lines, we can use the second TdS relation

$TdS = dH - Vdp$ and show

$$\Rightarrow S_2 - S_1 = mc_p\ln\frac{T_2}{T_1} - mR\ln\frac{p_2}{p_1}$$

8/20

Evaluation of Change in Entropy-III

For the system with incompressible substance

- For incompressible substances the expansion work is absent that makes $c_p = c_v = c$

$$TdS = dU + pdV$$

$$\Rightarrow TdS = dU + p\cancel{dV}$$

$$\Rightarrow TdS = mcdT$$

$$\Rightarrow dS = mc\frac{dT}{T}$$

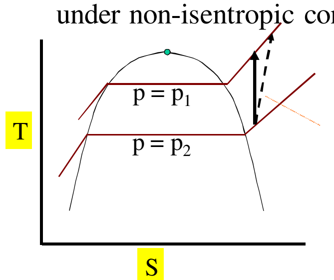
$$\Rightarrow \int_1^2 dS = mc\int_1^2\frac{dT}{T}$$

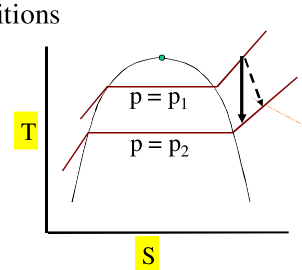
$$\Rightarrow S_2 - S_1 = mc\ln\frac{T_2}{T_1}$$

9/20

Isentropic efficiency-I

- Turbines, compressors and nozzles, perform ideally under isentropic conditions
- Turbine output decreases, Compressor input increases and nozzle KE developed decreases, under non-isentropic conditions





10/20

Isentropic efficiency-II

- We define isentropic efficiency as

$$\eta_{s-Turbine/Nozzle} = \frac{h_i - h_e}{h_i - h_{e*}}$$

$$\eta_{s-comp/pump} = \frac{h_{e*} - h_i}{h_e - h_i}$$

11/20

Availability Balance Equation-I

- We can generate an availability balance equation by combining first and second laws.

$$\int_1^2 dQ - \int_1^2 dW = \Delta E \qquad \text{First Law} \qquad (1)$$

$$\int_1^2 \frac{dQ}{T} + S_p = \Delta S \qquad \text{Second Law} \qquad (2)$$

- Eq. (2) x T_0 implies

$$\int_1^2 T_0 \frac{dQ}{T} + T_0 S_p = T_0 \Delta S \qquad (3)$$

- Eq. (1) - Eq. (3) implies

$$\int_1^2 \left(1 - \frac{T_0}{T}\right) dQ - W - T_0 S_p = \Delta E - T_0 \Delta S \quad = \Delta A - p_0 \Delta V \qquad (4)$$

12/20

Availability Balance Equation-II

- Eq. (4) can be recast as

$$\Delta A = \int_1^2 \left(1 - \frac{T_0}{T}\right) dQ - \left(W - p_0 \Delta V\right) - T_0 S_p \qquad (5)$$

Change of availability	Availability transfer due to heat exchange	Availability transfer due to work exchange	Work Lost Also called Irreversibility

- The above equation is useful to directly compute maximum useful work that can be extracted

13/20

Exergy Analysis-I

- Combined first and second law analysis for flow systems is called Exergy Analysis
- The governing equation for the same can be derived as follows:

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} + \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right) \quad (1)$$

$$\frac{dS_{CV}}{dt} = \frac{\dot{Q}_{CV}}{T} + \dot{m}_i s_i - \dot{m}_e s_e + \dot{S}_p \quad (2)$$

- Eq. (1) - Eq. (2) x T_0 implies

14/20

Exergy Analysis-II

$$\frac{d(E_{CV} - T_0 S_{CV})}{dt} = \frac{d(A_{CV} - p_0 V_{CV})}{dt} = \dot{Q}_{CV}\left(1 - \frac{T_0}{T}\right) - \dot{W}_{CV} - T_0\dot{S}_p + \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i - T_0 S_i\right) - \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e - T_0 S_e\right) \quad (3)$$

$$\underbrace{\frac{d(A_{CV})}{dt}}_{A} = \underbrace{\dot{Q}_{CV}\left(1 - \frac{T_0}{T}\right)}_{B} - \underbrace{(\dot{W}_{CV} - p_0\dot{V}_{CV})}_{C} - \underbrace{T_0\dot{S}_p}_{D} + \underbrace{\dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i - T_0 s_i\right)}_{E} - \underbrace{\dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e - T_0 s_e\right)}_{F}$$

15/20

Exergy Analysis-III

- Each of the term marked in the previous slides are as follows:
  - (A) Rate of change of Availability in the control volume
  - (B) Rate of change of Availability due to heat flow
  - (C) Rate of change of Availability due to work transfer
  - (D) Rate of production of Irreversibility
  - (E) Rate of Availability transported in
  - (E) Rate of Availability transported out

16/20

Exergy Analysis-IV

- The expression $h + \frac{V^2}{2} + gz - T_0 s$ is called Exergy and is usually denoted by b
- The final availability balance equation can be stated as:

$$\frac{d(A_{CV})}{dt} = \dot{Q}_{CV}\left(1 - \frac{T_0}{T}\right) - (\dot{W}_{CV} - p_0\dot{V}_{CV}) - T_0\dot{S}_p + \dot{m}_i b_i - \dot{m}_e b_e$$

17/20

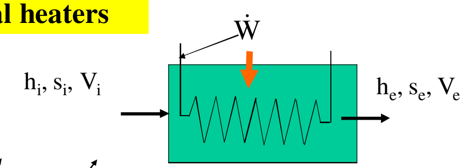
Second Law Efficiency-I

- Ideal processes have no entropy production. However, in many real processes, entropy production is inevitable, for e.g., heat exchangers, flash chambers, etc.
- Systems and processes that generate minimum entropy are preferred as they conserve energy resources
- An indicator is often desirable to compare processes and second law efficiency is an indicator in this direction
- Different definitions are given for various systems and we shall see some of them to get a flavour

18/20

Second Law Efficiency-II

Electrical heaters



$$\frac{d(A_{CV})}{dt} = \dot{Q}_{CV}\left(1-\frac{T_0}{T}\right) - (\dot{W}_{CV} - p_0\dot{V}_{CV}) - T_0\dot{S}_p + \dot{m}_i b_i - \dot{m}_e b_e$$

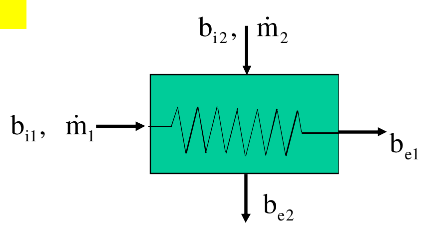
$$-\dot{W}_{CV} = T_0\dot{S}_p + \dot{m}(b_e - b_i)$$

$$\eta_{II} = \frac{\dot{m}(b_e - b_i)}{-\dot{W}_{CV}}$$

19/20

Second Law Efficiency-III

Heat Exchangers



$$\frac{d(A_{CV})}{dt} = \dot{Q}_{CV}\left(1-\frac{T_0}{T}\right) - (\dot{W}_{CV} - p_0\dot{V}_{CV}) - T_0\dot{S}_p + \sum \dot{m}_i b_i - \sum \dot{m}_e b_e$$

$$\Rightarrow \sum \dot{m}_e b_e + T_0\dot{S}_p = \sum \dot{m}_i b_i$$

$$\eta_{II} = \frac{\dot{m}_{cold}(b_{e-cold} - b_{i-cold})}{\dot{m}_{hot}(b_{i-hot} - b_{e-hot})}$$

To get this value less than 1, η_{II} is defined thus

20/20

Second Law Efficiency-IV

Turbine

$$\frac{d(A_{CV})}{dt} = \dot{Q}_{CV}\left(1-\frac{T_0}{T}\right) - (\dot{W}_{CV} - p_0\dot{V}_{CV}) - T_0\dot{S}_p + \dot{m}_i(b_i - b_e)$$

$$\dot{W}_{CV} = -T_0\dot{S}_p + \dot{m}(b_i - b_e)$$

$$\eta_{II} = \frac{\dot{W}_{CV}}{\dot{m}(b_i - b_e)}$$

Compressor

- The equation is same, but the definition is reversed to keep the efficiency to be less than 1

$$-\dot{W}_{CV} = T_0\dot{S}_p + \dot{m}(b_e - b_i) \qquad \eta_{II} = \frac{\dot{m}(b_e - b_i)}{-\dot{W}_{CV}}$$