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ME 209
Basic Thermodynamics
Introduction to Compressible Flow-I

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Compressible Fluid Flow-I

- The compressibility refers to the change of density of the fluid
- Density can change due to a change in pressure or temperature of a fluid
- In a Liquid, the density is a very weak function of pressure and but it can change perceptibly with temperature.
- In gas, the density is a strong function of temperature and pressure

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Compressible Fluid Flow-II

Applications of Compressible Flow

- Gas
  - Gas and Steam Turbines
  - Rocket Nozzles,
  - I.C. Engine ports,
  - Combustion chambers
  - Re-entry vehicles
- Liquids
  - Hydraulic Penstocks
  - High pressure hydraulic circuits

- In liquids normally, it is only the transient that calls for compressible flow analysis
- In gases both steady and transient flow may call for compressible analysis

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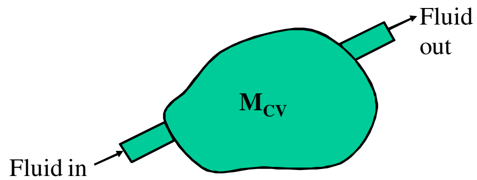
Compressible Fluid Flow-III

- It is a vast and complex subject
- Under some cases, we can treat the subject purely based on thermodynamic laws of mass and energy
- However, some concepts of momentum conservation is needed at places, which we shall consciously minimise.

Conservation of Momentum-I

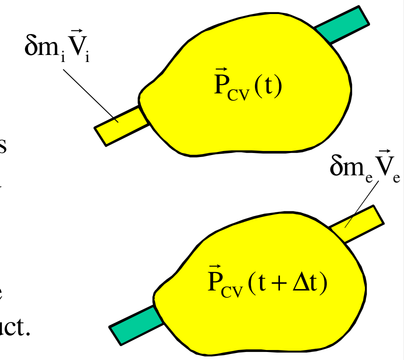
- Consider an arbitrary control volume as shown through which mass crosses (flowing from ducts)



- The aim shall be to Convert Newton's Second Law for a control mass to a flow system
- We shall now look at two snapshots one at t and other at t+Δt

Conservation of Momentum-II

- Let us consider the same mass of fluid as shown in yellow
- At time, t, the fluid fills the control volume and a portion of inlet duct
- The same fluid at t+Δt fills the control volume and a portion of exit duct.



$$\vec{P}_{CM}(t+\Delta t) = \vec{P}_{CV}(t+\Delta t) + \delta m_e \vec{V}_e$$
$$\vec{P}_{CM}(t) = \vec{P}_{CV}(t) + \delta m_i \vec{V}_i$$

Conservation of Momentum-III

- Subtracting the above two equations, we get

$$\vec{P}_{CM}(t+\Delta t) - \vec{P}_{CM}(t) = \vec{P}_{CV}(t+\Delta t) - \vec{P}_{CV}(t) + \delta m_e \vec{V}_e - \delta m_i \vec{V}_i$$

- Dividing both sides by Δt and then shrinking Δt to 0, we get

$$\frac{d\vec{P}_{CM}}{dt} = \frac{d\vec{P}_{CV}}{dt} + \dot{m}_e \vec{V}_e - \dot{m}_i \vec{V}_i$$

- Newton's Ssecond law implies

$$\frac{d\vec{P}_{CM}}{dt} = \vec{F} = \vec{F}_S + \vec{F}_B$$

Conservation of Momentum-IV

$$\Rightarrow \frac{d\vec{P}_{CV}}{dt} + \dot{m}_e \vec{V}_e - \dot{m}_i \vec{V}_i = \vec{F}_S + \vec{F}_B$$

$$\text{Or } \frac{d\vec{P}_{CV}}{dt} = \dot{m}_i \vec{V}_i - \dot{m}_e \vec{V}_e + \vec{F}_S + \vec{F}_B$$

- At steady state

$$0 = \dot{m}_i \vec{V}_i - \dot{m}_e \vec{V}_e + \vec{F}_S + \vec{F}_B$$

- If we put the above equation in words, we can write

Rate of momentum entering CV - Rate of momentum exiting CV + Sum of all forces = 0

Conservation of Momentum-V

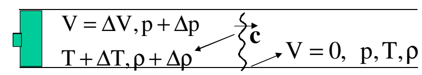
- The equation derived above can be extended to a steadily moving control volume as follows

$$0 = \dot{m}_{i-Rel}\vec{V}_{i-Rel} - \dot{m}_{e-Rel}\vec{V}_{e-Rel} + \vec{F}_S + \vec{F}_B$$

- In the above equation all quantities refer to quantities with respect to relative frame of reference.
- Its application will make it clear in the following derivation

Pressure Pulse Propagation-I

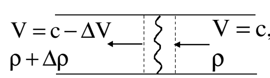
- Pressure pulses propagate in a compressible fluid with a characteristic speed.
- This is what we commonly call as speed of sound
- This speed is a property of the medium
- Consider a cylinder piston filled with a compressible fluid
- Let the piston be moved instantly
- This will set a pressure wave moving at a speed c



Undisturbed fluid

Pressure Pulse Propagation-II

- To derive a relation between the speed of propagation and system properties, let an observer ride on the wave. In this moving coordinate the fluid will be in steady state
- For the moving coordinate the properties are as shown



Mass balance $\Rightarrow (\rho + \Delta\rho)A(c - \Delta V) = \rho A c$

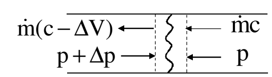
$\Rightarrow \rho c + \Delta\rho c - \rho\Delta V - \Delta\rho\Delta V) = \rho c$

Second order

$\therefore \Delta V = \dfrac{c\Delta\rho}{\rho}$ (1)

Pressure Pulse Propagation-III

Momentum balance



Positive direction

Momentum balance $\Rightarrow \dot{m}c - \dot{m}(c - \Delta V) + (-\Delta p A) = 0$ **No friction**

in **out** **force**

$\Rightarrow \dot{m}\Delta V - \Delta p A = 0$ $\quad \Rightarrow \Delta V = \dfrac{\Delta p A}{\dot{m}} = \dfrac{\Delta p A}{\rho A c} = \dfrac{\Delta p}{\rho c}$

$\Rightarrow \Delta V = \dfrac{\Delta p}{\rho C}$ (2)

- Eqs. (1) and (2) ➡ $\dfrac{c\Delta\rho}{\rho} = \dfrac{\Delta p}{\rho c}$ ➡ $c^2 = \dfrac{\Delta p}{\Delta\rho} = \dfrac{dp}{d\rho}$

Pressure Pulse Propagation-IV

- For ideal gas $dp = \left.\frac{\partial p}{\partial \rho}\right|_s d\rho + \left.\frac{\partial p}{\partial s}\right|_\rho ds$ Assuming the process to be adiabatic $\rightarrow ds = 0$

$$\therefore \frac{dp}{d\rho} = \left.\frac{\partial p}{\partial \rho}\right|_s = c^2$$

Newton had assumed the process to be Isothermal

$$s = \text{cons}\tan t \Rightarrow \frac{p}{\rho^\gamma} = \text{cons}\tan t \qquad \Rightarrow \ln(p) - \gamma \ln(\rho) = \text{cons}\tan t$$

$$\Rightarrow \frac{dp}{p} - \gamma\frac{d\rho}{\rho} = 0 \qquad \Rightarrow \frac{dp}{d\rho} = \gamma\frac{p}{\rho} \qquad \therefore c^2 = \gamma\frac{p}{\rho} = \gamma RT \qquad \Rightarrow c = \sqrt{\gamma RT}$$ (3)

- At 300 K $c = \sqrt{1.4X287X300} = 347 \ m/s$

Note that c is independent of p and depends only on T

Pressure Pulse Propagation-V

- For Solids and Liquids

$$\text{Bulk Modulus } E_v = \frac{dp}{d\rho/\rho} \qquad \Rightarrow \frac{dp}{d\rho} = \frac{E_V}{\rho} = c^2 \qquad \Rightarrow c = \sqrt{\frac{E_V}{\rho}}$$ (4)

For Water 20 °C, $E_v = 2.24 \text{ x } 10^9 \text{ N/m}^2$, $\rho = 998 \text{ kg/m}^3$

$$\therefore c = \sqrt{\frac{2.2x10^9}{998}} \approx 1500 \ m/s$$

For Steel 20 °C, $E_v = 200 \text{ x } 10^9 \text{ N/m}^2$, $\rho = 7830 \text{ kg/m}^3$

$$\therefore c = \sqrt{\frac{200x10^9}{7830}} \approx 5050 \ m/s$$