

ME 209

Basic Thermodynamics (Lecture-6)

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Review of Lecture 5

- Introduced Zeroth law
- Understood the two-point and single point scales
- Understood error in thermometers due to the non-linearity of the sensing property with temperature
- Introduced the International practical temperature scale which can be used for calibration.

Review of concepts

- Substances that have only one work mode are called simple substances
- Air and water can be classified as a simple compressible substance as the only relevant work mode is pdV
- If water is only in the liquid phase, then it can be assumed incompressible (approximation) and hence there is degeneracy. So all properties, except pressure depend only on temperature
- While we shall deal with water later, let us look at gases, which can be assumed as ideal as a first approximation

Ideal Gas Model - I

- Many gases can be treated as ideal gas as a first approximation
- The equation of state can be approximated by

$$pV = mRT \quad \text{Or} \quad p\underline{v} = RT \quad \begin{array}{l} \text{Specific} \\ \text{Volume} \end{array}$$

- R is gas constant and is given by R_u/M , where R_u is the universal gas constant and M is the relative molecular mass
- $R_u = 8.314 \text{ J/mol-K} = 8.314 \text{ kJ/kmol-K}$
- $R_{\text{air}} = (8.314 \text{ kJ/kmol-K})/29 \text{ kg/kmol} = 0.287 \text{ kJ/kg-K}$

Ideal Gas Model - II

- For a gas, p , v , ρ and T are properties
- For a gas at rest and at standard elevation, the total energy is in the form of internal energy (u) and from first law it is a property
- We can define another property called enthalpy (h) as, $h = u + pv$
- From state principle $h(p,T)$, $u(p,T)$
- We shall show later in the course that u for ideal gas is only a function of temperature, i.e., $u = u(T)$

Ideal Gas Model - III

- Since $h = u + pv = u(T) + RT = h(T)$, thereby we can conclude that h is only a function of temperature

- The specific heats are defined as,

$$c_v = \left. \frac{\partial u}{\partial T} \right|_v = \frac{du}{dT}, \quad \text{Only a function of temperature, as } u = u(T)$$

$$c_p = \left. \frac{\partial h}{\partial T} \right|_p = \frac{dh}{dT}, \quad \text{Only a function of temperature, as } h = h(T)$$

- However, $R = c_p - c_v = \text{constant}$
- Also, the ratio of specific heats $c_p / c_v = \gamma(T)$

Ideal Gas Model - IV

- For a process, change in internal energy can be written as

$$\int_1^2 du = \int_1^2 c_v(T) dT = \bar{c}_v (T_2 - T_1)$$

- For a moderate change in temperature, \bar{c}_v may be taken as constant, otherwise, the mean specific heat at constant volume shall be defined as

$$\bar{c}_v = \frac{\int_1^2 c_v(T) dT}{(T_2 - T_1)}$$

Ideal Gas Model - IV

- Similarly, the change in enthalpy for a process shall be,

$$\int_1^2 dh = \int_1^2 c_p(T) dT = \bar{c}_p (T_2 - T_1)$$

- While for a small change in temperature, \bar{c}_p may be taken as constant, otherwise, the mean specific heat at constant pressure shall be defined as

$$\bar{c}_p = \frac{\int_1^2 c_p(T) dT}{(T_2 - T_1)}$$

Ideal Gas Model - V

Reference State and its Consequence

- For a process where the specific heat can be assumed as constant, change in internal energy from a reference state can be written as

$$\int_{T(\text{Ref})}^T du = u(T) - u(T(\text{Ref})) = \int_{T(\text{Ref})}^T c_v(T) dT = c_v(T - T(\text{Ref}))$$

- For an ideal gas, we can postulate that its internal energy shall be zero at 0 K, hence the reference state is taken as 0 K. Thus in case of an ideal gas with constant specific heat model, we can write

$$u(T) = c_v T \quad \text{Note: T in K}$$

Ideal Gas Model - VI

- Similarly, we can write for change in enthalpy from 0 K

$$h(T) = c_p T + h_{\text{ref}}$$

$$\text{Where, } h_{\text{ref}} = u_{\text{ref}} + p_{\text{ref}} v_{\text{ref}} = 0$$

as, $u_{\text{ref}}, v_{\text{ref}} = 0$

- Hence, we can write

$$h(T) = c_p T$$

- The above relations for h and u appear to define absolute quantities. However, in thermodynamics, we always relate change of states.

Processes with Ideal Gas - I

Frictionless Adiabatic Process

- For an ideal gas with constant specific heat and no friction, we can use first law to arrive at the process equation

$$\Delta u = dQ - dW$$

$$\Rightarrow c_v dT = 0 - p dv \quad \mathbf{1}$$

From Equation of State

$$pv = RT \Rightarrow pdv + vdp = R dT \Rightarrow dT = \frac{pdv + vdp}{R}$$

Hence Eq. (1) can be rewritten as

$$c_v \frac{(pdv + vdp)}{R} = -pdv \Rightarrow c_v \frac{(pdv + vdp)}{c_p - c_v} = -pdv$$

Processes with Ideal Gas - II

$$\Rightarrow \frac{(pdv + vdp)}{\gamma - 1} = -pdv$$

$$\Rightarrow p \cancel{dv} + vdp = -\gamma p dv + p \cancel{dv}$$

- The above can be rewritten as

$$\frac{dp}{p} = -\gamma \frac{dv}{v}$$

- The above equation integrated between two states gives

$$p_1 v_1^\gamma = p_2 v_2^\gamma$$

Processes with Ideal Gas - II

- Several processes with ideal gas can be generalized as follows

$$pv^n = \text{constant}$$

$n = 1$ Isothermal process

$n = 0$ Isobaric process

$n = \gamma$ Adiabatic frictionless process

$n = n$ Polytropic process

$V = \text{constant}$ Isochoric process