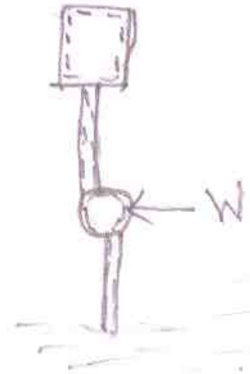
Problem-4

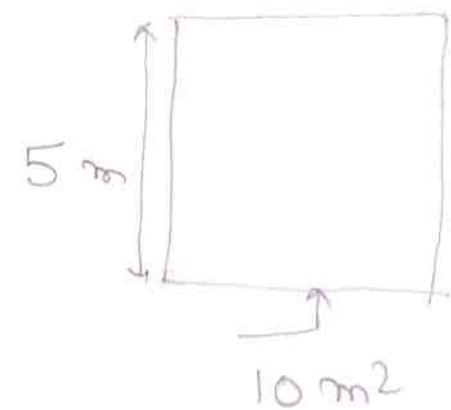


In this problem, the control volume shall be the tank, pump and connecting lines till the river.

The contents are air + water. During the process of filling, the mass of air is constant and mass of water is varying.

Let us first focus on air as a constant mass system in the tank.

State 1



$V_i = 50\ m^3, \quad p_i = 100\ kPa$

As water fills air gets compressed to $10 \times 1.25 = 12.5\ m^3$

$$W = \int p\,dV = m\int_1^2 \frac{RT}{V}\,dV$$

$$= mRT \ln \frac{V_2}{V_1}$$

[$\because$ T = constant]

$$= p_i V_i \ln \frac{V_2}{V_1}$$

$$= 100\ kPa \times 50\ m^3 \times \ln \frac{12.5}{50}$$

$$= -6.93 \times 10^3\ \underline{KJ}$$

Applying I-Law — $\cancel{\Delta U}^{\,0} = Q - W$

$$\Rightarrow Q = W = -6.93 \times 10^3\ \underline{KJ}$$

Now for variable mass system consisting of water and air

Mass bal. $\frac{dm_{cv}}{dt} = \dot{m}_{in} - \cancel{\dot{m}_{out}} \to 0$

$\Rightarrow M_{cv}\big|_2 - M_{cv}\big|_1 = \int \dot{m}_{in}\, dt$

Since air mass is constant $=$ Mair

$\cancel{M_{air}} + M_{water}\big|_2 - \cancel{M_{air}} = \int_0^t \dot{m}_{in}\, dt$

$M_{water} = 10\,m^2 \times 3.75\ m \times 1000\ \frac{kg}{m^3} = 37500\ kg$

$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \dot{m}_i h_i - \cancel{\dot{m}_e h_e} \to 0$

on integration

$E_{cv2} - E_{cv-1} = Q - W + h_i \int_0^t \dot{m}_i\, dt$

$\Delta E\big|_{air} + \Delta E\big|_{water} = Q - W + h_i M_{water}$

$M_{air}\left[C_{v\,air}\,(\cancel{T_2 - T_1}) \to 0 + \cancel{g(H_2 - H_1)} \to \text{negligible} + \cancel{\frac{V_2^2 - V_1^2}{2}}\right] = \underline{\Delta E\big|_{air}}$

$M_{water}\left[C_{water}\, T_w + g\left(40 + \frac{5\times 3}{8}\right) + \cancel{\frac{V_2^2}{2}} \to 0\right] - 0 = \underline{\Delta E\big|_{water}}$

$$Q = -6.93 \text{ MJ}$$

$$h_i M_w = M_{water} C_{water} T_w$$

$$\Rightarrow M_{water}\left[\cancel{C_{water} T_w} + 9.81 \, m/s^2 \times (41.875 \, m)\right]$$

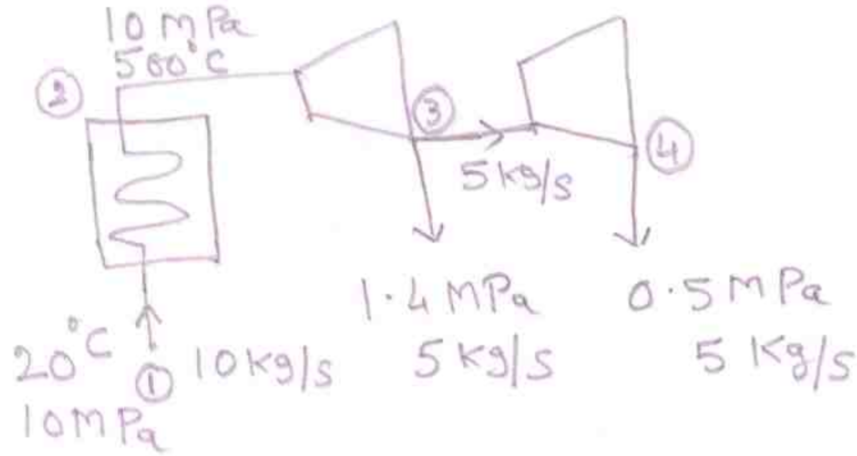
$$= -6.931 \text{ MJ} - W + \cancel{M_{water} C_{water} T_w}$$

$$\therefore W = -6.931 \text{ MJ} - 37500 \text{ kg} \times 9.81 \, m/s^2 \times 41.875 \, m$$

$$= -6.931 \text{ MJ} - 15.405 \text{ MJ}$$

$$\boxed{W = -22.3 \text{ MJ}}$$

(5)

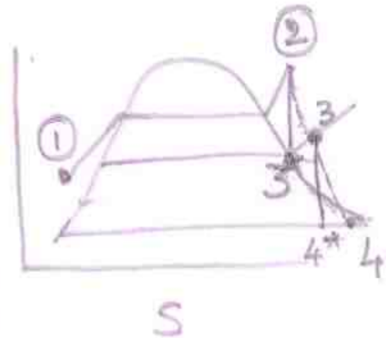


State-1 10 MPa, 20°C

$h_1 = u_f(20) + 10000\, v_f(20)$

$= 84 \frac{kJ}{kg} + 10^4\, kPa\; 0.001002 \frac{m^3}{kg}$

$= \underline{94.02\ kJ/kg}$



State-2 10 MPa, 500°C → Superheated

$h_g = 3373.7 \frac{kJ}{kg}$ $s_g = 6.597 \frac{kJ}{K}$

State 3* $p = 1.4\ MPa$, $s = 6.597$ → Superheated

T_g lies between 200°C – 250°C

200°C ⇒ $s_g = 6.498\ kJ/K$ 250°C → $s_g - 6.747 \frac{kJ}{K}$

$h_g = 2803.3\ kJ/kg$ $h_g - 2927.2 \frac{kJ}{kg}$

$$\Rightarrow \frac{6.597 - 6.498}{6.747 - 6.498} = \frac{h_3^* - 2803.3}{2927.2 - 2803.3}$$

$$(0.397) = \frac{h_3^* - 2803.3}{123.9}$$

$$\Rightarrow h_3^* = 2852.6$$

$$\eta_s = 0.85 = \frac{h_2 - h_3}{h_2 - h_3^*} = \frac{3373.7 - h_3}{3373.7 - 2852.6}$$

$\Rightarrow h_3 = 2930.8$

$$\text{Power} = \dot{m}(h_2 - h_3) = 10\ \text{kg/s}\ (3373.7 - 2930.8)\ \frac{\text{KJ}}{\text{kg}}$$

$$\underline{\dot{W}_1 = 4429\ \text{KW}}$$

s_3 also has to be evaluated by interpolation
As h_3 beyond both entries, we need 300°c data

$$\frac{s - 6.747}{6.953 - 6.747} = \frac{2930.8 - 2927.2}{3040.4 - 2927.2}$$

$$s = 6.754\ \frac{\text{KJ}}{\text{K}}$$

State 4^* $\rightarrow$ 0.5 MPa, $s = 6.754$

At 5 bar $s_g = 6.821\ \frac{\text{KJ}}{\text{K}}$ $s_f = 1.861\ \frac{\text{KJ}}{\text{K}}$

$$\Rightarrow x^* = \frac{6.754 - 1.861}{6.821 - 1.861} = 0.986$$

$$\Rightarrow h_4^* = \underset{h_f}{640.2} + \underset{x}{0.986} \times \underset{h_{fg}}{2108.5}$$

$$= 2720.2\ \frac{\text{KJ}}{\text{kg}}$$

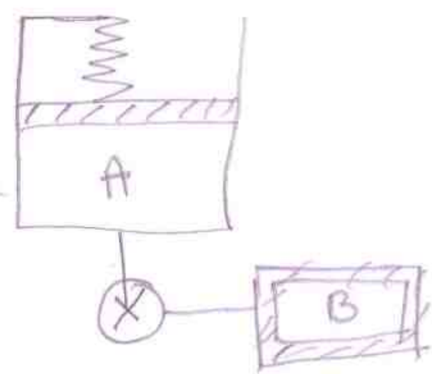
$$\therefore \frac{h_3 - h_4}{h_3 - h_4^*} = 0.85 \Rightarrow (h_3 - \underset{2751.8\ \text{KJ/kg}}{h_4}) = 179.0\ \frac{\text{KJ}}{\text{kg}}$$

$$\therefore \dot{W}_2 = 5 \times 179.0 = \underline{895\ \text{KW}}$$

$$\dot{Q} = 10(3373.7 - 94.02) = 32797\ \text{KW}$$

(6)

Tank B shall have a varying mass. Hence better dealt with transient equation.



System — Fluid in Tank B
connecting line including valve.

$$\dot{E} = \dot{Q} - \dot{W} + \dot{m}_i h_i - \cancel{\dot{m}_e} h_e$$

Integrating for time 0–t we get

$$\Delta E = \cancel{Q} - \cancel{W} + h_i \int_0^t \dot{m}_i \, dt \quad — (1)$$

Mass balance $\Rightarrow \dfrac{dm_{cv}}{dt} = \dot{m}_{in} - \cancel{\dot{m}_{out}}$

$$\Rightarrow M_{cv-2} - \cancel{M_{cv-1}} = \int_0^t \dot{m}_{in} \, dt$$

$$\therefore \int_0^t \dot{m}_{in} \, dt = M_{cv-2} \quad — (2)$$

(1) + (2) $\Rightarrow$

$$E_{cv-2} - \cancel{E_{cv-1}} = h_i M_{cv-2}$$

$$\therefore \cancel{M_{cv-2}} C_v T_2 = C_p T_i \cancel{M_{cv-2}}$$

$$\Rightarrow T_2 = \gamma T_i = \gamma T_A$$

Initial mass in Tank A

$$M_{A-1} = \frac{pV}{RT} = \frac{3500 \text{ kPa} \times 0.15 \text{ m}^3}{8.314 \frac{\text{kJ}}{} \times 293}$$

$= 6.249 \text{ Kg}$

Mass in Tank-B after the process

$$M_B = \frac{pV}{RT} = \frac{1500 \text{ kPa} \times 0.15 \text{ m}^3}{\frac{8.314}{29} \frac{\text{kJ}}{\text{kg-K}} \times 293 \times 1.4}$$

$$= 1.913 \text{ Kg}$$

∴ Final Mass in cylinder A

$$= 6.249 - 1.913 = 4.336 \text{ Kg}.$$

$$p = \frac{MRT}{V} = \frac{4.336 \times \frac{8.314}{29} \times 293}{V}$$

$$p = \frac{364.2}{V} = \frac{364.2}{V_0 + \Delta V} \quad (1)$$

(Note $V - V_0 = \Delta V$)

Also we have a linear spring. If F is spring force and Δs is deflection

$$F = F_0 + K\Delta s, \text{ where } K - \text{Spring constant}$$

Dividing by area

$$\frac{F}{A} = \frac{F_0}{A} + \frac{K}{A}\Delta s \Rightarrow p = p_0 + \frac{K}{A}\Delta s$$

$$\Rightarrow p = p_0 + \frac{K}{A}\frac{\Delta V}{A} = p_0 + \frac{K}{A^2}\Delta V \quad - (2)$$

Equating (1) + (2) $\Rightarrow$ $\frac{364.2}{V_0 + \Delta V} = p_0 (\text{kPa}) + \frac{40}{(0.03)^2}\Delta V$ (kPa)

$$\Rightarrow \frac{364.2}{0.15+\Delta V} = 3500\,(\mathrm{kPa}) + 44444\,\Delta V$$

$$\Rightarrow 364.2 = 3500 \times (0.15 + \Delta V) + 44444\,\Delta V\,(0.15+\Delta V)$$

$$= 364.2 = 525 + \Delta V\{3500 + 44444 \times 0.15\} + 44444\,\Delta V^2$$

$$364.2 = 525 + 10166\,\Delta V + 44444\,\Delta V^2$$

$$\Rightarrow 44444\,\Delta V^2 + 10166\,\Delta V + 160.8 = 0$$

$$\Delta V^2 + 0.228\,\Delta V + 3.618 \times 10^{-3} = 0$$

$$\Delta V = \frac{-0.228 \pm \sqrt{0.228^2 - 4 \times 3.618E\text{-}3}}{2}$$

$$= \frac{-0.228 \pm 0.193}{2} = -0.2105\ m^3 \text{ or } = -17.5 \times 10^{-3}\ m^3$$

Since $-0.210\ m^3$ is not possible, the correct $\Delta V = -17.5 \times 10^{-3}\ m^3$.

Note Neglection of ΔV^2 term will give

$$\Delta V = \frac{3.618 \times 10^{-3}}{0.228} = -15.8 \times 10^{-3}\ m^3 \quad [\text{close}]$$

Final $V = 150 - 17.5 = 132.5\,L$

Final $p = \frac{364.2}{V} = \frac{364.2}{0.1325} = 2748\ \mathrm{KPa} = \underline{2.748\ \mathrm{mPa}}$

For Tank - A

(c) Integrating $\dot{E} = \dot{Q} - \dot{W} + \cancel{\dot{m}_i h_i} - \dot{m}_e h_e$

$$\Delta E = Q - W - h_e \int_0^t \dot{m}_e \, dt$$

$$= Q - W - h_e \, 1.913 \text{ (kg)} \quad \left[\text{from part 2}\right]$$

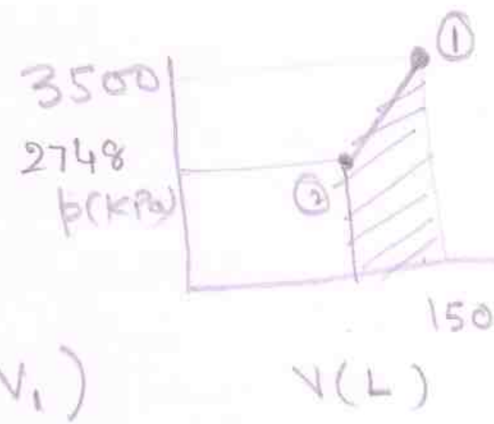
$$m_2 C_v T_A - m_1 C_v T_A = Q - W - C_p T_A \times 1.913$$

$$293 \times \frac{R}{\gamma - 1} \underbrace{\{m_2 - m_1\}}_{-1.913} = Q - W - \frac{\gamma R}{\gamma - 1} \times 1.913 \times 293$$

$$\Rightarrow Q - W = 293 \times \frac{8.314}{0.4} \left\{-1.913 + 1.4 \times 1.913\right\}$$

$$Q - W = 4660 \text{ KJ}$$

$$Q = 4660 + W$$



The process for the piston

piston work for gas

$$\int_1^2 p\,dV = \left(\frac{p_1 + p_2}{2}\right)(V_2 - V_1)$$

$$= \left(\frac{3500 + 2748}{2}\right)\left(\frac{150 - 132.5}{1000}\right)$$

$$= -54.67 \text{ KJ}$$

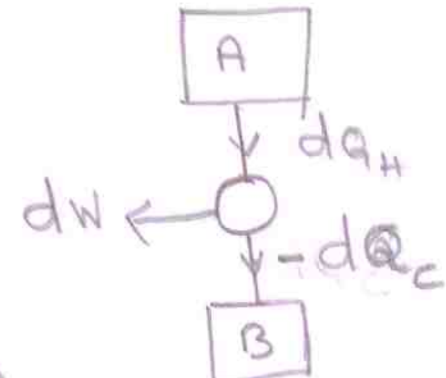
$$\therefore Q = 4660 - 54.67 = \underline{\underline{4605.3 \text{ KJ}}}$$

Heat flows in

7. Engine is reversible.

I-Law for Engine

$dQ_H - dQ_c = dW$ (1)



The above is for an infinitesimal process

Applying I-Law for Block A

$dU = dQ - \cancel{dW}^{\,0}$

$\therefore \; dQ = M_A C \, dT_A$

$dQ_H = -dQ = -M_A C \, dT_A$ (2)

Applying I-Law for Block B $\Rightarrow$

$dU = dQ - \cancel{dW}^{\,0} \Rightarrow M_B C \, dT_B$

As $-dQ_c = -dQ \Rightarrow dQ_c = M_B C \, dT_B$ (3)

From (1) (2) + (3) we get

$dW = -M_A C \, dT_A - M_B C \, dT_B$

combining A, B and engine as one system, and applying II Law

$\Delta S = \cancel{\int \frac{dQ}{T}}_{\,0} + \cancel{S_p}^{\,0}$ as process is <u>reversible</u>

No heat interaction with Surroundings

$\Rightarrow \Delta S = 0$

$\Rightarrow \Delta S_A + \cancel{\Delta S_E}^{\,0} + \Delta S_B = 0$

Reversible engine + cyclic

From $T\,ds = mc\,dT$ {incompressible}

$\Delta S = mc \ln \frac{T_f}{T_i}$

$\Rightarrow \Delta S_A + \Delta S_B = m_A c_A \ln \frac{T_f}{T_{iA}} + m_B c_B \ln \frac{T_f}{T_{iB}} = 0$

$2 \times 1 \ln \frac{T_f}{373} + 1 \times 1 \ln \frac{T_f}{303} = 0$

$\ln \left(\frac{T_f}{373}\right)^2 + \ln \frac{T_f}{303} = 0$

$\Rightarrow \frac{T_f^3}{373^2 \times 303} = 1 \Rightarrow T_f = \underline{348.0\,K}$

Applying I-Law to Combined System

$\Delta E = \cancel{Q}^{\,0} - W \Rightarrow W = -\Delta E$

$W = -(\Delta E_A + \cancel{\Delta E_E}^{\,0} + \Delta E_B)$

$= -\left[m_A c (T_f - 373) + m_B c (T_f - 303) \right]$

$= -\left[2 \times 1 (348 - 373) + 1 \times 1 (348 - 303) \right]$