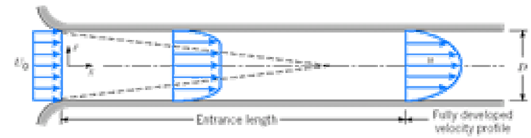
Heat Transfer in Circular ducts

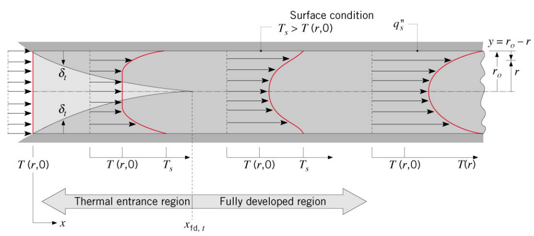
- Flow inside passages is the most common application of Fluid Mechanics and heat transfer
- Flow and heat transfer in between rod bundles are the most common application in Nuclear Engineering
- When velocity increases beyond a critical value, several whirls called vortices are formed
- This is called Turbulent Flow. In this case the velocity and temperature continuously fluctuate with time
- The transition to turbulent is governed by the Reynolds number
- Its value in circular ducts is typically 2300

Fully Developed Flow-I



- It implies that the velocity profile does not change along length
- The non-dimensional entrance length (L_h /D) is ~ 0.06 Re
- This is small in turbulent flow (L_h /D ~ 6-10)
- Since velocity profile is same, it implies that wall shear is same or friction factor is constant along length

Fully Developed Flow-II



- The entrance length (L_t /D) is ~ 0.05 Re Pr
- The entrance length is large for oils (Pr>>1)
- For turbulent flow L_t /D ~ 10
- In the fully developed region Nu is constant (h = constant)

Thermodynamic Mean Temperature-I

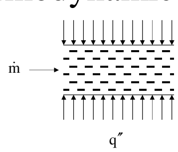
- This is also called bulk coolant temperature or mixing cup temperature or mixed mean temperature
- Since average mean temperature will be mass weighted

$$T_B = \frac{\int_A \rho u C T dA}{\int_A \rho u C dA} = \frac{\int_A C T d\dot{m}}{\int_A C d\dot{m}}$$

- If C is constant then

$$T_B = \frac{\int_A T d\dot{m}}{\dot{m}}$$

Thermodynamic Mean Temperature-II



- Integration over a length of pipe gives

$$\Rightarrow \int_{outlet} \rho c_p u T dA = \int_{inlet} \rho c_p u T dA + q'' P \Delta x$$

$$\Rightarrow \cancel{\dot{m} c_p T_B \Big|_{inlet}} + \frac{d(\dot{m} c_p T_B)}{dx}\Big|_{inlet} \Delta x + HOT = \cancel{\dot{m} c_p T_B \Big|_{inlet}} + q'$$

- For steady flow and constant c_p, when we shrink the length to 0, we get $\frac{dT_B}{dx} = \frac{q'}{\dot{m} c_p}$

Features for Constant Heat Flux Case-I

- The last equation in previous slide implies that the mean temperature varies linearly for constant heat flux case.
- In ducts since velocity of fluid at wall is zero

$$q''_{out} = -k \frac{dT}{dr} \quad \Rightarrow q''_{in} = k \frac{dT}{dr} = h(T_{wall} - T_{fluid})$$

- This will permit experimental evaluation of h
- For constant heat flux case, in fully developed region $T_{wall} - T_B$ = constant
- The above implies that for constant heat flux case the wall temperature would also vary linearly

Features for Constant Heat flux Case-II

- With the assumption of fully developed flow and h being constant, we can derive the temperature distribution

$$\frac{dT_B}{dx} = \frac{q'}{\dot{m} c_p}$$

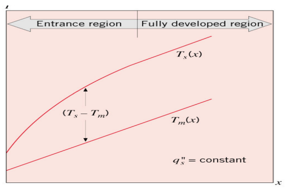
Boundary condition that $T_B = T_{Bo}$ at $x = 0$

$$\Rightarrow T_B = T_{Bo} + \frac{q' x}{\dot{m} c_p}$$

Note that q' = q" P

$$\Rightarrow T_W = T_B + \frac{q''}{h}$$

If h is known, T_w can be determined



Features for Constant Temperature Case

- For constant temperature case

$$\frac{dT_B}{dx} = \frac{q'}{\dot{m} c_p} = \frac{q'' P}{\dot{m} c_p} = \frac{h(T_W - T_B)P}{\dot{m} c_p}$$

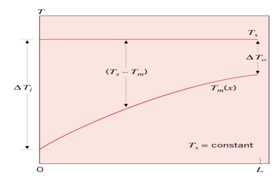
Boundary condition
$T_B = T_{Bo}$ at x = 0

- Defining $T_W - T_B = \theta$, we get,

Boundary condition
$\theta = \theta_o$ at x = 0

$$-\frac{d\theta}{dx} = \frac{hP\theta}{\dot{m} c_p}$$

$$\Rightarrow \theta = \theta_0 e^{-\frac{hPx}{\dot{m} c_p}}$$



Laminar Flow in Pipes-I

- V = V(r,z)

Continuity Equation

$$\frac{1}{r}\frac{\partial(rV_r)}{\partial r} + \frac{\partial V_z}{\partial z} = 0$$

⇒
- rV_r is independent of r
- Since V_r at r = R is 0, V_r is 0 everywhere
- Hence there is only $V_z = V_z(r)$

Laminar Flow in Pipes-II

r - Momentum Equation

$$\rho\left(\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + + V_z\frac{\partial V_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV_r)}{\partial r}\right) + + \frac{\partial^2 V_r}{\partial z^2}\right)$$

⇒
- p is only a function of z $\Rightarrow \frac{\partial p}{\partial z} = \frac{dp}{dz}$

Laminar Flow in Pipes-III

z - Momentum Equation

$$\rho\left(\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + + V_z\frac{\partial V_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{\partial^2 V_z}{\partial z^2}\right)$$

$$\Rightarrow \frac{\partial p}{\partial z} = \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)$$

f(z) f(r)

⇒ **LHS = RHS = constant**

$$\frac{dp}{dz} = \mu\frac{1}{r}\frac{d}{dr}\left(r\frac{dV_z}{dr}\right) = \text{Cons tan t}$$

Laminar Flow in Pipes-IV

Transposing r, we can write

$$\frac{d}{dr}\left(r\frac{dV_z}{dr}\right) = r\frac{1}{\mu}\frac{dp}{dz}$$

Integrating once with r

$$r\frac{dV_z}{dr} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r^2}{2} + C_1$$

Using the boundary condition that flow is symmetric as r→0

$$r\frac{dV_z}{dr} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r^2}{2} + C_1 \Rightarrow C_1 = 0 \quad \therefore r\frac{dV_z}{dr} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r^2}{2}$$

Laminar Flow in Pipes-V

Transposing r, we can write

$$\therefore \frac{dV_z}{dr} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r}{2}$$

Integrating with r

$$V_z = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r^2}{4} + C_2$$

Using the boundary condition $V_z = 0$ at $r = R$

$$C_2 = -\left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{R^2}{4} \qquad \therefore V_z = \frac{1}{4\mu}\left(-\frac{dp}{dz}\right)\left(R^2 - r^2\right) = \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$V_z = \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(1 - \frac{r^2}{R^2}\right)$$

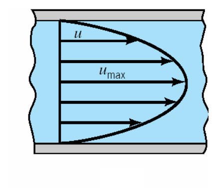
Laminar Flow in Pipes-VI

- Velocity distribution is parabolic

$V_z = V_z$ (max) at r = 0

$$\therefore V_z(\max) = \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)$$

$$\Rightarrow \quad V_z = V_z(\max)\left(1 - \frac{r^2}{R^2}\right)$$



Laminar Flow in Pipes-VIII

Average Velocity

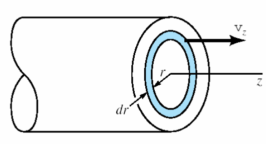
$$\overline{V}_z = \frac{1}{\pi R^2}\int_0^R v_z\, 2\pi r\, dr = \frac{2\pi}{\pi R^2}\int_0^R v_z\, r\, dr = \frac{2}{R^2}\int_0^R v_z\, r\, dr$$

$$\overline{V}_z = \frac{2V_z(\max)}{R^2}\int_0^R\left[1 - \frac{r^2}{R^2}\right] r\, dr = \frac{2V_z(\max)}{R^2}\int_0^R\left[r - \frac{r^3}{R^2}\right] dr$$

$$\overline{V}_z = \frac{2V_z(\max)}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$

$$\overline{V}_z = \frac{2V_z(\max)}{R^2}\frac{R^2}{4} = \frac{V_z(\max)}{2}$$

$$= \frac{-R^2}{8\mu}\frac{dP}{dz}$$



(b)

Laminar Flow in Pipes-IX

Shear Stress

$$V_z = V_z(\max)\left(1 - \frac{r^2}{R^2}\right)$$

$$\tau_{rz} = \tau_{rz} = \mu\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right) = \mu V_z(\max)\left(\frac{-2r}{R^2}\right)$$

$$= \mu\frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(\frac{-2r}{R^2}\right) \qquad \because V_z(\max) = \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)$$

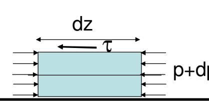
$$= -\frac{r}{2}\left(-\frac{dp}{dz}\right)$$

Direction is Negative as dp/dz is negative

Force Balance

$$2\pi r dz\tau + \pi r^2 dp = 0 \qquad \Rightarrow \tau = \frac{r}{2}\left(-\frac{dp}{dz}\right)$$

Note the answer is same, as direction of τ is implicitly assumed in force balance



Laminar Flow in Pipes-X

Fanning Friction Factor

- Fanning Friction factor can be calculated as follows

$$f = \frac{|\tau_w|}{0.5\rho\overline{V}_z^2}$$

Further $\overline{V}_z = \frac{R^2}{8\mu}\left(-\frac{dp}{dz}\right)$ Refer two slides back

and $\tau_w = \tau|_{r=R} = -\frac{R}{2}\left(-\frac{dp}{dz}\right)$ From previous slide

$$\Rightarrow f = \frac{(R/2)/(dp/dz)}{0.5\rho\overline{V}_z\left(R^2/8\mu\right)(dp/dz)} = \frac{8\mu}{\rho\overline{V}_z R} = \frac{16\mu}{\rho\overline{V}_z D} = \frac{16}{Re}$$

Laminar Flow in Pipes-XI

Pressure Drop

From force balance

$$\tau_w = \frac{R}{2}\left(-\frac{dp}{dz}\right)$$

$$\Rightarrow \left(-\frac{dp}{dz}\right) = \frac{2\tau_w}{R} = \frac{4\tau_w}{D} = \frac{4\left(0.5\rho\overline{V}_z^2 f\right)}{D} = \frac{4\left(\rho\overline{V}_z^2 f\right)}{2D}$$

$$= \frac{4\rho\overline{V}_z^2}{2D}\left(\frac{16\mu}{\rho\overline{V}_z D}\right) = \frac{32\mu\overline{V}_z}{D^2}$$

Linear dependence with velocity

$$\therefore -\Delta p = \frac{32\mu\overline{V}_z L}{D^2}$$

Some Aspects of Fully Developed Thermal Conditions-I

- Mathematically the fully developed state is represented by

$$\frac{\partial}{\partial z}\left(\frac{T_W - T}{T_W - T_B}\right)=0 \quad \Rightarrow \frac{(T_W-T)}{(T_W-T_B)} \neq f(z) \qquad \frac{\partial}{\partial r}\left(\frac{T_W-T}{T_W-T_B}\right)\neq f(z)$$

- Since T_W and T_B are not functions of r, we can write

$$\frac{1}{T_W-T_B}\frac{\partial}{\partial r}(-T)\neq f(z) \qquad \Rightarrow \frac{1}{T_W-T_B}k\frac{\partial}{\partial r}(-T)\neq f(z)$$

$$\frac{q''_{out}}{k(T_W-T_B)}\neq f(z) \qquad \Rightarrow \frac{h}{k}\neq f(z)$$

Heat transfer coefficient is not a function of z, if k is constant

Laminar Heat Transfer in Pipes-I

Assumptions

- We will derive this for constant heat flux case
- Fully developed velocity and temperature
- Constant fluid properties
- Axi-symmetric flow
- No body force

Laminar Heat Transfer in Pipes-II

- For constant heat flux, and h not a function of z implies

$$\frac{q''_{out}}{h}=(T_W-T_B)= \text{Constant} \qquad \Rightarrow \frac{\partial T_W}{\partial z}=\frac{\partial T_B}{\partial z}$$

- For fully developed flow, we had stated that

$$\frac{\partial}{\partial z}\left(\frac{T_W-T}{T_W-T_B}\right)=0 \quad \Rightarrow \frac{\partial}{\partial z}(T_W-T)=0 \quad \Rightarrow \frac{\partial T_W}{\partial z}=\frac{\partial T}{\partial z}$$

$$\therefore \frac{\partial T_W}{\partial z}=\frac{\partial T_B}{\partial z}=\frac{\partial T}{\partial z}$$

- As we had shown that the bulk coolant temperature is linear, this implies that the first derivative is constant and second derivative is zero

$$\Rightarrow \frac{\partial^2 T_W}{\partial z^2}=\frac{\partial^2 T_B}{\partial z^2}=\frac{\partial^2 T}{\partial z^2}=0$$

Laminar Heat Transfer in Pipes-III

- The governing energy equation is

$$\rho c_p\left(\cancel{\frac{\partial T}{\partial t}}+\cancel{V_r\frac{\partial T}{\partial r}}+V_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\cancel{\frac{\partial^2 T}{\partial z^2}}\right)+\cancel{q'''}$$

$$\left(V_z\frac{\partial T}{\partial z}\right)=\alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)$$

- Employing the velocity distribution derived earlier

$$\left(2\cancel{\bar{V}_z}\left(1-\frac{r^2}{R^2}\right)\frac{q''P}{\rho A\cancel{\bar{V}_z}c_p}\right)=\alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)$$

$$\Rightarrow\left(\frac{2}{\alpha}\left(1-\frac{r^2}{R^2}\right)\frac{q''}{\rho c_p}\frac{2}{R}\right)=\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)$$

Laminar Heat Transfer in Pipes-III

$$\Rightarrow\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)=\frac{4q''}{kR}\left(1-\frac{r^2}{R^2}\right) \qquad \Rightarrow\left(\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)=\frac{4q''}{kR}\left(r-\frac{r^3}{R^2}\right)$$

$$\Rightarrow\left(r\frac{\partial T}{\partial r}\right)=\frac{4q''}{kR}\left(\frac{r^2}{2}-\frac{r^4}{4R^2}\right)+c_1$$

Using the condition that at r=0, ∂T/ ∂r = 0 implies c_1 = 0

$$\Rightarrow\frac{\partial T}{\partial r}=\frac{4q''}{kR}\left(\frac{r}{2}-\frac{r^3}{4R^2}\right) \qquad \Rightarrow T=\frac{4q''}{kR}\left(\frac{r^2}{4}-\frac{r^4}{16R^2}\right)+c_2$$

- Using the condition, $T = T_W$ at $r = R$, we get

$$\Rightarrow T_W=\frac{4q''}{kR}\left(\frac{R^2}{4}-\frac{R^4}{16R^2}\right)+c_2$$

Laminar Heat Transfer in Pipes-IV

$$\Rightarrow T=\frac{4q''}{kR}\left(\frac{r^2}{4}-\frac{r^4}{16R^2}-\frac{3R^2}{16}\right)+T_W$$

- Since T_W is not explicitly not known, we will link it with T_B, which is known at any axial location

$$T_B=\frac{\int_A \rho V_z C T dA}{\dot{m}C}=\frac{\int_A \cancel{\rho} V_z \cancel{C} T 2\cancel{\pi} r dr}{\cancel{\rho}\bar{V}_z \cancel{\pi} R^2 \cancel{C}}=\frac{2\int_A V_z T r dr}{\bar{V}_z R^2}$$

$$\Rightarrow T_B=\frac{2\int_0^R\left[2\cancel{\bar{V}_z}\left(1-\frac{r^2}{R^2}\right)\right]\left[\frac{4q''}{kR}\left(\frac{r^2}{4}-\frac{r^4}{16R^2}-\frac{3R^2}{16}\right)+T_W\right]r dr}{\cancel{\bar{V}_z}R^2}$$

Laminar Heat Transfer in Pipes-V

$$\Rightarrow T_B = \int_0^R \left[\frac{16q''}{kR^3} \left\{ \left(\frac{r^3}{4} - \frac{r^5}{16R^2} - \frac{3R^2 r}{16} \right) - \left(\frac{r^5}{4R^2} - \frac{r^7}{16R^4} - \frac{3r^3R^2}{16R^2} \right) \right\} + \frac{4}{R^2} \left\{ T_W r - T_W \frac{r^3}{R^2} \right\} \right] dr$$

$$\Rightarrow T_B = \left[\frac{16q''}{kR^3} \left\{ \left(\frac{r^4}{16} - \frac{r^6}{96R^2} - \frac{3R^2 r^2}{32} \right) - \left(\frac{r^6}{24R^2} - \frac{r^8}{128R^4} - \frac{3r^4}{64} \right) \right\} + \frac{4}{R^2} \left\{ T_W \frac{r^2}{2} - T_W \frac{r^4}{4R^2} \right\} \right]_0^R$$

$$\Rightarrow T_B = \left[\frac{16q''}{kR^3} \left\{ \left(\frac{R^4}{16} - \frac{R^4}{96} - \frac{3R^4}{32} \right) - \left(\frac{R^4}{24} - \frac{R^4}{128} - \frac{3R^4}{64} \right) \right\} + \frac{4}{R^2} \left\{ T_W \frac{R^2}{2} - T_W \frac{R^2}{4} \right\} \right]$$

$$\Rightarrow T_B = \left[\frac{11}{24} \frac{q''R}{k} + T_W \right] \qquad \Rightarrow T_W - T_B = \frac{11}{24} \frac{q''R}{k}$$

$$\Rightarrow \frac{24}{11} = \frac{q''R}{k(T_W - T_B)} \qquad \Rightarrow \frac{48}{11} = \frac{hD}{k}$$

$\Rightarrow Nu_D = 4.36$

For constant heat flux

Laminar Heat Transfer in Pipes-VI

- For constant temperature case it is a bit more messy
- We shall state without going through this messy proof that

$\Rightarrow Nu_D = 3.66$ For constant temperature case

- The above cases were for laminar heat transfer
- The heat transfer coefficient increases as turbulent sets in
- We use empirical equations obtained through experiments

Turbulent Heat Transfer in Pipes-I

- We had shown in Nucl 350 that using the Karman's university velocity profile friction factor was shown to be

$$\sqrt{\frac{1}{f}} = 2.05 \log\left(Re\sqrt{f}\right) - 1.1$$

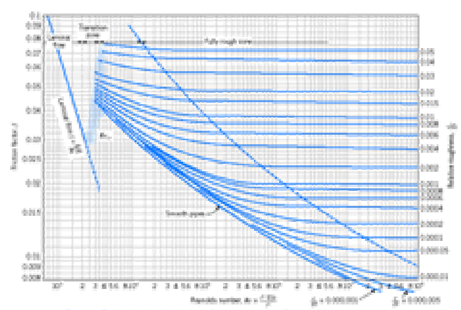
- Using large experimental data this was modified as

$$\sqrt{\frac{1}{f}} = \left(2.0 \log Re\sqrt{f} - 0.8\right)$$

- A composite relation was derived by Colebrook for both rough and smooth pipes as

$$\sqrt{\frac{1}{f}} = -2.0 \log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right)$$

Turbulent Heat Transfer in Pipes-II



Moody's Chart

Turbulent Heat Transfer in Pipes-III

- For computer calculations the following relations are useful

$$f = \frac{64}{Re} \qquad \text{For } Re < 2300$$

$$\sqrt{\frac{1}{f}} = -1.8 \log\left(\left(\frac{\varepsilon / D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right) \qquad \text{For } Re > 2300$$

- The second equation can be approximated for smooth pipes as

$$f = 0.316 Re^{-0.25} \qquad \text{For } 2300 < Re < 2\text{x}10^5$$

$$f = 0.184 Re^{-0.2} \qquad \text{For } Re > 2\text{x}10^5$$

Turbulent Heat Transfer in Pipes-IV

- We had introduced Reynolds and modified Reynolds analogy earlier for laminar flows
- The friction coefficient introduced is same as Fanning friction factor and is four times smaller than Darcy's friction factor

$$C_f = f_{Fanning} = \frac{f_{Darcy}}{4}$$

- If we use the second equation for the turbulent flows and employ modified Reynolds analogy, we get

$$\frac{C_f}{2} = \frac{f}{8} = \frac{0.184 Re^{-0.2}}{8} = 0.023 Re^{-0.2} = \frac{Nu}{Re Pr^{1/3}}$$

Turbulent Heat Transfer in Pipes-V

$$\Rightarrow Nu = 0.023 Re^{0.8} Pr^{1/3}$$

- The above equation is modified to give the Dittus-Boelter Equation and is the most common correlation used in turbulent flows

$$\Rightarrow Nu = 0.023 Re^{0.8} Pr^{n}$$

n = 0.4 for heating ($T_W > T_B$), n = 0.3 for cooling ($T_W < T_B$)

- The properties are calculated at Mean Bulk coolant temperature
- The validity of the above has been checked for

$0.7 < Pr < 160$
$Re_D > 10{,}000$
$L/D > 10$

It is about +/- 15%

Turbulent Heat Transfer in Pipes-VI

- For High temperature difference between the wall and the bulk fluid, Sieder-Tate equation is mostly used

$$Nu = 0.027 Re^{0.8} Pr^{1/3} \left(\frac{\mu}{\mu_W}\right)^{0.14}$$

- The properties are calculated at Bulk Coolant temperature, except for μ_w that is taken at wall Temp.
- The validity of the above has been checked for

$0.7 < Pr < 16{,}700$
$Re_D > 10{,}000$
$L/D > 10$

It is about +/- 15%

For more accurate equations, one can refer your book and other quoted literature

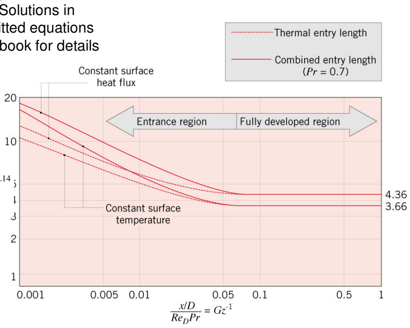
Heat Transfer at Entrance Region

In the developing region heat transfer coefficient is larger. Solutions in graphical form and fitted equations are available. Refer book for details

$$\overline{Nu}_D = 1.86 \left(\frac{Re_D Pr}{L/D}\right)^{1/3} \left(\frac{\mu}{\mu_W}\right)^{0.14}$$

For constant T and

$$\left(\frac{Re_D Pr}{L/D}\right)^{1/3} \left(\frac{\mu}{\mu_W}\right)^{0.14} \geq 2$$



Review of Heat Transfer in Internal Passages

- In internal passages, boundary layers develop and merge
- This leads to fully developed regions
- In the developed region, both friction factor and Nusselt number are constant
- In laminar flow, the values of Nu = 4.36 for constant wall heat flux case and is 3.66 for constant wall temperature case.
- Modified Reynolds analogy predicts the Nusselt number in turbulent case. Correlations have been presented

Heat Transfer in Complex Passages

- In fluid mechanics we have seen that we could use circular tube correlation by introducing the concept of hydraulic diameter,

$$D_h = \frac{4\ \text{Area}}{\text{Wetted Perimeter}}$$

- In turbulent flow the method works reasonably well
- However, specialized correlations exist for several shapes in literature and they may be used for better prediction

Convective Mass Transfer

- The concepts developed for heat transfer hold good for mass transfer as the governing equations are identical
- The bulk vapor density is defined as

$$\rho_{A,m} = \frac{\int_A \rho_A V_z dA}{\overline{V}_z A}$$

- Local mass flux is defined as

$$n'' = h_m (\rho_{A,s} - \rho_{A,m})$$

- Relations for Sherwood numbers can be expressed as

$Sh_D = 3.66$ For laminar flow

$Sh_D = 0.023 Re^{0.8} Sc^{0.4}$ For turbulent flow