ME 210
Heat Transfer

Kannan Iyer
Kiyer@me.iitb.ac.in

Department of Mechanical Engineering
Indian Institute of Technology, Bombay

What is Heat Transfer?

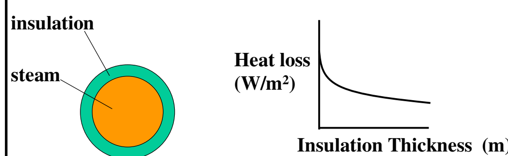
- The science of quantification of rate of heat interaction between system and surroundings.
- The laws of thermodynamics are inadequate for this purpose as it deals with equilibrium process
- Thus governing conservation equation of energy is not capable of quantifying the rate of interaction locally.
- It turns out that empirical laws are required that play a key role.
- These laws along with the conservation equations of mass momentum and energy quantify the total heat transfer from engineering equipment

Why Study Heat Transfer?

- Most man made objects during their evolution chain undergoes heat transfer process.
- Cooling or heating is an everyday occurrence
  - Computer needs to be cooled,
  - Automobiles need to be cooled/heated,
  - Rooms have to be heated/cooled, etc., etc.
- Without advancement in this field, there will be
  - No space shuttle,
  - No superconducting magnet,
  - No nuclear reactors, etc., etc.

How is it Useful?

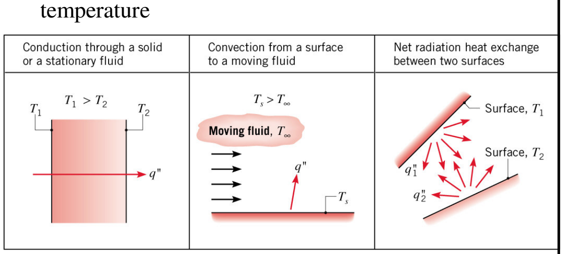
- It enables taking decisions on sizing of equipment



- Higher the heat loss, higher heating cost
- Thicker the insulation, higher the capital cost
- So a optimum can be chosen for a give life of insulation

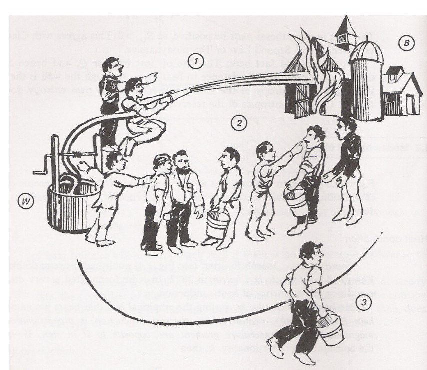
Modes of Heat Transfer

- Heat transfer occurs when a system at one temperature interacts with a system at another temperature



Molecular Diffusion **Bulk Motion** **Electromagnetic waves**

Acknowledgement: Taken from Lienhard's text book



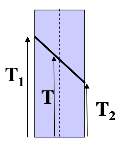
Heat Transfer by Conduction

- Fourier in 1822 came up with an empirical law

$$\vec{q}'' = -k\nabla T$$

W/m² W/m-K K/m

- In one dimensional slab case



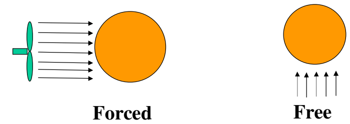
  - Heat flux crossing any plane must be constant at steady state

$$\Rightarrow \vec{q}'' = -k\frac{dT}{dx} = c \qquad \Rightarrow \frac{dT}{dx} = \frac{T_1 - T_2}{L}$$

  - The above implies that temperature varies linearly as shown

Heat Transfer by Convection-I

- Heat transfer by bulk motion is called convection

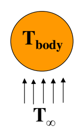


- Convection heat transfer is a cumulative effect of molecular diffusion and bulk motion
- The concepts involved require solution of Navier-Stokes equations along with Fourier's law hence complicated

Heat Transfer by Convection-II

- To get a feel, it is easier to invoke Newton's law of cooling. Newton (1701) suggested

$$\frac{dT_{body}}{dt} \alpha \ (T_{body} - T_{\infty})$$



- First law of thermodynamics states that

$$Mc\frac{dT_{body}}{dt} = \dot{Q} - \dot{W} \qquad \Rightarrow \dot{Q} \ \alpha \frac{dT_{body}}{dt}$$

$$\Rightarrow \dot{Q} \ \alpha \ (T_{body} - T_{\infty}) \qquad \Rightarrow q'' \ \alpha \ (T_{body} - T_{\infty})$$

$$\Rightarrow q'' = h(T_{body} - T_{\infty})$$ **h is called heat transfer coefficient**

- Unit of h ➡ $h = q''/(T_{body} - T_{\infty})$ **W/m²-K**

Heat Transfer by Radiation-I

- Stefan (1879) established experimentally a law, which was given a firm footing by Boltzmann (1884) and is called the Stefan-Bltzmann Law
- It states that every black body emits radiation at all times at a rate given by

$$E_b(W/m^2) = \sigma(T_{body}(K))^4 = q''_{emitted}$$

Where, $\sigma = 5.67 \times 10^{-8}$ (W/m²-K⁴)

- Real surfaces emit a fraction of what a black body emits

$$E(W/m^2) = \varepsilon \ \sigma(T_{body}(K))^4 = q''_{emitted}$$

Heat Transfer by Radiation-II

- The constant ε introduced in the last equation is called emissivity and is dimensionless
- Just as the body emits radiation, the surroundings also emit radiation which the body receives.
- For a simple case of a small body interacting with another surface that fully encloses it, then the net heat transfer (q''_{out}-q''_{in}) can be written as (This will be proved later in the course)

$$q''_{net} = \varepsilon \ \sigma(T^4_{body} - T^4_{surroundings})$$

- It is pointed out that radiation is a surface phenomenon and the temperatures refers to the surfaces exchanging radiation

Heat Diffusion Equation

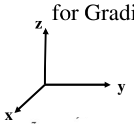
- Heat Diffusion Equation, sometimes simply called Heat Equation is the conservation of energy equation with conduction as the only mode of heat transfer within the domain.
- Prior to its derivation, it will be useful to elaborate on the Fourier law of heat conduction

$$\vec{q}'' = -k\nabla T$$

- In the above equation, The direction of heat flux is opposite to the direction of the gradient
- Physically, the direction of gradient, is the direction along which the temperature changes maximum

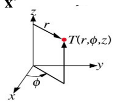
Gradient

- From Vector calculus, we can write the expression for Gradient as



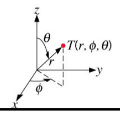
$$\nabla = \vec{e}_x \frac{\partial}{\partial x} + \vec{e}_y \frac{\partial}{\partial y} + \vec{e}_z \frac{\partial}{\partial z}$$

Cartesian System (rectangular coordinates)



$$\nabla = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\phi \frac{1}{r}\frac{\partial}{\partial \phi} + \vec{e}_z \frac{\partial}{\partial z}$$

in Cylindrical coordinate system



$$\nabla = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \vec{e}_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

in spherical coordinate system

Heat Diffusion Equation-I

- As stated earlier, Heat equation is the first law of thermodynamics with only conduction
- To allow the analysis for heat generating media such as nuclear fuels, we will account for a heat generation term
- Conceptually stated for a control volume,

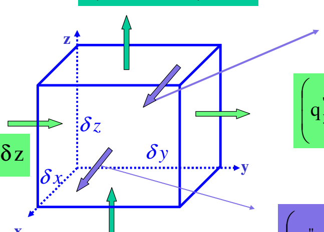
Rate of increase of energy in the control volume = **Net Rate of energy flowing into the control volume by conduction** + **Rate of energy generated within the control volume**

$$\dot{E}_{CV} = \dot{E}_{net} + \dot{E}_{gen}$$

Heat Diffusion Equation-II

$$\left(q_z'' + \frac{\partial q_z''}{\partial z}\delta z\right)\delta x \delta y$$

$$q_x'' \, \delta y \, \delta z$$



$$\left(q_y'' + \frac{\partial q_y''}{\partial y}\delta y\right)\delta x \, \delta z$$

$$q_y'' \delta x \, \delta z$$

$$q_z'' \, \delta x \delta y$$

$$\left(q_x'' + \frac{\partial q_x''}{\partial x}\delta x\right)\delta y \, \delta z$$

Heat Diffusion Equation-III

- Looking at the net rate of energy diffusing in, we can write

$$\dot{E}_{net} = \dot{Q}_{in} - \dot{Q}_{out}$$

$$\dot{Q}_{in} = q_x'' \, \delta y \, \delta z + q_y'' \delta x \, \delta z + q_z'' \, \delta x \delta y$$

$$\dot{Q}_{out} = \left(q_x'' + \frac{\partial q_x''}{\partial x}\delta x\right)\delta y \, \delta z + \left(q_y'' + \frac{\partial q_y''}{\partial y}\delta y\right)\delta x \, \delta z + \left(q_z'' + \frac{\partial q_z''}{\partial z}\delta z\right)\delta x \delta y$$

$$\therefore \dot{E}_{net} = -\left(\frac{\partial q_x''}{\partial x} + \frac{\partial q_y''}{\partial y} + \frac{\partial q_z''}{\partial z}\right)\delta x \delta y \delta z$$

- Rate of increase of energy in control volume can be expressed as

$$\dot{E}_{CV} = \frac{\partial(\rho c T \delta x \delta y \delta z)}{\partial t} = \frac{\partial(\rho c T)}{\partial t}\delta x \delta y \delta z$$

Heat Diffusion Equation-IV

- Rate of energy generated within the control volume can be expressed as

$$\dot{E}_{gen} = q''' \delta x \delta y \delta z$$

$$\dot{E}_{CV} = \dot{E}_{net} + \dot{E}_{gen} \quad \Rightarrow \quad \frac{\partial(\rho c T)}{\partial t} = -\left(\frac{\partial q_x''}{\partial x} + \frac{\partial q_y''}{\partial y} + \frac{\partial q_z''}{\partial z}\right) + q'''$$

$$\Rightarrow \frac{\partial(\rho c T)}{\partial t} = -\left(\frac{\partial}{\partial x}\left(-k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(-k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(-k\frac{\partial T}{\partial z}\right)\right) + q'''$$

$$\Rightarrow \frac{\partial(\rho c T)}{\partial t} = \left(\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)\right) + q'''$$

Heat Diffusion Equation-V

- For a constant property system

$$\Rightarrow \rho c \frac{\partial(T)}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + q'''$$

Cartesian

- In general for a constant property system

$$\rho c \frac{\partial(T)}{\partial t} = k\nabla^2 T + q'''$$

- In general, for a variable property system

$$\rho c \frac{\partial(T)}{\partial t} = \nabla.(k\nabla T) + q'''$$

Heat Diffusion Equation-VI

- For a constant property system

$$\rho c\frac{\partial (T)}{\partial t}=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}\right)+q'''$$

Cylindrical

$$\rho c\frac{\partial (T)}{\partial t}=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)\right)+q'''$$

Spherical

- For a variable property system

$$\rho c\frac{\partial (T)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+q'''$$

Cylindrical

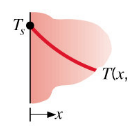
Heat Diffusion Equation-VII

$$\rho c\frac{\partial (T)}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+q'''$$

Spherical

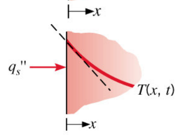
- Mathematically heat equation at steady state falls under the class of Boundary Value Problems
- Simply stated, for obtaining the solution within the domain, we need to specify the boundary values completely
- It has three class of boundary conditions

Heat Diffusion Equation-VIII



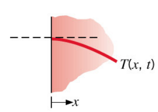
$T(0,t)=T_S$

Temperature specified

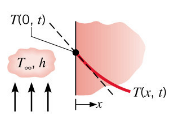


$-k\frac{\partial T}{\partial n}(0,t)=q_s''$

Flux specified



Special case: $q_s''=0$

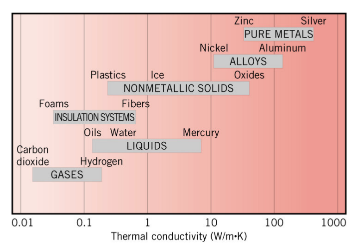


$-k\frac{\partial T}{\partial n}(0,t)=h\left(T_\infty-T(0,t)\right)$

Convective boundary

Properties

Thermal conductivity k (W/m-K)



- In a constant property system, during transients another property α=k/(ρc) comes into play
- This is called thermal diffusivity
- We shall refer to it later