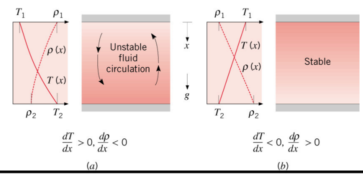
Free Convection

- Fluid motion generated due to density changes is called free convection
- The word free refers to absence of a mechanical device to drive the flow.
- Free convection or natural convection plays an important role in the safety of nuclear reactors as it is a passive means of heat removal
- We will see the important case of flow in a vertical plate and study the empirical correlations for some common configurations

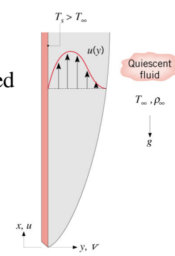
Onset of Natural Convection

- The fluid convection is started due to instability of flow
- Heavier fluid sitting on top of a lighter fluid is inherently unstable and any perturbation tumbles the fluid.
- The topic of instability is quite involved and so will be deferred to a graduate level course



The Governing Equations-I

- The velocity distribution has a characteristic shape
- Since velocity and temperature are intimately coupled, there is one boundary layer thickness
- Boundary layer assumptions are valid
- One of the standard assumption invoked is called Boussinessq approximation
- This states that the density variation with temperature need to be accounted only in the body force term of the momentum equation



The Governing Equations-II

- Thus the governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho_\infty\left(u\frac{\partial (u)}{\partial x} + v\frac{\partial (u)}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right) - \rho g, \qquad \frac{\partial p}{\partial y} = 0$$

$$\left(u\frac{\partial (T)}{\partial x} + v\frac{\partial (T)}{\partial y}\right) = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right)$$

- Since there is no pressure gradient normal to the flow, the longitudinal pressure gradient inside the Boundary layer will be same as out side

$$\frac{\partial p}{\partial x} = \frac{\partial p_\infty}{\partial x} = -\rho_\infty g$$

The Governing Equations-III

- Thus the x momentum equation can be simplified as

$$\rho_\infty\left(u\frac{\partial (u)}{\partial x} + v\frac{\partial (u)}{\partial y}\right) = \mu\left(\frac{\partial^2 u}{\partial y^2}\right) - (\rho - \rho_\infty)g$$

- The above approximation is valid only for moderate changes in temperatures
- As temperature changes are not high, the equation of state is written in a linear form as follows

$$\rho = \rho_\infty + \left.\frac{\partial \rho}{\partial T}\right|_{T_\infty}(T - T_\infty)$$

The Governing Equations-IV

- Introducing the definition of the volumetric expansion coefficient (isobaric)

$$\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T} \qquad \beta|_{T_\infty} = -\frac{1}{\rho_\infty}\left.\frac{\partial \rho}{\partial T}\right|_{T_\infty}$$

$$\Rightarrow \rho = \rho_\infty - \beta\rho_\infty(T - T_\infty) \qquad \Rightarrow \frac{\rho - \rho_\infty}{\rho_\infty} = -\beta(T - T_\infty)$$

- Therefore momentum equation can be written as

$$\left(u\frac{\partial (u)}{\partial x} + v\frac{\partial (u)}{\partial y}\right) = \mu\left(\frac{\partial^2 u}{\partial y^2}\right) + \beta(T - T_\infty)g$$

The Governing Equations-V

- Thus the governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\left(u\frac{\partial (u)}{\partial x}+v\frac{\partial (u)}{\partial y}\right)=\nu\left(\frac{\partial^2 u}{\partial y^2}\right)+g\beta(T-T_\infty) \qquad \frac{\partial p}{\partial y}=0$$

$$\left(u\frac{\partial (T)}{\partial x}+v\frac{\partial (T)}{\partial y}\right)=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)$$

- If we non-dimensionalize the equations using

$x^* = x/L$, $y^* = y/L$, $u^* = u/u_{ref}$, $v^* = v/v_{ref}$, $T^* = (T-T_\infty)/(T_W-T_\infty)$

We get,

The Governing Equations-VI

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$$

$$\left(u^*\frac{\partial (u^*)}{\partial x}+v^*\frac{\partial (u^*)}{\partial y}\right)=\frac{1}{Re_L}\left(\frac{\partial^2 u^*}{\partial y^{*2}}\right)+\frac{g\beta(T_W-T_\infty)L}{u_{Ref}^2}T^*$$

$$\left(u^*\frac{\partial (T^*)}{\partial x^*}+v^*\frac{\partial (T^*)}{\partial y^*}\right)=\frac{1}{Re_L\,Pr}\left(\frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

- Since we do not have a reference velocity, the scale chosen is, $u_{ref} = \nu/L$

$$\Rightarrow \frac{g\beta(T_W-T_\infty)L}{u_{Ref}^2}T^*=\frac{g\beta(T_W-T_\infty)L^3}{\nu^2}T^*=Gr_L T^*$$

The Governing Equations-VII

- Gr_L is called the Grashof Number and represents the ratio of buoyancy force to the viscous force
- From the governing equations we can deduce that $Nu_L = f(Re_L, Pr, Gr_L)$
- With the definition of the velocity scale we introduced, Re_L will take the form

$$Re_L=\frac{\nu}{L}\frac{L}{\nu}=1$$

- Hence, $Nu_L = f(Gr_L, Pr)$

The Governing Equations-VIII

- If however, velocity scale existed then the parameter Gr_L/Re_L^2 represents the importance of gravity term
- If $Gr_L/Re_L^2 >> 1$ gravity term is dominant and is called free convection
- If $Gr_L/Re_L^2 << 1$ gravity term is weak and can be neglected and we call this forced convection
- If $Gr_L/Re_L^2 \sim 1$ we call this as mixed convection as forced and free effects are present

The Governing Equations-IX

- The equation can be solved by both Similarity method as well as integral method
- Both are messy and involves considerable algebra
- So we shall look at the correlations and shall only apply them

Correlations for Natural Convection-I

- The similarity solution leads to the following Correlation

$$Nu_x=\left(\frac{Gr_x}{4}\right)^{0.25} f(Pr)$$

- Where, the term f(Pr) is given by:

$$f(Pr)=\frac{0.75\,Pr^{0.5}}{\left(0.609+1.221\,Pr^{0.5}+1.238\,Pr\right)^{0.25}}$$

Correlations for Natural Convection-II

- Often for design purposes, we need the average Nusselt number. This can be obtained by integration as we have done for Forced convection

$$\overline{Nu}_L = \frac{4}{3} Nu_L$$

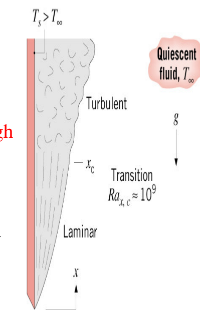
- A very common empirical equation used in laminar flow is

$$\overline{Nu}_L = 0.68 + \frac{0.67\, Ra_L^{0.25}}{\left(1 + (0.492 / Pr)^{9/16}\right)^{4/9}}$$

Correlations for Natural Convection-III

- Transition to Turbulence
  - Amplification of disturbances depends on relative magnitudes of buoyancy and viscous forces.
  - Transition occurs at a Critical Rayleigh Number. $Ra_{x,c} \sim 10^9$
  - Rayleigh Number is defined as

$$Ra_x = Gr_x\, Pr = \frac{g\beta (T_W - T_\infty) x^3}{\nu^2} \frac{\nu}{\alpha} = \frac{g\beta (T_W - T_\infty) x^3}{\nu\alpha}$$



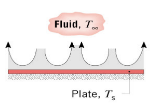
Correlations for Natural Convection-IV

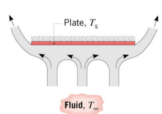
- Empirical Correlation (Churchill and Chu)

$$\overline{Nu}_L = \left[0.825 + \frac{0.387\, Ra_L^{1/6}}{\left(1 + (0.492 / Pr)^{9/16}\right)^{8/27}}\right]^2$$

- In horizontal plates, the physics is a bit different

Hot face upward





Hot face downward

Correlations for Natural Convection-V

- The correlation for the heated wall upward is

$$\overline{Nu}_L = 0.54\, Ra_L^{1/4} \qquad \text{For } 10^4 < Ra_L < 10^7$$

$$\overline{Nu}_L = 0.15\, Ra_L^{1/3} \qquad \text{For } 10^7 < Ra_L < 10^{11}$$

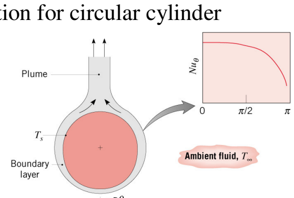
- The correlation for the heated wall downward is

$$\overline{Nu}_L = 0.27\, Ra_L^{1/4} \qquad \text{For } 10^5 < Ra_L < 10^{10}$$

- Similar correlations are available for inclined plates

Correlations for Natural Convection-VI

- Correlation for circular cylinder



$$\overline{Nu}_D = \left[0.60 + \frac{0.387\, Ra_L^{1/6}}{\left(1 + (0.559 / Pr)^{9/16}\right)^{8/27}}\right]^2 \qquad Ra_D < 10^{12}$$

- Similarly correlations are available for many geometries