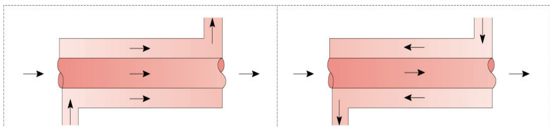
Heat Exchangers-I

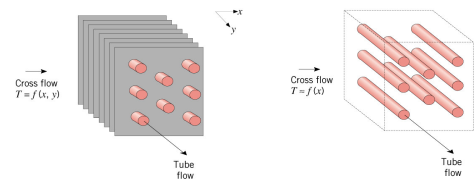
- Heat exchangers are the most common devices used in energy conversion and utilization.
- They involve heat exchange between two fluids separated by a solid and encompass a wide range of flow configurations.
- Concentric pipe heat exchangers is the simplest of the configurations



Parallel-Flow   Counter-Flow

Heat Exchangers-II

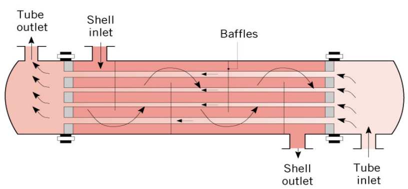
- Cross-flow Heat Exchangers
  - ➢ For cross-flow over the tubes, fluid motion, and hence mixing, in the transverse direction (y) is prevented for the finned tubes, but occurs for the unfinned condition.
  - ➢ Heat exchanger performance is influenced by mixing.



Finned (Both Fluids Unmixed)   Unfinned (One Fluid Mixed and the Other Unmixed)

Heat Exchangers-III

- Shell-and-Tube Heat Exchangers

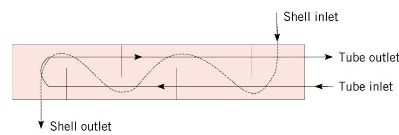


One Shell Pass and One Tube Pass

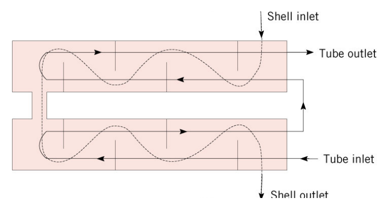
- ➢ Baffles are used to establish a cross-flow and to induce turbulent mixing of the shell-side fluid, both of which enhance convection.
- ➢ The number of tube and shell passes may be varied

Heat Exchangers-IV

One Shell Pass, Two Tube Passes

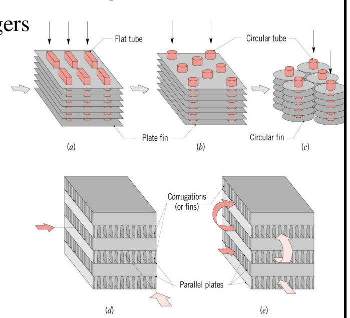


Two Shell Passes, Four Tube Passes



Heat Exchangers-V

- Compact Heat Exchangers
  - ➢ Widely used to achieve large heat rates per unit volume, particularly when one or both fluids is a gas.
  - ➢ Characterized by large heat transfer surface areas per unit volume, small flow passages, and laminar flow.



Heat Exchangers-VI

- Each exchanger design involves several empirical equations
- Instead dealing with specific shapes, we shall look at the general theoretical features
- The methods discussed should give a reasonable picture so that you should be able understand specific design procedure and special cases in later life, if required
- We shall begin with pipe in pipe heat exchanger

Heat Exchangers-VII

- The assumption usually made is that the heat transfer coefficient does not change along the length
- We have already defined an overall heat transfer coefficient earlier

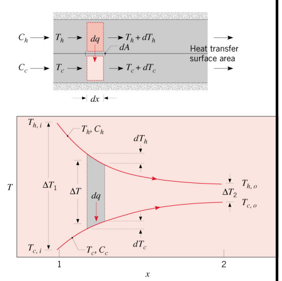
$$\Rightarrow \frac{1}{UA} = \frac{1}{h_i A_i} + \frac{\ln(R_o / R_i)}{2\pi L k_w} + \frac{1}{h_i A_i}$$

- The heat transfer coefficients are arrived from suitable correlations
- In many cases, the wall resistance is negligible
- U can be based on inner diameter and can be written as

$$\Rightarrow \frac{1}{U_i} = \frac{1}{h_i} + \frac{1}{h_o (d_o / d_i)}$$

Heat Exchangers-VIII

- Consider parallel flow
- Energy balance implies



$$dq = \dot{m}_c c_{p,c} dT_c = -\dot{m}_h c_{p,h} dT_h$$

$$\frac{dq}{C_c} = dT_c, \text{ where } C_C = \dot{m}_c c_{p,c}$$

$$\frac{dq}{-C_h} = dT_h, \text{ where } C_h = \dot{m}_h c_{p,h}$$

$$-dq\left(\frac{1}{C_h} + \frac{1}{C_c}\right) = dT_h - dT_c$$

$$\Rightarrow -dq\left(\frac{1}{C_h} + \frac{1}{C_c}\right) = d(T_h - T_c)$$

Also $dq = U_i P_i dx\,(T_h - T_c)$

Heat Exchangers-IX

$$\Rightarrow -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) = \frac{d(T_h - T_c)}{dq} = \frac{d(T_h - T_c)}{U_i P_i (T_h - T_c) dx}$$

$$\Rightarrow -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) dx = \frac{d(T_h - T_c)}{U_i P_i (T_h - T_c)}$$

- Integration from inlet to outlet we get,

$$\Rightarrow -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \int_0^L dx = \frac{1}{U_i P_i} \int_{inlet}^{outlet} \frac{d(T_h - T_c)}{(T_h - T_c)}$$

$$\Rightarrow -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) L = \frac{1}{U_i P_i} \ln\left(\frac{(T_h - T_c)_{outlet}}{(T_h - T_c)_{inlet}}\right)$$

Heat Exchangers-X

- Overall energy balance gives

$$q = C_c (T_{c,outlet} - T_{c,inlet}) = -C_h (T_{h,outlet} - T_{h,inlet})$$

$$\Rightarrow \frac{q}{(T_{c,outlet} - T_{c,inlet})} = C_c \quad -\frac{q}{(T_{h,outlet} - T_{h,inlet})} = C_h$$

$$\Rightarrow -\left(-\frac{(T_{h,outlet} - T_{h,inlet})}{q} + \frac{(T_{c,outlet} - T_{c,inlet})}{q}\right) L = \frac{1}{U_i P_i} \ln\left(\frac{(T_h - T_c)_{outlet}}{(T_h - T_c)_{inlet}}\right)$$

$$\Rightarrow \left(\frac{T_{h,outlet} - T_{c,outlet}}{q} - \frac{T_{h,inlet} - T_{c,inlet}}{q}\right) L = \frac{1}{U_i P_i} \ln\left(\frac{(T_h - T_c)_{outlet}}{(T_h - T_c)_{inlet}}\right)$$

$$\Rightarrow \frac{(T_h - T_c)_{outlet} - (T_h - T_c)_{inlet}}{\ln\left(\frac{(T_h - T_c)_{outlet}}{(T_h - T_c)_{inlet}}\right)} U_i A_i = q$$

Heat Exchangers-XI

$$\therefore q = U_i A_i \mathrm{LMTD} = U_i A_i \Delta T_m$$

Logarithmic Mean Temperature Difference

$$\mathrm{LMTD} = \Delta T_m = \frac{(T_h - T_c)_{outlet} - (T_h - T_c)_{inlet}}{\ln\left(\frac{(T_h - T_c)_{outlet}}{(T_h - T_c)_{inlet}}\right)}$$

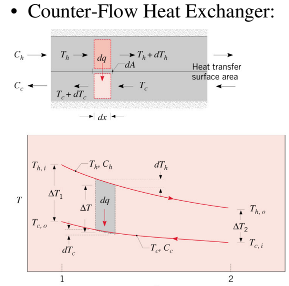
- Usually, this is written as

$$\mathrm{LMTD} = \Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

- The final equation is identical for counter-flow heat exchanger also

Heat Exchangers-XII

- Counter-Flow Heat Exchanger:



For the same inlet and exit conditions, counter-flow heat exchanger gives a higher LMTD and hence more compact

Heat Exchangers-XIII

- For the special case of counter flow when $C_c = C_h$

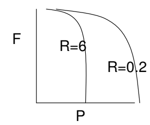
$$LMTD = \Delta T_m = \Delta T_2 = -\Delta T_1$$

- For shell and tube heat exchangers, the LMTD is modified by a correction factor,

$$\Delta T_m = F \Delta T_{m-counter-flow}$$

- These are available in the form of figures; F(R,P)

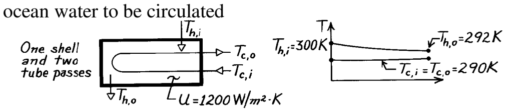
$$R = \frac{T_{Si} - T_{So}}{T_{to} - T_{ti}} \qquad P = \frac{T_{to} - T_{ti}}{T_{Si} - T_{ti}}$$



F factors are available for cross-flow heat exchangers

Problem 11.47

Design a two-pass shell and tube heat exchanger for Ocean Thermal Energy Conversion. The Power is to generate 2 MW(e) at an efficiency of 3%. Ocean water enters tubes at 300 K, desired outlet Temperature is 292 K. The working fluid boils at 290 K and U = 1200 W/m^2-K. Calculate the mass flow rate of ocean water to be circulated



PROPERTIES: *Table A-6*, Water ($\bar{T}_m$ = 296 K): c_p = 4181 J/kg·K.

$$q = \frac{2\,MW}{0.03} = 66.7\,MW \qquad LMTD_{CF} = \frac{(300 - 290) - (292 - 290)}{\ln\left(\frac{(300 - 290)}{(292 - 290)}\right)} = 5$$

$$R = \frac{T_{Si} - T_{So}}{T_{to} - T_{ti}} = \infty \qquad P = \frac{T_{to} - T_{ti}}{T_{Si} - T_{ti}} = 0 \qquad \Rightarrow F = 1$$

$$A = \frac{q}{U \times F \times LMTD_{CF}} = \frac{6.67 \times 10^7}{1200 \times 1 \times 5} = 11{,}100\ m^2$$

Water flow rate

$$\dot{m} = \frac{q}{c_p \times (T_{h,i} - T_{h,o})} = \frac{6.67 \times 10^7}{4181 \times 8} = 1994\ kg/s$$

Effectiveness-NTU Method-I

- Limitations of the LMTD Method
  - ➢ The LMTD method may be applied to design problems for which the fluid flow rates and inlet temperatures, as well as a desired outlet temperature of one fluid, are prescribed.
  - ➢ For a specified HX type, the required size (surface area), as well as the other outlet temperature, are readily determined.
  - ➢ If the LMTD method is used in performance calculations for which both outlet temperatures must be determined from knowledge of the inlet temperatures, for a given heat exchanger, the solution procedure is iterative.
  - ➢ For both design and performance calculations, the effectiveness-NTU method may be used without iteration.

Effectiveness-NTU Method-II

- Definitions
  - Heat exchanger effectiveness (ε)

$$\varepsilon = \frac{q}{q_{max}}$$

  - Maximum possible heat rate:

$$q_{max} = C_{min}(T_{hi} - T_{ci})$$

Where, C_{min}= Min (C_h or C_c)

  - Number of Transfer Units, (NTU)

$$NTU = \frac{UA}{C_{min}}$$

  - ➢ A dimensionless parameter whose magnitude influences HX performance:

Effectiveness-NTU Method-III

- Consider a parallel flow heat exchanger

$$q = C_c(T_{co} - T_{ci}) = C_h(T_{hi} - T_{ho})$$
$$q_{max} = C_{min}(T_{hi} - T_{ci})$$

No attention is paid to sign. Both are positive

- Let us consider the case in which C_h be equal to C_{min}
- By definition

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h(T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{ci})} = \frac{(T_{hi} - T_{ho})}{(T_{hi} - T_{ci})}$$

- From the first equation above, we can write

$$\frac{C_h}{C_C} = \frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ho})} = \frac{C_{min}}{C_{max}} \qquad T_{co} = T_{ci} + \frac{C_{min}}{C_{max}}(T_{hi} - T_{ho})$$

Effectiveness-NTU Method-IV

- In our LMTD derivation, we showed that

$$-\left(\frac{1}{C_h}+\frac{1}{C_c}\right)L = \frac{1}{U_iP_i}\ln\left(\frac{(T_h-T_c)_{outlet}}{(T_h-T_c)_{inlet}}\right)$$

$$\Rightarrow -\frac{1}{C_h}\left(1+\frac{C_h}{C_c}\right)U_iA_i = \ln\left(\frac{(T_h-T_c)_{outlet}}{(T_h-T_c)_{inlet}}\right)$$

$$\Rightarrow -\frac{U_iA_i}{C_{min}}\left(1+\frac{C_{min}}{C_{max}}\right) = \ln\left(\frac{(T_{ho}-T_{co})}{(T_{hi}-T_{ci})}\right)$$

$$\Rightarrow \left(\frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}}\right) = \exp\left[-\frac{U_iA_i}{C_{min}}\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

- In the expression for ε in previous slide, we do not have T_{co}, hence we shall eliminate it

Effectiveness-NTU Method-V

$$\Rightarrow \left(\frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}}\right) = \exp\left[-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

$$\Rightarrow \left(\frac{T_{ho}}{T_{hi}-T_{ci}}-\frac{T_{co}}{T_{hi}-T_{ci}}\right) = \exp\left[-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

Substituting for T_{co} from 2 slides back

$$\Rightarrow \left(\frac{T_{ho}}{T_{hi}-T_{ci}}-\frac{T_{ci}+C_{min}/C_{max}\,(T_{hi}-T_{ho})}{T_{hi}-T_{ci}}\right) = \exp\left[-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

$$\Rightarrow \left(\frac{T_{ho}-T_{ci}}{T_{hi}-T_{ci}}-\varepsilon\frac{C_{min}}{C_{max}}\right) = \exp\left[-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

$$\Rightarrow \left(\frac{T_{ho}-T_{hi}+T_{hi}-T_{ci}}{T_{hi}-T_{ci}}-\varepsilon\frac{C_{min}}{C_{max}}\right) = \exp\left[-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

Effectiveness-NTU Method-VI

$$\Rightarrow \left(\frac{T_{ho}-T_{hi}+T_{hi}-T_{ci}}{T_{hi}-T_{ci}}-\varepsilon\frac{C_{min}}{C_{max}}\right) = \exp\left[-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

$$\Rightarrow \left(-\varepsilon+1-\varepsilon\frac{C_{min}}{C_{max}}\right) = \exp\left[-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

$$\Rightarrow 1-\varepsilon\left(1+\frac{C_{min}}{C_{max}}\right) = \exp\left[-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

$$\Rightarrow \varepsilon = \frac{1-\exp\left[-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right]}{\left(1+\frac{C_{min}}{C_{max}}\right)}$$

Effectiveness-NTU Method-VII

- Thus, we can write

$$\varepsilon = f(NTU, C_r)$$

Where, $C_r = C_{min}/C_{max}$)

- Rearranging the effectiveness equation, we can write

$$\Rightarrow NTU = \frac{-\ln\left[1-\varepsilon\left(1+\frac{C_{min}}{C_{max}}\right)\right]}{\left(1+\frac{C_{min}}{C_{max}}\right)}$$

- Thus, we can write

$$NTU = f(\varepsilon, C_r)$$

- Similar equations are available for other heat exchanger Configurations. These are summarized in Tables 11.3 and 11.4

Effectiveness-NTU Method-VIII

- Steps in Problem Solution
  - For the given flow rates and fluid properties, compute C_h, C_c
  - Identify the C_{min} and compute C_r
  - From the given data and suitable correlations, compute U
  - If the temperatures are known, compute ε
  - Otherwise, if area is known, compute UA and NTU
  - Using either ε (NTU, C_r) or NTU(ε, C_r) determine NTU or ε respectively
  - Using the above result, either the area or the temperatures can be found out

Example-I

A parallel flow double pipe heat exchanger heat exchanger has hot and cold water flowing at 10 and 25 kg/min respectively. The inlet water temperatures are 70°C and 25°C. Calculate the area of the exchanger, if (a) h = 1600 W/m²-K on both sides, exit temperature on the hot side is required to be 50°C. Take c_{ph} = 4179 J/kg-K, c_{pc}=4174 J/kg-K

$$C_C = \dot{m}_c c_{pc} = \frac{25}{60}\times 4174 = 1739\ W/k$$

$$C_h = \dot{m}_c c_{ph} = \frac{10}{60}\times 4179 = 696.5\ W/k$$

$$\therefore C_r = \frac{696.5}{1739} = 0.4$$

$$\varepsilon = \frac{(T_{hi} - T_{ho})}{(T_{hi} - T_{ci})} = \frac{(70 - 50)}{(70 - 25)} = 0.444$$

$$NTU = \frac{-\ln\left[1 - \varepsilon\left(1 + \frac{C_{min}}{C_{max}}\right)\right]}{\left(1 + \frac{C_{min}}{C_{max}}\right)} = \frac{-\ln\left[1 - 0.444\,(1 + 0.4)\right]}{(1 + 0.4)} = 0.695$$

$$\therefore UA = NTU \times C_{min} = 0.695 \times 696.5 = 484\ W/K$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{1600} + \frac{1}{1600} = \frac{1}{800}$$

$$\therefore A = \frac{484}{800} = 0.605\ m^2$$

Example-II

Now imagine that in the HX in the previous example, the hot flow rate is doubled and hence the heat transfer coefficient on that side increases by a factor of $2^{0.8}$. Estimate the exit temperatures of hot and cold streams

$$h_o = 1600 \times 2^{0.8} = 2785.8\,W/m^2 - K$$

$$\Rightarrow \frac{1}{U_i} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{1600} + \frac{1}{2785.8}$$

$$\Rightarrow U_i = 1016.3\,W/m^2 - K$$

$$C_C = \dot{m}_c c_{pc} = \frac{25}{60} \times 4174 = 1739\ W/k$$

$$C_h = \dot{m}_c c_{ph} = 696.5 \times 2 = 1393\ W/k$$

$$\therefore C_r = \frac{1393}{1739} = 0.8$$

$$\therefore NTU = \frac{UA}{C_{min}} = \frac{1016 \times 0.605}{1393} = 0.44$$

$$\Rightarrow \varepsilon = \frac{1 - \exp\left[-NTU\left(1 + \frac{C_{min}}{C_{max}}\right)\right]}{\left(1 + \frac{C_{min}}{C_{max}}\right)} = \frac{1 - \exp\left[-0.44\,(1 + 0.8)\right]}{(1 + 0.8)} = 0.304$$

$$\varepsilon = \frac{(T_{hi} - T_{ho})}{(T_{hi} - T_{ci})} = 0.304 = \frac{(70 - T_{ho})}{(70 - 25)}$$

$$\therefore T_{ho} = 70 - 0.304\,(70 - 25) = 56.3^{o}C$$

$$T_{co} = T_{ci} + \frac{C_{min}}{C_{max}}(T_{hi} - T_{ho}) = 25 + 0.8(70 - 56.3) = 36.0^{o}C$$