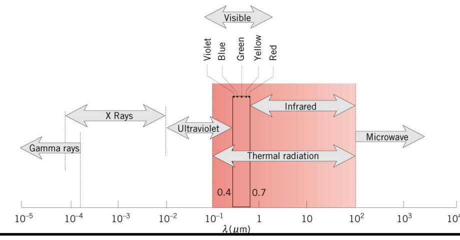
# Introduction to Radiation-I

- All bodies at T > 0 K emit electro magnetic radiation
- This is due to oscillations and transitions of electrons from different energy levels
- Emissions reduce the thermal energy levels and would continuously cool unless sustained by energy input
- Radiation can be intercepted and absorbed and this will account for increase in energy
- Emission for gas or semi-transparent solids and liquids is a volumetric phenomenon. However, for a opaque solid and liquid it is a surface phenomenon

# Introduction to Radiation-II

- Electromagnetic Radiation Characteristics
  - Electromagnetic radiation can be viewed as particles as well as waves condition.
  - When considered as waves, the wavelength and frequency characterizes the radiation

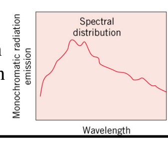



# Introduction to Radiation-III

- Wavelength (λ) and frequency (ν) are related to the velocity (c) by the expression

$$c = \lambda\nu$$

- Energy of the radiation is related to frequency. Higher the frequency, higher the energy
- The velocity of the electromagnetic is constant irrespective of the frequency and is equal to 2.998 x $10^8$ m/s
- Thermal radiation wavelengths range from infrared to ultraviolet regions $0.1 < \lambda < 100$ µm
- The amount of radiation emitted by an opaque surface varies with wavelength and can be described by the spectrum of the radiation




# Definitions-I

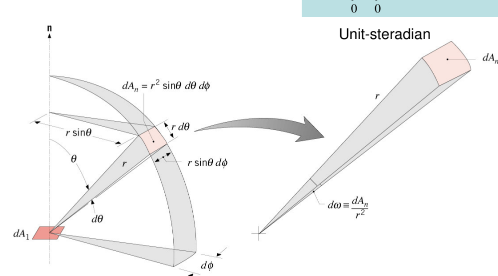
- Solid Angle

$$d\omega = \frac{dA_n}{r^2} = \frac{r\sin\theta d\phi\ r d\theta}{r^2}$$

Total hemispherical solid angle for a surface

$$\omega = \int_0^{\pi/2}\int_0^{2\pi}\sin\theta d\theta d\phi = 2\pi$$

Unit-steradian

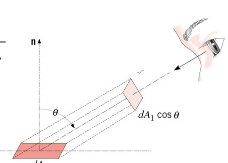



# Definitions-II

- Spectral Intensity
  - It is defined as rate of radiant energy emitted of wave length between λ and λ+dλ in a direction $(\theta, \phi)$ and $(\theta + d\theta, \phi + d\phi)$ per unit area of the emitting surface normal to this direction per unit solid angle

$$I_{\lambda,e}(\lambda,\theta,\phi) = \frac{dq}{dA_1\cos\theta d\omega d\lambda}$$

  - The above definition can be used to find q" when $I_{\lambda,e}$ is known, as follows




$$q'' = \left.\frac{dq}{dA_1}\right|_{Integrated} = \int_0^{\pi/2}\int_0^{2\pi}\int_0^{\infty} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi d\lambda$$

# Definitions-III

- Emissive Power
  - This is to define the radiation emitted by a surface and the so called hemispherical emissive power is identical to what is given in the previous slide

$$E(W/m^2) = \int_0^{\pi/2}\int_0^{2\pi}\int_0^{\infty} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi d\lambda$$

  - The spectral hemispherical emissive power is defined as

$$E_\lambda\left(\frac{W}{m^2 - \mu m}\right) = \frac{dE}{d\lambda} = \int_0^{\pi/2}\int_0^{2\pi} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

- Diffuse Emitter
  - It has been observed that many surfaces behave such that the intensity of radiation emission is independent of direction

## Definitions-IV

$$E_\lambda\left(\frac{W}{m^2-\mu m}\right) = I_{\lambda,e}(\lambda)\int_0^{\pi/2}\int_0^{2\pi}\cos\theta\sin\theta\, d\theta\, d\phi = I_{\lambda,e}(\lambda)\pi$$

- The spectral emissive power can be integrated to give

$$E\left(\frac{W}{m^2}\right) = \int_0^\infty I_{\lambda,e}(\lambda)\pi\, d\lambda = \pi I_e \quad \text{where} \quad I_e = \int_0^\infty I_{\lambda,e}(\lambda)\, d\lambda$$

- E and $I_e$ are called Total emissive power and total intensity respectively
- It may be noted that E is the total heat flux, whereas, I is the heat flux directed in different directions

## Definitions-V

- Similar to the radiation emission out from a surface, we can define incident radiation called Irradiation

$$G_\lambda = \int_0^{\pi/2}\int_0^{2\pi} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

$$G = \int_0^\infty G_{\lambda,}(\lambda)\, d\lambda$$

- Finally, we can define the term Radiosity, which represents the total radiation (emitted +reflected)

$$J_\lambda = \int_0^{\pi/2}\int_0^{2\pi} I_{\lambda,e+r}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

$$J = \int_0^\infty J_{\lambda,}(\lambda)\, d\lambda$$

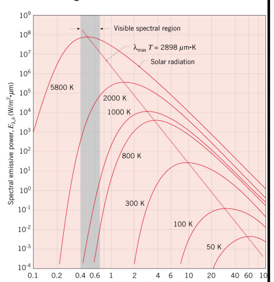
## Black Body Emission-I

- It is an idealized body with the following characteristics
  - ➢ Absorbs all radiation incident on it
  - ➢ Emits maximum energy (for a given Temperature)
  - ➢ It is a diffused emitter
- The spectral distribution of the blackbody emissive power (determined theoretically and confirmed experimentally) is

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left(\exp(C_2/\lambda T)-1\right)}$$

$$C_1 = 3.742\times10^8\ W(\mu m)^4/m^2$$

$$C_2 = 1.439\times10^4\ \mu m - K$$




## Black Body Emission-II

- The maximum wavelength of the spectrum can be described by Wien's Displacement Law

$$\lambda_{max}T = 2898\ \mu m - K$$

- The total emissive power of a black body can be obtained as

$$E_b(T) = \pi I_b(T) = \int_0^\infty \frac{C_1}{\lambda^5\left(\exp(C_2/\lambda T)-1\right)}\, d\lambda = \sigma T^4$$

Stefan-Boltzman Law | Where $\sigma = 5.67\times10^{-8}\ W/m^2\text{-}k^4$

- The fraction of radiation emitted between wavelength (0-λ) is given by

$$F_{(0-\lambda)} = \frac{\int_0^\lambda E_{\lambda,b}\, d\lambda}{\sigma T^4} = f(\lambda T)$$

Tabulated in Book

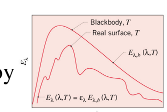
## Emissivity

- The Surface emission from a real surface is connected to a black surface using the term emissivity
- Its definition varies depending on context
- The spectral directional emissivity is defined by

$$\varepsilon_{\lambda,\theta,\phi}(\lambda,\theta,\phi,T) = \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_b(\lambda,T)}$$

- Total directional emissivity is defined by

$$\varepsilon_{\theta,\phi}(\theta,\phi,T) = \frac{I_{\lambda,e}(\theta,\phi,T)}{I_b(T)}$$




- The simplest of all, the total hemispherical emissivity

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda,T)E_{\lambda,b}(\lambda,T)\, d\lambda}{E_b(T)}$$

## Kirchoff's law

- One form of Kirchoff's law states that

$$\varepsilon_\lambda = \alpha_\lambda$$

- We shall not worry about its proof in this course
- The surfaces that satisfy the condition ($\varepsilon = \alpha$) is called a Gray Surface
- If a surface is opaque, then $\tau = 0$

$$\Rightarrow \alpha + \rho = 1 \qquad \Rightarrow \rho = 1-\alpha \qquad \Rightarrow \rho = 1-\varepsilon$$

# Problem 12-2

A diffused surface horizontal $A_1$ ($10^{-4}$ m$^2$) has a total emissive power of 5 x $10^4$ W/m$^2$. The radiation from this surface is intercepted by another vertical surface $A_2$ (5 x $10^{-4}$ m$^2$) as shown. Find Irradiation G on $A_2$

$I_{e,1} = E/\pi$

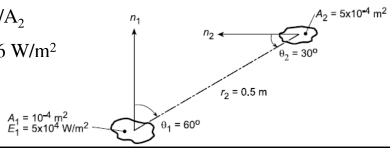
Amount of heat from $A_1$ to $A_2$ = $q_{1-2} = I_{e,1} A_1 \cos\theta_1 \, \omega_{2-1}$

$\omega_{2-1} = (A_2 \cos\theta_2)/r_2^2$

Hence $q_{1-2} = I_{e,1} A_1 \cos\theta_1 (A_2 \cos\theta_2)/r_2^2$

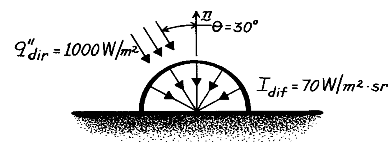
$G_2 = q_{1-2}/A_2$

$G_2 = 2.76$ W/m$^2$




# Problem 12-6

Consider the solar radiation that falls in the form of direct radiation of 1000 W/m$^2$ incident at an angle of 30° and a diffused radiation of 70 W/m$^2$-Sr What is the solar irradiation on the surface of earth




The total irradiation consists of two parts; direct and diffused

Since the direct radiation consists of parallel rays

$$G_{dir} = q''_{dir} \cos\theta = 1000 \cos(30) = 866 \text{ W/m}^2$$

The contribution from the diffuse radiation is

$$G_{dif} = \pi I_{dif} = \pi \times 70 = 219.9 \text{ W/m}^2$$

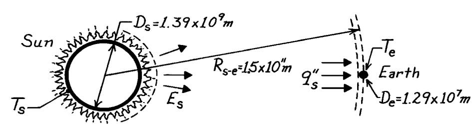
Therefore total radiation is

$$866 + 219.9 = 1085.9 \text{ W/m}^2$$

# Problem 12-18

The radiation flux on the outer surface of the earth is measured to be 1335 W/m$^2$. Given that the diameters of the earth and sun are respectively 1.29 x $10^7$ m and 1.39 x $10^9$ m respectively, find

a) Emissive power of the sun
b) Assuming that sun can be considered as black, estimate the temperature of the sun
c) What is the wavelength at which the spectral power is maximum
d) If earth can be considered as a black surface and sun is the only source of energy, find the equilibrium surface temperature




Energy balance on two concentric spheres, one on the surface of sun

$$E_s \pi D_S^2 = 4\pi (R_{s-e} - R_e)^2 q''_{earth}$$

$$E_s = 4\frac{(R_{s-e} - R_e)^2}{D_S^2} q''_{earth} = 4\frac{(1.5 \times 10^{11} - 0.654 \times 10^7)^2 1353}{(1.39 \times 10^9)^2}$$

$$= 6.302 \times 10^7 \text{ W/m}^2$$

From Stefan-Boltzmann law

$$T_s = \left(\frac{E_s}{\sigma}\right)^{1/4} = \left(\frac{6.302 \times 10^7}{5.67 \times 10^{-8}}\right)^{1/4} = 5774 \text{ K}$$

From Wien's Displacement law

$$\lambda_{max} = \left(\frac{2898}{T}\right) = \left(\frac{2898}{5774}\right) = 0.5 \mu m$$

As earth is considered black, it absorbs all the radiation and reradiates it. Now applying energy balance Stefan-Boltzman law, we can write

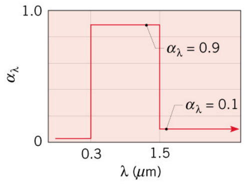
$$E_e \pi D_e^2 = q''_{earth} \frac{\pi D_e^2}{4} \Rightarrow E_e = \frac{q''_{earth}}{4} = \sigma T_e^4$$

$$T_s = \left(\frac{E_e}{\sigma}\right)^{1/4} = \left(\frac{1353}{4 \times 5.67 \times 10^{-8}}\right)^{1/4} = 278 \text{ K}$$

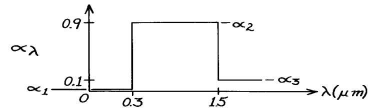
The temperature is warmer than this due to Greenhouse Effect as atmosphere is transparent to short wave but not long wave radiation

# Problem 12-49

If the spectral hemispherical absorptivity is as given in the figure, compute the solar absorptivity. If the temperature of the surface is 300 K, compute the total hemispherical emissivity for the Diffuse Surface




- For solar radiation the spectral emissivity can be assumed as Planck's distribution with T = 5800 K
- The absorptivity $\alpha(T) = \dfrac{\int_0^\infty \alpha_\lambda(\lambda,T)E_{\lambda,b}(\lambda,T)d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T)d\lambda}$




$$\alpha(T) = \frac{\alpha_1 \int_0^{0.3} E_{\lambda,b}(\lambda,T)d\lambda + \alpha_2 \int_{0.3}^{1.5} E_{\lambda,b}(\lambda,T)d\lambda + \alpha_3 \int_{1.5}^{\infty} E_{\lambda,b}(\lambda,T)d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T)d\lambda}$$

$$= \frac{\left[\alpha_1 0 + \alpha_2\left(F_{(0-1.50)} - F_{(0-0.3)}\right) + \alpha_3\left(F_{(0-\infty)} - F_{(0-1.5)}\right)\right]\sigma T^4}{F_{(0-\infty)}\sigma T^4}$$

See next slide

For $\lambda T$= 0.3x5800 = 1740, $F_{(0\text{-}0.3)}$ = 0.0335,
For $\lambda T$ = 1.5x5800 = 8700, $F_{(0\text{-}1.5)}$ = 0.8805,
$F_{(0\text{-}\infty)}$ = 1

$$= \frac{\left[0.9(0.8805 - 0.0335) + 0.1(1 - 0.8805)\right]\sigma T^4}{1\sigma T^4} = 0.774$$

**TABLE 12.1 Blackbody Radiation Functions**

| $\lambda T$ ($\mu$m · K) | $F_{(0\to\lambda)}$ | $I_{\lambda,b}(\lambda,T)/\sigma T^5$ ($\mu$m · K · sr)$^{-1}$ | $\dfrac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$ |
|---|---|---|---|
| 200 | 0.000000 | $0.375034 \times 10^{-27}$ | 0.000000 |
| 400 | 0.000000 | $0.490335 \times 10^{-13}$ | 0.000000 |
| 600 | 0.000000 | $0.104046 \times 10^{-8}$ | 0.000014 |
| 800 | 0.000016 | $0.991126 \times 10^{-7}$ | 0.001372 |
| 1,000 | 0.000321 | $0.118505 \times 10^{-5}$ | 0.016406 |
| 1,200 | 0.002134 | $0.523927 \times 10^{-5}$ | 0.072534 |
| 1,400 | 0.007790 | $0.134411 \times 10^{-4}$ | 0.186082 |
| 1,600 | 0.019718 | 0.249130 | 0.344904 |
| 1,800 | 0.039341 | 0.375568 | 0.519949 |
| 2,000 | 0.066728 | 0.493432 | 0.683123 |
| 2,200 | 0.100888 | $0.589649 \times 10^{-4}$ | 0.816329 |
| 2,400 | 0.140256 | 0.658866 | 0.912155 |
| 2,600 | 0.183120 | 0.701292 | 0.970891 |
| 2,800 | 0.227897 | 0.720239 | 0.997123 |
| 2,898 | 0.250108 | $0.722318 \times 10^{-4}$ | 1.000000 |
| 3,000 | 0.273232 | $0.720254 \times 10^{-4}$ | 0.997143 |
| 3,200 | 0.318102 | 0.705974 | 0.977373 |
| 3,400 | 0.361735 | 0.681544 | 0.943551 |
| 3,600 | 0.403607 | 0.650396 | 0.900429 |
| 3,800 | 0.443382 | $0.615225 \times 10^{-4}$ | 0.851737 |
| 4,000 | 0.480877 | 0.578064 | 0.800291 |

- For the given surface $\varepsilon_\lambda = \alpha_\lambda$

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{0.3} E_{\lambda,b}(\lambda,T)d\lambda + \varepsilon_2 \int_{0.3}^{1.5} E_{\lambda,b}(\lambda,T)d\lambda + \varepsilon_3 \int_{1.5}^{\infty} E_{\lambda,b}(\lambda,T)d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T)d\lambda}$$

- On similar lines we get

$$= \frac{\left[\alpha_1 0 + \alpha_2\left(F_{(0-1.50)} - F_{(0-0.3)}\right) + \alpha_3\left(F_{(0-\infty)} - F_{(0-1.5)}\right)\right]\sigma T^4}{F_{(0-\infty)}\sigma T^4}$$

For $\lambda T$= 0.3x300 = 90, $F_{(0\text{-}0.3)}$ = 0.0,
For $\lambda T$ = 1.5x300 = 450, $F_{(0\text{-}1.5)}$ = 0.0,
$F_{(0\text{-}\infty)}$ = 1

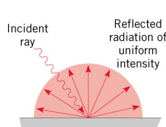
$$\Rightarrow \varepsilon = \frac{\left[0.1(1-0)\right]\sigma T^4}{1\sigma T^4} = 0.1$$

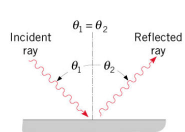
For the given surface ε would have been equal to α, if the spectral radiation would have been the same for irradiation and emission

# Absorption, Reflection, Transmission

- When radiation is incident on a surface, it can either be absorbed, reflected or transmitted
- Reflection can be specular or diffused
- The individual fractions are denoted by α, ρ and τ

$$\alpha + \rho + \tau = 1$$




# Radiation Exchange Between Surfaces

- We shall consider only the cases in which there is no participating media
- This implies the intervening media is either vacuum or gases that does not interact with radiation
- Gases like $H_2O$, $CO_2$ absorb radiation and would need special treatment
- Similarly presence of dust would scatter and would need special considerations
- The surfaces are assumed Isothermal, opaque, gray, diffuse

## View Factor-I

- Consider two black surfaces exchanging irradiation as shown. As black surface is a diffuse emitter

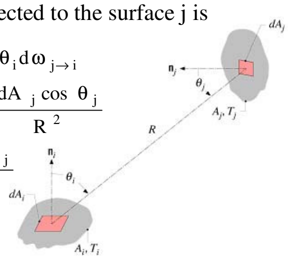
$$\Rightarrow I_{i,\theta i} = \frac{\sigma T_i^4}{\pi}$$

- The amount of heat directed to the surface j is

$$\Rightarrow dq_{i\to j} = I_{i,\theta i} dA_i \cos\theta_i d\omega_{j\to i}$$

$$\Rightarrow dq_{i\to j} = I_{i,\theta i} dA_i \cos\theta_i \frac{dA_j \cos\theta_j}{R^2}$$

$$= \frac{\sigma T_i^4}{\pi} \cos\theta_i dA_i \frac{dA_j \cos\theta_j}{R^2}$$




## View Factor-II

$$dq_{i\to j} = \sigma T_i^4 dA_i \frac{dA_j \cos\theta_i \cos\theta_j}{\pi R^2}$$

Heat going in all directions

Fraction going towards $dA_j$

- Therefore the total exchange from i to j shall be

$$q_{i\to j(net)} = \sigma T_i^4 \int_{A_i}\int_{A_j} \frac{dA_i dA_j \cos\theta_i \cos\theta_j}{\pi R^2}$$

Heat from I to j

$$\frac{q_{i\to j}}{\sigma T_i^4 A_i} = \frac{\int_{A_i}\int_{A_j} \frac{dA_i dA_j \cos\theta_i \cos\theta_j}{\pi R^2}}{A_i} = F_{ij}$$

Total heat emitted from i

Fraction, called View factor ($F_{ij}$)

## View Factor-III

- In the same way, the heat flowing from j to i can be written as

$$q_{j\to i} = \sigma T_j^4 A_j \frac{\int_{A_i}\int_{A_j} \frac{dA_i dA_j \cos\theta_i \cos\theta_j}{\pi R^2}}{A_j}$$

$F_{ji}$

- Note that

$$F_{ij} A_i = \int_{A_i}\int_{A_j} \frac{dA_i dA_j \cos\theta_i \cos\theta_j}{\pi R^2} = F_{ji} A_j$$

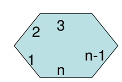
Reciprocity Relation

- The net heat transfer from i to j can be computed as

$$q_{i\to j(net)} = q_{i\to j} - q_{j\to i}$$

## View Factor-IV

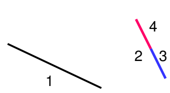
- View factor $F_{12}$ can be interpreted as the fraction of radiation leaving surface 1 and intercepted by surface 2
- View factor $F_{11}$ is not necessarily 0. For instance a concave surface will have a finite $F_{11}$
- For a closed enclosure with n-surfaces we can write $\sum_{j=1}^{n} F_{ij} = 1$



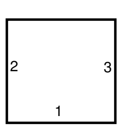

$F_{11} + F_{12} + .. + F_{1n} = 1; \ F_{21} + F_{22} + .. + F_{2n} = 1$, etc.

## View Factor-V

- View factor $F_{12} = F_{13} + F_{14}$, if surface 2 is made of surfaces 3 and 4. This means that view factor for the whole is sum of the view factors of the parts




- View factors with symmetric surfaces will be same




$F_{12} = F_{13}$

## Radiation Exchange Between Opaque, Diffuse and Gray Surfaces-I

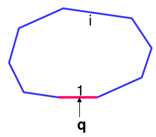
- We had just derived the radiation exchange between black surfaces as

$$q_{1-2} = \sigma\left(T_1^4 - T_2^4\right) A_1 F_{12}$$

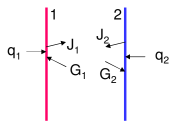
- Now we shall derive the same for real surfaces that have a finite reflectivity ($\rho = 1-\varepsilon \neq 0$)
- It is called Radiosity (J)-Irradience (G) method
- We shall first build the electrical analogy to get useful physical insight and then introduce matrix method

## Radiation Exchange Between Opaque, Diffuse and Gray Surfaces-I

- Consider N surfaces interacting with each other
- For every surface we shall define $q_i$ such that




$$q_i = A_i(J_i - G_i)$$




Note $q_i$ is external heat that has to be supplied to keep the surface at $T_i$

- Note that q will be positive for heat giving surface and negative for heat receiving surface

## Radiation Exchange Between Opaque, Diffuse and Gray Surfaces-II

- By definition

$$J_i = (E_i + \rho_i G_i) = (E_i + (1-\alpha_i)G_i)$$
$$= (\varepsilon_i E_{bi} + (1-\varepsilon_i)G_i)$$

- Since $G_i = \left(J_i - \frac{q_i}{A_i}\right)$ beacuse $q_i = A_i(J_i - G_i)$

$$\Rightarrow J_i = \left(\varepsilon_i E_{bi} + (1-\varepsilon_i)\left(J_i - \frac{q_i}{A_i}\right)\right)$$

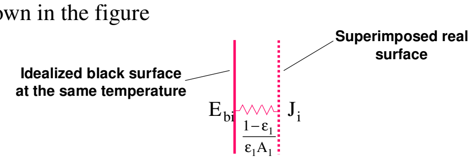
$$\Rightarrow J_i = \varepsilon_i E_{bi} + (1-\varepsilon_i)J_i - (1-\varepsilon_i)\frac{q_i}{A_i}$$

$$\Rightarrow \varepsilon_i(E_{bi} - J_i) = (1-\varepsilon_i)\frac{q_i}{A_i}$$

## Radiation Exchange Between Opaque, Diffuse and Gray Surfaces-II

$$\Rightarrow q_i = \frac{E_{bi} - J_i}{\left(\frac{(1-\varepsilon_i)}{\varepsilon_i A_i}\right)}$$

- The above equation suggests a network analogy as shown in the figure

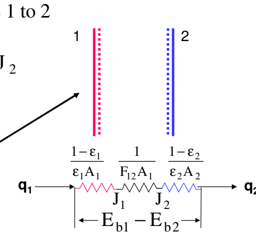



## Radiation Exchange Between Opaque, Diffuse and Gray Surfaces-III

- The fraction of $J_1$ leaving the surface and intercepted by the second surface will be $J_1 F_{12}$
- Similar amount of heat intercepted by surface 1 from $J_2$ will be $J_2 F_{21}$
- The net heat transfer from surface 1 to 2 can be written as

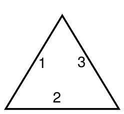
$$q_{12\,(net)} = A_1 F_{12} J_1 - A_2 F_{21} J_2$$
$$= A_1 F_{12}(J_1 - J_2)$$

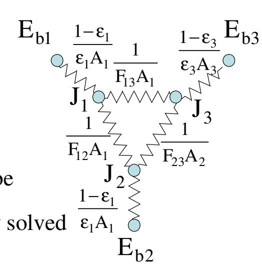
- Thus the network can be constructed as shown




## Radiation Exchange Between Opaque, Diffuse and Gray Surfaces-IV

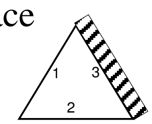
- The problem can now be extended to a N-surface interaction very easily
- As an illustration, three body interaction is shown




- The electrical network shall be
- These problems can be easily solved by matrix methods

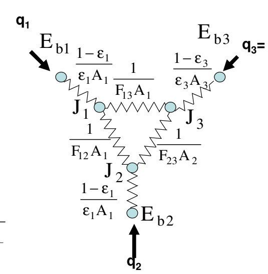
## Re-Radiating Surface




- Often in radiation heat transfer between two surfaces, we encounter a third surface, which simply re-radiates (there is no net heat transfer as far as that surface is considered)
- The net heat transfer $q_1 = -q_2$ can be found by

$$q_1 = \frac{E_{b1} - E_{b1}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2} + R}$$

where $\frac{1}{R} = \frac{1}{F_{12}A_1} + \frac{1}{\frac{1}{F_{13}A_1} + \frac{1}{F_{23}A_2}}$



## Matrix Method for N-surface interaction-I

- When the number of surfaces are large, matrix method is the ideal choice
- The algebra is straight forward, but the solution has to be obtained using numerical methods

$$q_i = A_i J_i - \sum_{j=1}^{N} F_{ji} J_j A_j$$

- Using reciprocity relation, we can write

$$q_i = A_i J_i - \sum_{j=1}^{N} F_{ij} J_j A_i = A_i \left( J_i - \sum_{j=1}^{N} F_{ij} J_j \right)$$

## Matrix Method for N-surface interaction-II

- Noting that sum of all $F_{ij} = 1$, we can write

$$\Rightarrow q_i = A_i \left( \sum_{j=1}^{N} F_{ij} J_i - \sum_{j=1}^{N} F_{ij} J_j \right) = \sum_{j=1}^{N} A_i F_{ij} (J_i - J_j) = \sum_{j=1}^{N} \frac{(J_i - J_j)}{\frac{1}{A_i F_{ij}}}$$

- We had shown previously that $q_i = \dfrac{E_{bi} - J_i}{\left(\frac{1-\varepsilon_i}{\varepsilon_i A_i}\right)}$
- Hence $\dfrac{E_{bi} - J_i}{\left(\frac{1-\varepsilon_i}{\varepsilon_i A_i}\right)} = \displaystyle\sum_{j=1}^{N} \frac{(J_i - J_j)}{\frac{1}{A_i F_{ij}}}$
- If all $T_i$ are known, then we can solve for $J_i$ and hence for $q_i$

## Matrix Method for N-surface interaction-III

- In case the heat is known as in the case of re-radiating surface or electrically heated surface, then for that surface, we can use

$$\Rightarrow q_i = \sum_{i=1}^{N} \frac{(J_i - J_j)}{\frac{1}{A_i F_{ij}}}$$

- The system of equation of the form shown below will be solved by numerical methods.

$$[\text{Coeff}]\{J_i\} = \{\text{RHS}\}$$

## Example-1

- Calculate the view factor of the inner surface of the conical cavity onto itself in terms of the semi-vertex angle θ
- Let the curved surface be labeled 1 and the base be labeled 2

$$F_{11} + F_{12} = 1$$

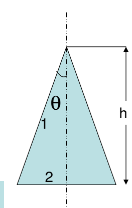
Similarly $F_{21} + F_{22} = 1$

But $F_{22} = 0 \Rightarrow F_{21} = 1$

Since $F_{21} A_2 = F_{12} A_1$

$$\Rightarrow F_{12} = F_{21} \frac{A_2}{A_1} = \frac{A_2}{A_1}$$

$$\therefore F_{11} = 1 - \frac{A_2}{A_1} = 1 - \frac{\pi (h \tan \theta)^2}{\pi (h \tan \theta)(h / \cos \theta)} = 1 - \sin \theta$$

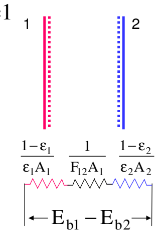



## Example-2

- Calculate the heat exchange by radiation per unit area between two parallel infinite surfaces maintained at $T_1$ and $T_2$. The surfaces can be assumed gray, diffuse each with emissivity ε
- Since the plates are infinitely long, $F_{12}=1$
- Constructing the electrical network and noting that $\varepsilon_1 = \varepsilon_2 = \varepsilon,\ A_1 = A_2 = A$

$$q_{1-2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon}{\varepsilon A} + \frac{1}{F_{12} A} + \frac{1-\varepsilon}{\varepsilon A}}$$

$$\frac{q_{1-2}}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{2}{\varepsilon} - 1}$$

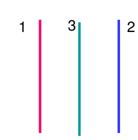



## Example-3

- If in example 2, we insert another identical plate, what is the effect of heat transfer per unit area
- Since the plates are infinitely long, $F_{13} = F_{32} = 1$
- From previous example

$$\frac{q_{1-3}}{A} = \frac{\sigma\left(T_1^4 - T_3^4\right)}{\frac{2}{\varepsilon} - 1}$$

$$\frac{q_{3-2}}{A} = \frac{\sigma\left(T_3^4 - T_2^4\right)}{\frac{2}{\varepsilon} - 1}$$

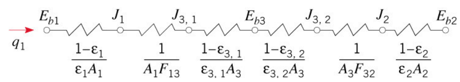



- Since $q_{1\text{-}3} = q_{3\text{-}2} \Rightarrow \left(T_1^4 - T_3^4\right) = \left(T_3^4 - T_2^4\right)$

$$\Rightarrow 2T_3^4 = T_1^4 + T_2^4 \qquad \Rightarrow T_3^4 = \frac{T_1^4 + T_2^4}{2}$$

$$\Rightarrow \frac{q_{1-3}}{A} = \frac{q_{3-2}}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)/2}{\frac{2}{\varepsilon} - 1}$$

- Heat cut down by half
- This is the principle of radiation shield
- The electrical network for a general case be drawn as shown below



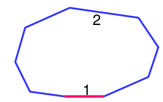

# Example-4

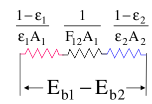
- Show that when a diffuse-gray surface of area $A_1$ with emissivity $\varepsilon_1$ and temperature $T_1$ is surrounded by a large isothermal surface at $T_2$ and emissivity $\varepsilon_2$, the net heat lost by the surface is given by

$$q_{1-2} = \sigma\varepsilon_1\left(T_1^4 - T_2^4\right)A_1$$

$$q_{1-2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{F_{12} A_1} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

$F_{12}=1$ $\quad$ $A_2$ Large




$$\therefore q_{1-2} = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} - 1 + 1} = \sigma\varepsilon_1\left(T_1^4 - T_2^4\right)A_1$$

# Example-5

- Calculate the heat exchange by radiation per unit area between two thick parallel infinite surfaces maintained at 40°C ($\varepsilon$=0.4) and 20°C ($\varepsilon$=0.5) The surfaces can be assumed gray, diffuse and with thickness and thermal conductivities of 30 cm (k=0.5 W/m-K) and 20 cm (1 W/m-K) respectively

- For slab-1

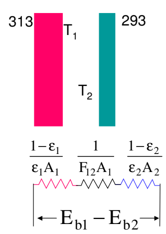
$$\frac{q_1}{A} = \frac{k_1(313 - T_1)}{L_1} = \frac{0.5(313 - T_1)}{0.3} \quad (1)$$

- For slab-2

$$\frac{q_2}{A} = \frac{k_2(T_2 - 293)}{L_2} = \frac{1(T_2 - 293)}{0.2} \quad (2)$$

- For radiation heat transfer

$$\frac{q_{1-2}}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2}} \quad (3)$$




- Note that $\frac{q_1}{A} = \frac{q_2}{A} = \frac{q_{1-2}}{A}$
- Equating the first two, we get

$$\frac{0.5(313 - T_1)}{0.3} = \frac{1(T_2 - 293)}{0.2} \qquad \Rightarrow T_1 = 1192 - 3T_2$$

- Substituting $T_1$ in Equation (3), we get

$$\frac{q_{1-2}}{A} = \frac{5.67\times10^{-8}\left((1192 - 3T_2)^4 - T_2^4\right)}{\frac{1-0.4}{0.4} + \frac{1}{1} + \frac{1-0.5}{0.5}}$$

$$= \frac{5.67\times10^{-8}\left((1192 - 3T_2)^4 - T_2^4\right)}{3.5} \quad (4)$$

- Iterating on $T_2$ to get same q/A in eqs (3) and (4) leads to

$T_2$ = 295.92 K, q = 14.6 W/m²