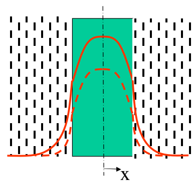
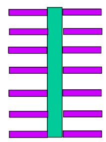
Extended Heat Transfer-I

- Consider a slab fuel with volumetric heat generation. If it is cooled by an efficient media, then the temperature profile is as shown in the dotted line
- If the heat transfer coefficient is poor, then the fluid temperature drop will be more and the centerline temperature will also increase as shown by the dark line
- This will lead to unacceptably high temperature. One way of reducing the temperature would be by attaching fins to the outside surface

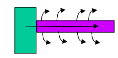




Extended Heat Transfer-II

- Addition of fins reduce convective resistance by increasing A

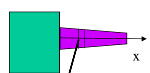
$$R_{conv} = \frac{1}{hA}$$



- Heat transfer by addition of surface area is called extended heat transfer or fin heat transfer
- Heat transfer in Fin is by both convection and conduction
- Though strictly not one dimensional, it can be approximated by one dimensional heat transfer
- Since it is a model used for analysis, we shall derive the fin equation from basics

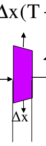
Governing equations for Fins-I

- In general, let the fin be of varying cross section as shown



$$hP\Delta x(T - T_\infty)$$

$$-kA\frac{dT}{dx} \longrightarrow \quad \longrightarrow -kA\frac{dT}{dx} + \frac{d}{dx}\left(-kA\frac{dT}{dx}\right)\Delta x$$



- In steady state, heat in should be equal to heat out

$$-kA\frac{dT}{dx} - \left[-kA\frac{dT}{dx} + \frac{d}{dx}\left(-kA\frac{dT}{dx}\right)\Delta x\right] - hP\Delta x(T - T_\infty) = 0$$

In **out** **out**

Governing equations for Fins-II

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - hP(T - T_\infty) = 0$$

- If k of the fin is constant, then,

$$\frac{d}{dx}\left(A\frac{dT}{dx}\right) - \frac{hP}{k}(T - T_\infty) = 0$$

- If the fin is prismatic, then, A is independent of x

$$\Rightarrow \frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0$$

- Defining $(T\text{-}T_\infty) = \theta$, and $(hP/kA) = m^2$, we have

$$\Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

Governing equations for Fins-III

- The governing equation is a second order differential equation and its solution can be expressed as

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

- The Boundary condition for the above equation is usually idealized conditions
- At the base of the fin, the temperature is usually specified as T_b

$$\Rightarrow \theta = \theta_b = T_b - T_\infty \text{ at } x = 0$$

- The commonly used boundary condition at the tip is what is known as insulated tip boundary

Governing equations for Fins-IV

- This is expressed as

$$\Rightarrow \frac{dT}{dx} = 0 \Rightarrow \frac{d\theta}{dx} = 0 \text{ at } x = L$$

- Boundary condition at x = 0, gives

$$\theta_b = C_1 + C_2$$

- Boundary condition at x = L, gives

$$0 = C_1 m e^{mL} + C_2(-m)e^{-mL}$$

$$\Rightarrow 0 = C_1 e^{mL} - (\theta_b - C_1)e^{-mL}$$

$$\Rightarrow C_1 = \theta_b \frac{e^{-mL}}{e^{mL} + e^{-mL}} \qquad C_2 = \theta_b - C_1 = \theta_b - \theta_b \frac{e^{-mL}}{e^{mL} + e^{-mL}}$$

Governing equations for Fins-V

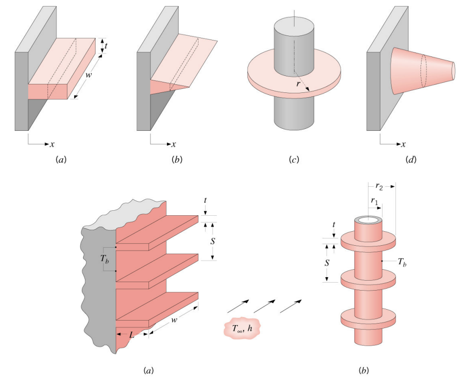
$$\Rightarrow C_2 = \theta_b \frac{e^{mL}}{e^{mL} + e^{-mL}}$$

$$\therefore \theta = \theta_b \frac{e^{-mL}e^{mx} + e^{mL}e^{-mx}}{e^{mL} + e^{-mL}}$$

$$= \theta_b \frac{e^{-m(L-x)} + e^{m(L-x)}}{e^{mL} + e^{-mL}}$$

$$\theta = \theta_b \frac{\cosh m(L-x)}{\cosh mL}$$

Fins

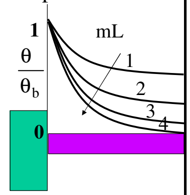


Fin Heat Transfer -I

- We Studied the case of Fin with insulated tip and obtained the solution as

$$\theta = \theta_b \frac{\cosh m(L-x)}{\cosh mL}$$

- The parameter mL is non-dimensional and decides the character of the fin



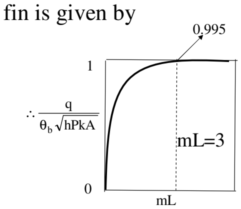
- mL=1 is kind of under-designed fin and mL=5 will be a over designed fin with material waste, mL=3 is considered a good number for fin design
- Let us get more information by obtaining the total heat transfered

Fin Heat Transfer -II

- Total heat transferred by the fin is given by

$$q = -kA\left.\frac{dT}{dx}\right|_{x=0} = -kA\left.\frac{d\theta}{dx}\right|_{x=0}$$

$$\frac{d\theta}{dx} = \theta_b \frac{\text{Sinh}\big(m(L-x)\big)(-m)}{\cosh mL}$$



$$\therefore \left.\frac{d\theta}{dx}\right|_{x=0} = -\theta_b m \frac{\text{Sinh}(mL)}{\cosh(mL)} = -\theta_b m \text{Tanh}(mL)$$

$$\therefore -kA\left.\frac{d\theta}{dx}\right|_{x=0} = kA\theta_b m \text{Tanh}(mL) = kA\theta_b \sqrt{\frac{hP}{kA}}\text{Tanh}(mL)$$

$$\therefore q = \theta_b \sqrt{hPkA}\,\text{Tanh}(mL)$$

Fin Heat Transfer -II

- As the boundary conditions are modified, or variable area fins are concerned, the subject gets more complicated and will be left to specialists
- However, to help non-specialists use complex results and interpret them, some simple parameters are defined.
- The first one is fin effectiveness ε_f

$$\varepsilon_f = \frac{\text{Heat transferred with fin}}{\text{Heat transferred without fin}} = \frac{q_{\text{with fin}}}{q_{\text{without fin}}}$$

- For the case we have analysed

$$\varepsilon_f = \frac{\theta_b \sqrt{hPkA}\,\text{Tanh}(mL)}{\theta_b hA} = \sqrt{\frac{kP}{hA}}\text{Tanh}(mL)$$

Fin Heat Transfer -III

- Another measure used is the fin efficiency η_f

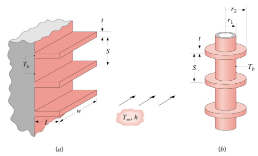
$$\eta_f = \frac{\text{Heat transferred with fin}}{\text{Heat transferred with entire fin at base temperature}}$$

$$\eta_f = \frac{\theta_b \sqrt{hPkA}\,\text{Tanh}(mL)}{\theta_b hPL} = \sqrt{\frac{kA}{hP}}\frac{\text{Tanh}(mL)}{L} = \frac{\text{Tanh}(mL)}{mL}$$

- The value of η_f is generally dependent on the non-dimensional parameter mL and expressions are available for complex geometries.
- The definitions of P and A have to be carefully understood. This you can do in the homework

Fin Heat Transfer -IV

- The real interest in engineering is not a single fin but a fin array such as the ones shown



- Let us now define the following areas
- A_{uf} is the total area that is not finned (unfinned)
- A_f is the each fin surface area (perimeter x Length)
- Heat transferred by the fins can be given by

$$q_f = NhPL\theta_b\eta_f = NhA_f\theta_b\eta_f$$

Fin Heat Transfer -V

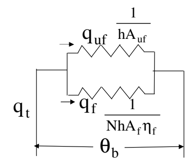
- Heat transferred by the unfinned area

$$q_{uf} = hA_{uf}\theta_b$$

- Total Heat transferred

$$q_t = q_f + q_{uf} = NhA_f\theta_b\eta_f + hA_{uf}\theta_b$$

- In terms of electrical analogy



Fin Heat Transfer -VI

- Many times the fins are not metallurgically bonded but shrink fitted or bonded with adhesives
- This adds contact resistance
- This can easily be accommodated using electrical analogy