

Review of Nuclear Physics-I

Chemical Energy \Rightarrow Rearrangement of Electrons
 Nuclear Energy \Rightarrow Rearrangement of Nucleons

Rutherford's Model
Protons, Neutrons, Electrons

	Mass	Charge
Proton	$1.67261 \times 10^{-24} \text{ g}$	$+1.60219 \times 10^{-19} \text{ Columbs}$
Electron	$9.10956 \times 10^{-28} \text{ g}$	$-1.60219 \times 10^{-19} \text{ Columbs}$
Neutron	$1.67492 \times 10^{-24} \text{ g}$	0

Review of Nuclear Physics-II

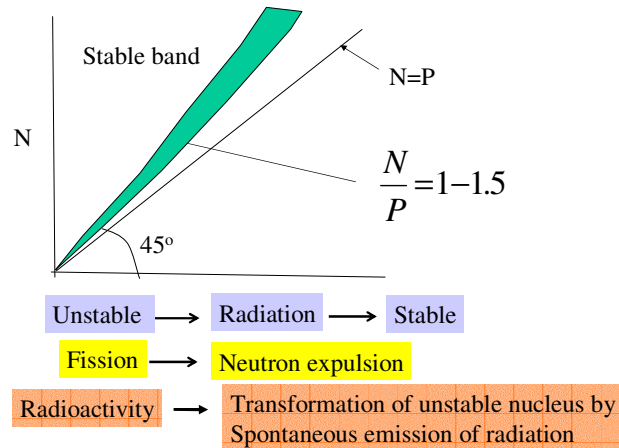
$\begin{matrix} A \\ Z \end{matrix} X$
Z - No of Protons
A - No of Protons + No. of Neutrons

Isotope – Same Z, different A

\Rightarrow Enormous Hollow space
 \Rightarrow Large number of neutrons for collisions of occur.

Radius of Nucleus	~	10^{-15} m
Radius of Atom	~	10^{-10} m
Density of Nucleus	~	10^{17} Kg/m^3
Density of Atom	~	10^3 Kg/m^3

Stability of Nucleus-I

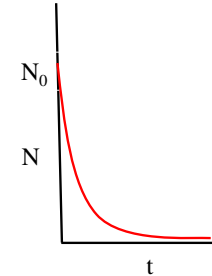


Stability of Nucleus-II

Postulate Probability of nuclear disintegration is constant.

Population	N
Time	dt
Disintegration	dN
$\frac{dN}{Ndt} = \text{Constant} = -\lambda$	(1)
I-order ODE	One condition
Initial Value	$N = N_0$ at $t = 0$

$N = N_0 e^{-\lambda t}$



Stability of Nucleus-III

Half life $T_{1/2} \Rightarrow \frac{N}{N_0} = 0.5$

$\therefore \frac{N}{N_0} = 0.5 = e^{-\lambda T_{1/2}}$

$\Rightarrow \lambda T_{1/2} = \ln 2$

or $T_{1/2} = \frac{0.693}{\lambda}$

$N = N_0 e^{\frac{-0.693 t}{T_{1/2}}}$

\Rightarrow 5-half lives $\Rightarrow \frac{N}{N_0} = 0.0313$

\Rightarrow 10-half lives $\Rightarrow \frac{N}{N_0} = 0.00978$

Stability of Nucleus-IV

Every disintegration \rightarrow Radiation

$\therefore \frac{dN}{dt} \rightarrow \text{Rate of emission of Radiation} = \lambda N$

Activity

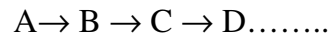
Units \rightarrow Becquerel = 1 dps or s^{-1}

1 - curie = 3.7×10^{10} Bq

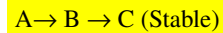
\equiv 1g of Radium

Analysis of Decay Chain-I

Fission Products are Radio active



- Practical chains are long
- 3 chain has all the characteristics



Population Balance

Rate of change = Production Rate – Destruction Rate

$$\frac{dN_A}{dt} = 0 - \lambda_A N_A \quad \frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \quad \frac{dN_C}{dt} = \lambda_B N_B - 0$$

Analysis of Decay Chain-II

$$\frac{dN_A}{dt} = 0 - \lambda_A N_A \quad \text{With } N_A = N_{A0} \text{ at } t = 0 \quad \Rightarrow N = N_0 e^{-\lambda t}$$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \quad \Rightarrow \frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_{A0} e^{-\lambda_A t}$$

Using $e^{\lambda_B t}$ as the integral factor

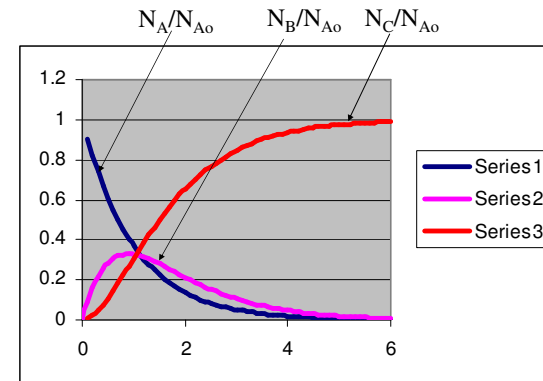
$$\begin{aligned} \frac{dN_B}{dt} e^{\lambda_B t} + \lambda_B N_B e^{\lambda_B t} &= \lambda_A N_{A0} e^{(\lambda_B - \lambda_A)t} \\ \Rightarrow d(N_B e^{\lambda_B t}) &= \lambda_A N_{A0} e^{(\lambda_B - \lambda_A)t} dt \\ \Rightarrow [N_B e^{\lambda_B t}]_{t=0}^{t=6} &= \lambda_A N_{A0} \left[\frac{e^{(\lambda_B - \lambda_A)t}}{(\lambda_B - \lambda_A)} \right]_0^t \end{aligned}$$

Analysis of Decay Chain-III

With $N_B = N_{B0}$ at $t = 0$

$$\begin{aligned} \Rightarrow N_B e^{\lambda_B t} - N_{B0} &= \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A} (e^{(\lambda_B - \lambda_A)t} - 1) \\ \text{or } N_B &= N_{B0} e^{-\lambda_B t} + \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \end{aligned}$$

Since $N_A + N_B + N_C = N_{A0} + N_{B0} + N_{C0}$, we can estimate N_C



$$N_{B0}/N_{A0} = 0, \quad N_{C0}/N_{A0} = 0, \quad \lambda_A/\lambda_B = 0.8$$

Special Cases -I

$$N_B = N_{B0}e^{-\lambda_B t} + \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A} [e^{-\lambda_A t} - e^{-\lambda_B t}]$$

For $\lambda_B > \lambda_A$ $e^{-\lambda_B t} \ll e^{-\lambda_A t}$

- After some time, term 1 vanishes
- In term 2, the second term vanishes

$$\Rightarrow N_B = \frac{\lambda_A}{\lambda_B - \lambda_A} N_{A0} e^{-\lambda_A t} = \frac{\lambda_A N_A}{\lambda_B - \lambda_A}$$

Or $\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B - \lambda_A}$ Transient Equilibrium

Special Cases -II

$$\text{For } \lambda_B \gg \lambda_A \quad \frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B} \Rightarrow \lambda_A N_A = \lambda_B N_B$$

The above implies that the activities are equal irrespective of the initial activities. This is called Secular equilibrium