ME 417/ME 419 Experimental Engineering III
Steam Power Laboratory
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Experiment 1
Compressible Flow through a Converging-Diverging Duct

Objectives

1. To determine the *limiting venturi condition* theoretically and experimentally.
2. To determine the *pressure distribution*s along the converging diverging duct for different flow regimes theoretically and compare with the experimental results.
3. To determine the theoretical and experimental *mass flow rates* for varying upstream stagnation pressures and constant downstream pressure.

Apparatus

Compressor, orifice plate, U-tube manometer, thermometer, CD nozzle assembly, pressure gauges.

Theory

Consider one-dimensional steady compressible flow though a converging-diverging duct.

When the pressures upstream and downstream of the duct are equal, there is no flow. Now, suppose the upstream (stagnation) pressure p_o is increased slightly, keeping the downstream (back) pressure p_b constant. The fluid will start to flow through the duct due to the imposed pressure difference across the duct. The fluid will accelerate in the converging portion of the duct with a corresponding decrease in pressure upto the throat of the duct. Thereafter, the fluid will decelerate in the diverging portion of the duct, with a corresponding increase in pressure. This is the usual *venturi flow* condition. The maximum velocity occurs at the throat, and the pressure gradient at the throat is zero.

If the upstream pressure is increased further, keeping the downstream pressure fixed, the velocity and Mach number at the throat will increase, while the pressure at the throat p_t will decrease. As the upstream pressure is increased, the Mach number at the throat will approach *unity* at a *critical value* of the pressure ratio $\left(p_t / p_o \right)$. The critical pressure ratio is $\left(p_t / p_o \right)_{cr} = \left(\frac{2}{\gamma+1} \right)^{\left(\frac{\gamma}{\gamma-1} \right)}$ for *isentropic flows*, where γ is the ratio of specific heats c_p / c_v. For air, $\gamma = 1.4$ and $\left(p_t / p_o \right)_{cr} = 0.5283$. Once the Mach number at the throat reaches one, pressure pulses cannot propagate upstream. Thus, further increase in the upstream pressure p_o will not alter the *dimensionless* pressure distribution $\left(p / p_o \right)$ (local pressure scaled by the upstream pressure) in the converging portion of the duct, and the flow remains *sonic* at the throat (i.e. the Mach number remains equal to one at the throat). Such flows are said to be *choked*. The pressure gradient at the throat is non-zero for choked flows, while the mass flow rate through the duct depends only on the upstream pressure p_o. The pressure variation in the diverging portion of the duct for choked flows depends on the pressure ratio $\left(p_b / p_o \right)$.

The flow is referred to as a *limiting venturi flow* if it is choked, and the pressure gradient at the throat is zero. In the limiting venturi condition, the diverging portion of the duct acts like a *diffuser*, and the flow is *subsonic* everywhere in the duct except at the throat, where it is *sonic*. In the other limiting condition, referred to as the *design condition*, the flow is choked and the pressure gradient at the throat is negative. In the design condition, the pressure continues to fall in the diverging part of the duct, and the fluid is accelerated isentropically to *supersonic* Mach numbers. Thus, in the design condition, the diverging portion of the duct acts like a *nozzle*.

At the limiting venturi condition P_{lv}, there is no shock anywhere in the diverging section. As the pressure ratio, $\left(p_b / p_o \right)$ decreases below the limiting venturi condition, a normal shock occurs in the diverging portion of the duct, the location of the shock depends on the pressure ratio, $\left(p_b / p_o \right)$. The flow in the diverging section is supersonic upto the shock and downstream of the shock, it becomes subsonic. As the pressure ratio is progressively decreased, the normal shock moves further away towards the exit. For a particular value of this pressure ratio, $\left(p_b / p_o \right)$ say, P_s, the shock is located at the exit of the duct.

If the pressure ratio lies between P_{des} and P_s, the flow is isentropic everywhere inside the duct. However, an *oblique shock* occurs outside the duct. If the pressure ratio is lower than P_{des}, the flow remains isentropic everywhere inside the duct, but expands non-isentropically outside the duct through a series of *expansion waves*.

The three key pressure ratios P_{lv}, P_{des} *and* P_s separate the nozzle exit conditions into four regions. The salient features of these four regions is summarised in Table 1.

Table 1: Regimes of Flow in Converging-Diverging Duct

Region	Pressure Ratio	Mach Number		Exit Pressure	Features
		Throat	Exit		
1	$P_{lv} < \left(p_b / p_o \right) < 1$	$M_t < 1$	$M_e < 1$	$p_e = p_b$	Isentropic flow throughout the duct
2	$P_s < \left(p_b / p_o \right) < P_{lv}$	$M_t = 1$	$M_e < 1$	$p_e = p_b$	Normal shock occurs in the diverging section of the duct
3	$P_{des} < \left(p_b / p_o \right) < P_s$	$M_t = 1$	$M_e > 1$	$p_e < p_b$	Oblique shock outside the duct
4	$\left(p_b / p_o \right) < P_{des}$	$M_t = 1$	$M_e > 1$	$p_e > p_b$	Expansion waves outside the duct

The salient features of the flow when the pressure ratio $\left(p_b / p_o \right)$ is equal to one of the key pressure ratios $\left(P_{lv}, P_{des}\ and\ P_s \right)$ is given in Table 2.

Table 2: Salient Features of Flow in Converging-Diverging Duct when the pressure ratio equals one of the key pressure ratios

Pressure Ratio	**Mach Number**		**Exit Pressure**	**Features**
	Throat	**Exit**		
$\left(p_b / p_o \right) = P_{lv}$	$M_t = 1$	$M_e < 1$	$p_e = p_b$	This is the minimum P_o for choked flow. The flow is isentropic everywhere. No shock anywhere.
$\left(p_b / p_o \right) = P_s$	$M_t = 1$	$M_{e1} > 1$, $M_{e2} < 1$	$p_{e1} < p_b$, $p_{e2} = p_b$	Normal shock at exit plane. 1 is upstream of shock, 2 is downstream of shock.
$\left(p_b / p_o \right) = P_{des}$	$M_t = 1$	$M_e > 1$	$p_e = p_b$	Isentropic everywhere.

The mass flow rate at any axial location, x, in the duct is given by $\dot{m} = \rho A v$, where A is the cross -sectional area of the duct, v is the average velocity and ρ is the average fluid density at that location. For steady compressible flows, the mass flow rate, $\dot{m}$, has the same value at all axial locations.

For steady adiabatic one-dimensional flows, the velocity can be determined from an energy balance, and is given by $v = \sqrt{2(h_o - h)}$, where h is the enthalpy at the given axial location x and h_o is the stagnation enthalpy. For a perfect gas, $h = c_p T$, and hence the velocity is $v = \sqrt{2c_p(T_o - T)}$, where T_o is the stagnation temperature. The temperature at any axial location, x, can be estimated from the pressure distribution using the relation $\frac{T}{T_o} = \left(\frac{p}{Fp_o} \right)^{\frac{\gamma-1}{\gamma}}$, where $F = \exp\left[-\frac{(s - s_o)}{R} \right]$. The factor F, which represents the ratio of the local stagnation pressure to the upstream stagnation pressure, depends on the entropy change (s - s_0) scaled by the gas constant R, where s is the entropy at section x, and s_0 is the upstream stagnation entropy. If frictional effects are negligible and the flow is adiabatic, the entropy does not change, and the factor F = 1. (Note that F cannot be assumed to be 1 at locations downstream of a shock since the entropy changes across a shock.) Thus, the velocity can be determined using the relation $V = \sqrt{(2c_p T_0)\left[1 - \left(\frac{p}{Fp_0} \right)^{\frac{\gamma-1}{\gamma}} \right]}$. The density at any location can be determined from the perfect gas law, and is given by $\frac{\rho}{\rho_o} = F\left(\frac{p}{Fp_o} \right)^{\frac{1}{\gamma}}$, where $\rho_o = \left(\frac{P_o}{RT_o} \right)$ is the density at upstream stagnation condition. Thus, the mass flow rate can be estimated using the relation

$$\dot{m} = \left(\frac{2\gamma}{\gamma-1}\right)^{\frac{1}{2}} \frac{Fp_o A}{(RT_o)^{\frac{1}{2}}} \left(\frac{p}{Fp_o}\right)^{\frac{1}{\gamma}} \left[1 - \left(\frac{p}{Fp_o}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{1}{2}} \quad (1)$$

The factor F may be set equal to 1 at sections upstream of a shock if the effects of friction are negligible. One can define a non-dimensional mass flow rate $m_{nd} = \frac{\dot{m}\sqrt{RT_o}}{p_o A_t}$, where A_t is the throat area. For isentropic flows, this dimensionless mass flow rate is given by

$$m_{nd} = \sqrt{\frac{2\gamma}{\gamma-1}} \left(\frac{p_t}{p_o}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_t}{p_o}\right)^{\frac{\gamma-1}{\gamma}}} \quad (2)$$

For choked flows, m_{nd} has a constant value equal to $\sqrt{\frac{2\gamma}{\gamma-1}} \left(\frac{p_t}{p_o}\right)_{cr}^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_t}{p_o}\right)_{cr}^{\frac{\gamma-1}{\gamma}}}$. For air, this value is 0.6847.

Experimental Setup

A schematic diagram of the experimental setup is shown in the Figure 1. Air from the laboratory compressor enters the setup through a 50 mm pipe. An orifice in the pipeline measures the flow-rate through the system. The details of the orifice plate are given below. The air velocity in the pipeline is gradually reduced as it moves through a gradually expanding section ending in a 300 mm diameter pipeline. This large diameter pipeline is about 600 mm and acts as a plenum chamber. The CD nozzle is attached to the plenum. The pressure and temperature of the air within the plenum are measured using a pressure gauge and 'mercury in glass' thermometer respectively.

A detailed diagram of the CD nozzle used in the experimental setup is given in the Figure 2. The nozzle, together with the flange is machined from a single 74 mm OD Perspex rod 120 mm long. The pressure is measured at the throat and two locations in the diverging portion of the nozzle as shown in the figure.

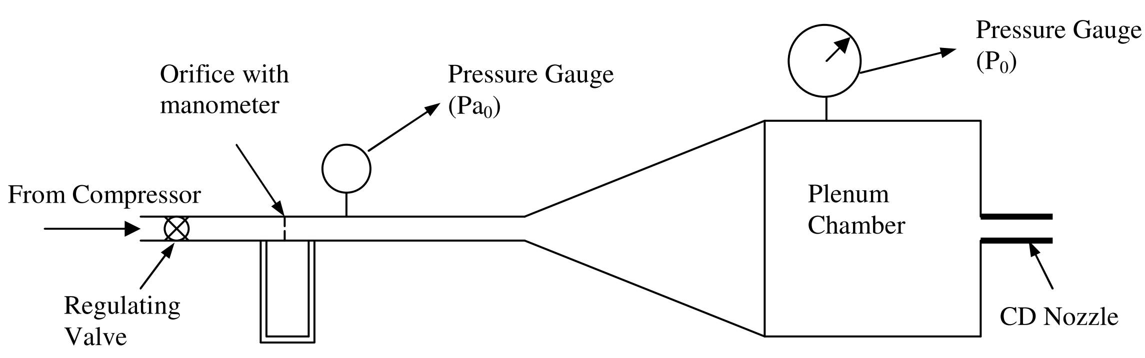


Figure 1: Schematic of experimental setup

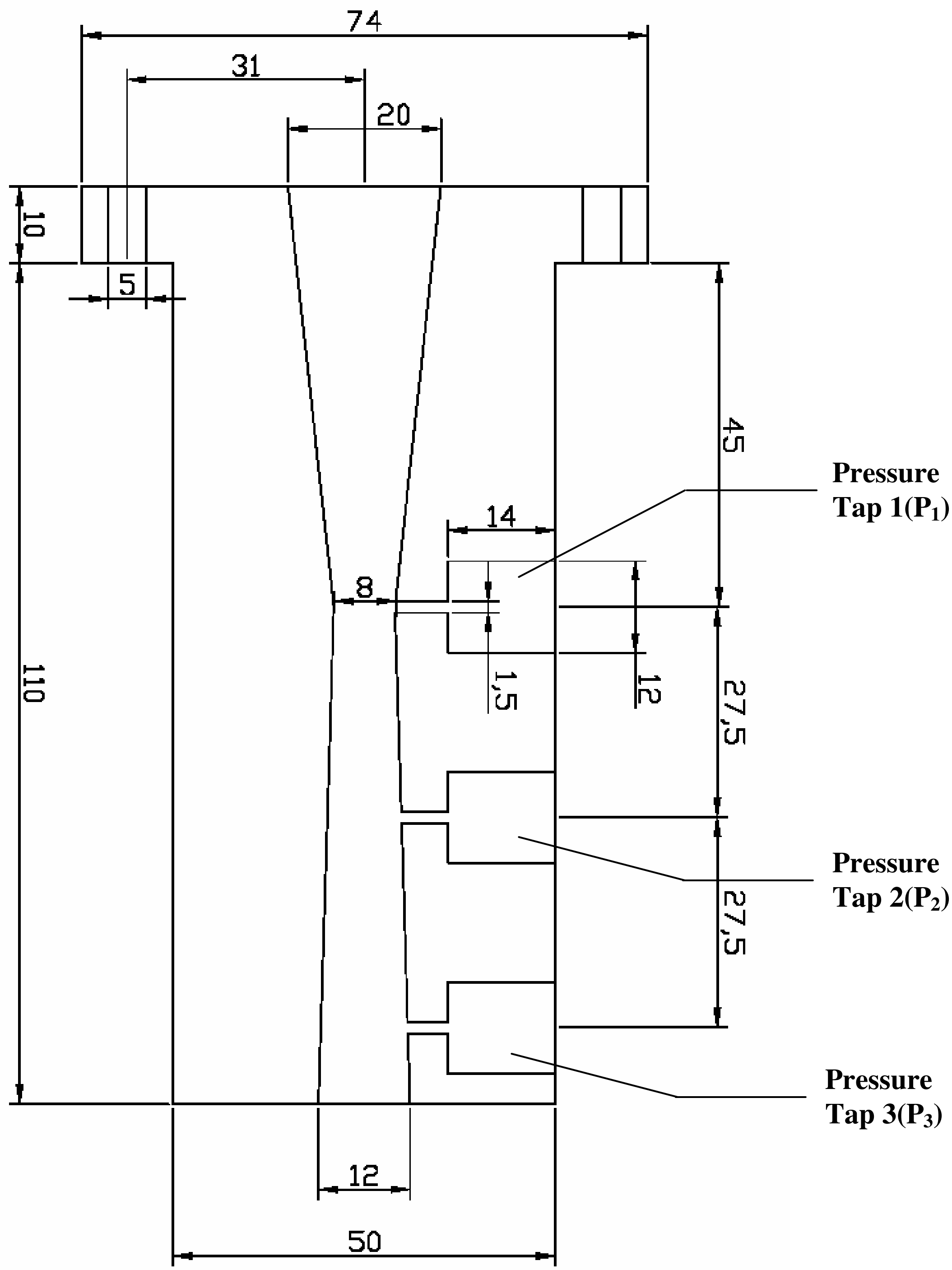


Figure 2: CD nozzle used in experiment

Procedure

The theoretical estimates of upstream stagnation pressures for different flow regimes are to be obtained before the experiment.

I Determination of upstream stagnation conditions and pressure distribution for different flow regimes (methodology explained in the ***appendix****)*

1. The inlet stagnation pressure corresponding to the following five conditions is to be determined using the isentropic and normal shock tables provided. (Each student will be assigned one condition for which he/she will have to do the calculation)

i. Limiting venturi condition and Design condition.
ii. Normal Shock at a distance of L/3 from the throat. (L is the length of the diverging section)
iii. Normal Shock at a distance of 2L/3 from the throat.
iv. Normal Shock at the exit of the duct.
v. Normal Shock beyond the exit. (Choose a P_o greater than that obtained for the exit shock case)

2. The pressure at the tapping locations is to be determined using the isentropic tables.

II Experiment

Each student is to do the experiment for the condition he/she has calculated.

i. The upstream stagnation pressure is set in the stagnation chamber (P_o) using the valve.
ii. The values to be noted are pressure at orifice (P_{ao}), temperature (T_o), deflection in the manometer (h), pressure at tappings (P_1, P_2, P_3).
iii. Increase the stagnation pressure and repeat the procedure. (Wait for the transients to die out before taking the readings)
iv. For experimental determination of the choking condition, experiment is to be done at P_o (54, 56, 58, 60, 62^o in the gauge) in addition to the above cases.

Note: Gauge measuring P_o and P_{ao} are calibrated in degrees with a maximum scale of 300^o degrees. Range of the gauges is P_o – 0 to 6 kgf/cm^2 (abs) and P_{ao} -0 to16 kgf/cm^2 (gauge).

Calculations

Calculation of mass flow rate using the orifice meter observations

Mass flow rate through the duct is obtained from the manometer deflection, h, using the relation

$$\dot{m} = c_d K \sqrt{\frac{2gh\rho_w}{\rho_{ao}}}\rho_{ao}, \qquad (3)$$

where c_d is the coefficient of discharge, g is the gravitational acceleration, ρ_w is the density of water (the manometer fluid), ρ_{ao} is the density of air at the location of the orifice, K is a constant given by $K = \frac{A_o}{\sqrt{1-\beta^4}}$, $\beta = \frac{d_o}{d_p}$, is the ratio of orifice diameter to pipe diameter and $A_o = \pi d_o^2/4$ is the area of the orifice. The density of air at the location of the orifice is obtained from the perfect gas law $\rho_{ao} = \frac{P_{ao}}{RT_{ao}}$, where P_{ao} and T_{ao} are the pressure and temperature at the location of the orifice, and R is the gas constant for air. The coefficient of discharge, c_d, depends on the Reynolds number of the flow in the pipe and β. For large Reynolds numbers, c_d approaches a constant value which depends on β. Thus, for large Reynolds numbers, the value of c_d is known *a priori*, and Equation (3) gives the mass flow rate. The assumption of large Reynolds number, however, needs to be checked by determining the Reynolds number after the mass flow rate has been calculated from Equation (3). For low or moderate Reynolds number flows, the mass flow rate needs to be determined iteratively since c_d is unknown.

Determination of the limiting venturi condition using experimental

The pressure ratio for limiting venturi condition, P_{lv}, can be determined experimentally by plotting m_{nd} as a function of $\left(P_b/P_o\right)$. When $\left(P_b/P_o\right)$ is 1, there is no flow, and m_{nd} is 0. As $\left(P_b/P_o\right)$ is reduced from 1, m_{nd} will increase initially, and then saturate to a constant value. The pressure ratio, $\left(P_b/P_o\right)$, at which the curve begins to saturate will correspond to P_{lv}.

Data used in the Calculations

Gas constant for air, $R = 287 J/kg-K$

Ratio of specific heats for air, $\gamma = 1.4$

Kinematic viscosity of air, $\nu = 1.5\times10^{-5} m^2/s$

Diameter of pipe, $d_p = 50mm$

Orifice diameter, $d_o = 38mm$

Ratio of orifice diameter to pipe diameter, $\beta = 0.76$

Coefficient of discharge for orifice meter, $C_d = 0.628$ (for large Reynolds numbers)

1 kg/cm^2 = 0.981 bar

Observations

Room temperature, T_a = _________°C = ___________ K

Ambient pressure $P_b = 1.013 bar$

Initial reading of gauge measuring stagnation pressure $P_o =$ _________ deg

Initial reading of gauge measuring $P_{ao} =$ _________ deg

Table 3: Experimental Observations 1

S.No.	P_o	T_o	h	P_{ao}	P_1	P_2	P_3
	deg	℃	*mm of* H_2O	*deg*	kgf/cm^2	kgf/cm^2	kgf/cm^2
1							
2							
3							
4							
5							
6							

Table 4: Experimental Observations 2

S.No.	P_o	T_o	h	P_{ao}
	deg	℃	*mm of* H_2O	*deg*
1	54			
2	56			
3	58			
4	60			
5	62			

Results

The following should be presented.

1. The stagnation conditions obtained for different flow regimes.
2. Sample calculation for theoretical, experimental and non-dimensional mass flow (for one observation only)
3. The following plots are to be made with all observations (Though calculations would be done for one observation only, plots all cases is to be done)
    i. Mass flow rate (m) vs P_o (experimental and theoretical)
    ii. Non-Dimensional mass flow (m_{nd}) vs P_o
    iii. P/P_o vs A/A^* (experimental and theoretical)
4. Discussions to be made on
    i. Comparison of theoretical and experimental pressure distribution
    ii. Comparison of theoretical and experimental mass flow rate
    iii. Comparison of theoretical and experimental limiting venturi condition.

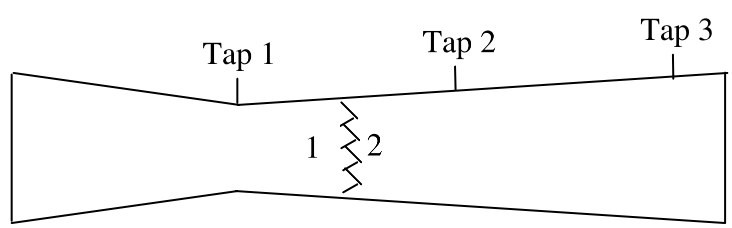
Appendix

I. Determination of inlet stagnation condition (P_o) for limiting venturi and the design condition

Determine the exit area to sonic area ratio (A/A^*). Using the isentropic tables determine the ratio of back pressure to the stagnation pressure (P_e/P_o). Note: The area ratio corresponding to sub sonic Mach number gives the limiting condition and that corresponding to the super sonic Mach number gives the design condition. (Here the sonic area is the throat area and the back pressure (P_e) is atmospheric. Thus having the P_o determined, find the pressures at the tappings. Find the respective (A/A^*) using which P/P_o can be read from the isentropic tables.

II Determination of inlet stagnation condition (P_o) for shock at L/3 from the throat (L is the length of diverging section)

Determine the area ratio (A_1/A_1^*) and find the Mach number at section 1 (M_1) from the isentropic tables (Here the sonic area A_1^* for upstream shock would be the throat area). Using M_1, read M_2 and the ratio of downstream stagnation condition to upstream stagnation condition (P_{o2}/P_{o1}) from the normal shock tables.

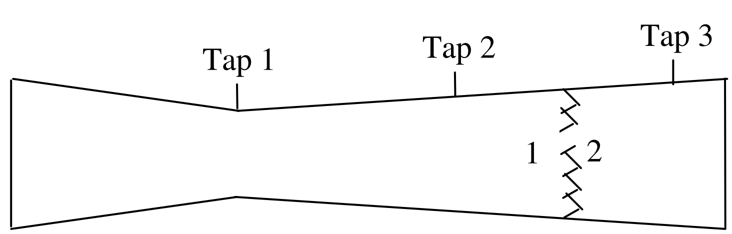


Using M_2, find the area ratio (A_2/A_2^*) from the isentropic tables. Here the sonic area A_2^* (downstream shock) is not known and is found out assuming $A_1=A_2$. Then find A_e/A_2^* to read P_e/P_{o2} from isentropic tables. (P_e is atmospheric and thus the P_{o2} and P_{o1} are determined.)

Having P_{o1} and P_{o2} determined, find the pressure at the tappings. (Use A/A^* to read P/P_o from isentropic tables. Use appropriate A^* and P_o upstream and downstream shock). Also find the pressure before and after the shock.

III Determination of inlet stagnation condition (P_o) for shock at 2L/3 from the throat (L is between length of the diverging section)

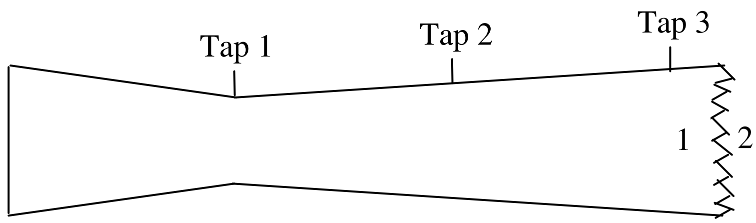
The calculation is very similar to the one given for shock at L/3.



IV Determination of inlet stagnation condition (P_o) for shock at exit

Determine the exit area to sonic area ratio (A_1/A_1^*) and find the Mach number upstream the shock (M_1). Use normal shock tables to find the downstream Mach number M_2 and the ratio P_{o2}/P_{o1}. Using M_2, read P_e/P_{o2} from isentropic tables. (P_e is atmospheric and thus the P_{o2} and P_{o1} are determined.).

Thus having the P_o determined, find the pressures at the tappings. Find the respective (A/A^*) using which P/P_o can be read from the isentropic tables.



Isentropic tables **Shock Tables**

M	P/P_o	A/A^*		M_1	M_2	P_{o2}/P_{o1}
0.2685	0.9511	2.250		1.7150	0.6367	0.8495
0.3017	0.9388	2.025		2.0691	0.5660	0.6886
0.3220	0.9307	1.911		2.3282	0.5311	0.5709
0.3637	0.9127	1.721				
0.4420	0.8745	1.469				
0.4733	0.8578	1.394				
0.5311	0.8252	1.285				
0.5549	0.8113	1.248				
0.5660	0.8046	1.232				
0.6367	0.7612	1.148				
1.0000	0.5283	1.000				
1.7150	0.1981	1.352				
1.8267	0.1671	1.469				
2.0691	0.1148	1.789				
2.2113	0.0919	2.025				
2.3282	0.0765	2.250				