Experiment 2
To characterise and model leakage flow through Labyrinth seals

Objective

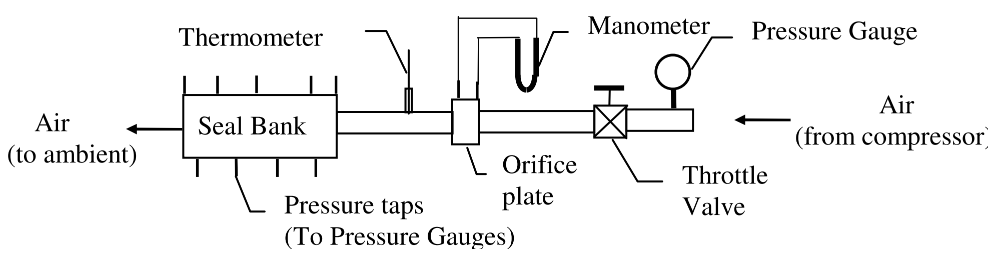
To study the leakage behaviour of a 8-stage labyrinth seal bank and to compare it with two idealised models.

Apparatus

Compressed air supply, Throttle Valve, Orifice plate, U-tube manometer, Thermometer, Seal bank with suitable pressure Taps, Pressure gauges

Test Procedure

The schematic of the Experimental Set-up is as shown in the figure below. A 25 kW, two stage reciprocating air-compressor (not shown in the figure) supplies the necessary air pressure for the experiment. The various steps that need to be followed are given below:



1. Adjust the bypass valves available in the lab to adjust the pressure at the inlet of throttle valve to 3 bar
2. Open the valve fully. This will allow maximum flow across the seals.
3. Note the level of water in each column of the U-tube manometer.
4. Note the temperature of the air from the thermometer
5. Note down the pressure at each stage of the seal bank.
6. Now repeat the steps 3-5 for three or four (one per each student) other throttle valve positions such that the pressure drop in the U-tube manometer varies approximately from 80% to 20% of the values obtained when the valve was fully open
7. Plot the necessary curves as required

Observations

The observations shall be given in the following order

Diameter of the process pipe:	2.54 cm
Diameter of the orifice:	1.65cm

Discharge Coefficient of the Orifice Plate	0.7
Dimensions of the seal	6.585 cm OD
	6.525 cm ID

Experimental Data

S. No.	Manometer Readings			Pressure Gauge Readings							
	Left (mm)	Right (mm)	Δh (mm)	P_1 kgf/cm^2	P_2 kgf/cm^2	P_3 kgf/cm^2	P_4 kgf/cm^2	P_5 kgf/cm^2	P_6 kgf/cm^2	P_7 kgf/cm^2	P_8 kgf/cm^2
1.											
2											
3											
4											

Theoretical Models

Two idealised models shall be used for modelling the seal performance. These are described later in this section. In both the models, the overall process the fluid undergoes is assumed to be isenthalpic (as no work is extracted, heat transfer is negligible, kinetic and potential effects between the chambers across a seal can be neglected). Further, if one assumes that the fluid can be treated as ideal gas with constant properties, then the process would become isothermal (dh=0 implies dT=0). This will lead to the overall process equation for the fluid as:

$$pv = p_0 v_0 \tag{1}$$

where, p_0 and v_0 correspond to the stagnation conditions before the fluid enters the seal assembly and p and v correspond to the properties of air at any stage (in the chamber) in the seal bank.

Incompressible fluid model

The assumptions used in this model are:

1. The flow across a seal is locally incompressible
2. The average pressure of the fluid for the computation of density of the fluid flowing across a seal is the mean of the pressure values on either side of the seal
3. Even for the case of choking condition existing in the last seal, incompressible flow approximation is valid for flows across upstream seals.

With the above assumptions for a flow across i^{th} seal across which the pressures are p_{i-1} and p_i respectively, we can write:

$$\dot{m}^2 = A^2\left(\frac{2\Delta p}{\bar{v}}\right) = A^2\left(\frac{2\Delta p\,\bar{p}}{p_0 v_0}\right) = A^2\left(\frac{2(p_{i-1}-p_i)\frac{(p_{i-1}+p_i)}{2}}{p_0 v_0}\right) = \frac{(p_{i-1}^2 - p_i^2)}{p_0 v_0} \tag{2}$$

We can rewrite this equation using equation of state as:

$$(p_{i-1}^2 - p_i^2) = \frac{\dot{m}^2}{A^2} R T_0 \tag{3}$$

Summing up for N seals we can write:

$$(p_0^2 - p_N^2) = N\frac{\dot{m}^2}{A^2}RT_0 \quad \text{or} \quad \dot{m} = A\left[\frac{(p_0^2 - p_N^2)}{NRT_0}\right]^{0.5} \tag{4}$$

Using this value of mass flow rate, the pressure at the end of the ith stage can be computed as:

$$p_i = \left[p_0^2 - \frac{\dot{m}^2}{A^2} iRT_0\right]^{0.5} \tag{5}$$

The above expressions have been arrived at under the incompressible flow assumption. If, however, P_{N-1} is such that the flow will choke at the exit, i.e., P_N/P_{N-1} is less than $(2/(n+1))^{(n/(n-1))}$, then the mass flow rate cannot be given by Eq. (4). However, if we assume that flow is incompressible in the first N-1 stages, then we can still write

$$\dot{m} = A\left[\frac{(p_0^2 - p_{N-1}^2)}{(N-1)RT_0}\right]^{0.5} \tag{6}$$

If we assume isentropic flow in the last stage, we can write the mass flow rate as (refer CD-Nozzle write up-Eq. (3))

$$\dot{m} = 0.6847A\frac{p_{N-1}}{\sqrt{RT_0}} \tag{7}$$

Elimination of mass flow rate from Eqs. (6 &7) will lead to the solution of p_{N-1}. This can be shown to be

$$p_{N-1} = \frac{p_0}{\sqrt{1 + (N-1)0.6847^2}} \tag{8}$$

Having obtained p_{N-1}, one can get mass flow rate from either Eq. (7) or Eq. (6). The pressure at all other stages shall be obtained by using this mass flow rate in Eq. (5).

Compressible fluid model

In this model flow across a seal is modelled as isentropic flow. In such circumstances it can be shown that an elliptic approximation,

$$\frac{\dot{m}^2}{\dot{m}_*^2} + \frac{(\varepsilon - \varepsilon_*)^2}{(1-\varepsilon_*)^2} = 1, \tag{9}$$

very closely approximates the exact solution. In the above equation ε represents p/p_0, the suffix star implies values at sonic condition and suffix 0 implies stagnation condition. The above equation is valid for any stage k.

In further analysis, the nomenclature chosen is that for a stage k, the downstream pressure is p_{k+1} and the upstream pressure is p_K. The mass flow rate from that stage is $\dot{m}_k$. The stagnation pressure for stage 1 shall be p_1. Due to visous effects in the seal the stagnation pressures for every successive stage will decrease. Equation (9) for any stage k, can be written as,

$$\frac{\dot{m}^2}{\dot{m}_{*k}^2} + \frac{(p_{k+1}/p_k - \varepsilon_*)^2}{(1-\varepsilon_*)^2} = 1 \tag{10}$$

It may be noted that no stage index is set for ε_* as it is a constant. The above equation may be rewritten as

$$(1-\varepsilon_*)^2 \frac{\dot{m}^2}{\dot{m}_{*k}^2} + (p_{k+1}/p_k - \varepsilon_*)^2 (1-\varepsilon_*)^2 = (1-\varepsilon_*)^2 \quad (11)$$

It should be noted that mass flow rate $\dot{m}$ will be same at every stage of the seal assembly at steady state. Further, from the analysis presented in the CD Nozzle experiment the choked mass flow rate is linearly proportional to the stagnation pressure. This implies,

$$\dot{m}_{*k} \ \alpha \ p_k, \quad \dot{m}_{*1} \ \alpha \ p_1 \quad (12)$$

Thus $\dot{m}_{*k} = \dot{m}_{*1}\varepsilon_k$, (13)

where $\varepsilon_k = p_k/p_1$ (14)

Using Eqs. (13) and (14), Equation (11) can be rewritten as,

$$(1-\varepsilon_*)^2 \frac{\dot{m}^2}{\dot{m}_{*1}^2 \varepsilon_k^2} + (1-\varepsilon_*)^2 \left(\frac{\varepsilon_{k+1}^2}{\varepsilon_k^2} - \varepsilon_*\right)^2 = (1-\varepsilon_*)^2 \quad (15)$$

$$\Rightarrow (1-\varepsilon_*)^2 \frac{\dot{m}^2}{\dot{m}_{*1}^2} + (1-\varepsilon_*)^2 (\varepsilon_{k+1}^2 - \varepsilon_* \varepsilon_k^2)^2 = (1-\varepsilon_*)^2 \varepsilon_k^2$$

$$\Rightarrow (1-\varepsilon_*)^2 \frac{\dot{m}^2}{\dot{m}_{*1}^2} = (1-\varepsilon_*)^2 \varepsilon_k^2 - (1-\varepsilon_*)^2 (\varepsilon_{k+1}^2 - \varepsilon_* \varepsilon_k^2)^2$$

The above equation after some manipulation leads to

$$(1-\varepsilon_*)^2 \frac{\dot{m}^2}{\dot{m}_{*1}^2} = (1-\varepsilon_*)(\varepsilon_k^2 - \varepsilon_{k+1}^2) - \varepsilon_* (\varepsilon_k - \varepsilon_{k+1})^2 \quad (16)$$

Since during steady state mass flow rate will be a constant across each seal, the LHS is a constant. Summing over N seals, we can write,

$$N(1-\varepsilon_*)^2 \frac{\dot{m}^2}{\dot{m}_{*1}^2} = (1-\varepsilon_*)(\varepsilon_1^2 - \varepsilon_N^2) - \varepsilon_* \sum_{k=1}^{N} (\varepsilon_k - \varepsilon_{k+1})^2 \quad (17)$$

Approximating the ε_k to have a linear profile,

$$(\varepsilon_k - \varepsilon_{k+1}) = \frac{(\varepsilon_1 - \varepsilon_N)}{N} \quad (18)$$

$$\Rightarrow N(1-\varepsilon_*)^2 \frac{\dot{m}^2}{\dot{m}_{*1}^2} = (1-\varepsilon_*)(\varepsilon_1^2 - \varepsilon_N^2) - \frac{\varepsilon_*}{N} (\varepsilon_1 - \varepsilon_N)^2 \quad (19)$$

Thus, we can estimate the mass flow from the seal bank using Eq. (19). It may be noted that for air $\varepsilon_* = 0.528$, $\varepsilon_1 = 1.0$ and $\dot{m}_{1*} = 0.6847 A_{seal} \frac{p_1}{\sqrt{RT_1}}$.Further, The normalised pressure after k^{th} seal, ε_k , can be estimated by using the mass flow computed for N seals previously and using Eq. (10) with the index k replacing N.

For criticality, $d\dot{m}/d\varepsilon_N = 0$. This will happen when $\varepsilon_N = \varepsilon_{N*}$. Differentiating the above equation leads to,

$$\varepsilon_{N*} = \frac{\varepsilon_*}{N(1-\varepsilon_*) + \varepsilon_*} \quad (20)$$

If $\varepsilon_N \geq \varepsilon_{N*}$, then there is no choking, otherwise the last seal chokes and the mass flow rate can be obtained by replacing ε_N by ε_{N*} in Eq. (19). similarly, the normalised pressure after k^{th} seal, ε_k, can be estimated by using the mass flow computed for N seals previously and using Eq. (19) with index k replacing N.

Results and Discussions

In this section the following should be presented:
(1) Plot the variation of experimental mass flow rate with stagnation pressure. Also mark the uncertainty in this estimate. In the same curve, plot the prediction by both the models.
(2) Plot the experimental pressure distribution as a function of the seal stages. Compare these with that obtained from the two models.
(3) For both (1) and (2), one sample calculation must be shown. Each student shall compute for different readings.
(4) Discuss the results obtained.