EN 634 Nuclear Reactor Thermal Hydraulics and Safety

Governing Equations for Single-phase Flows

Kannan Iyer
Kiyer@iitb.ac.in

Department of Mechanical Engineering
Indian Institute of Technology, Bombay

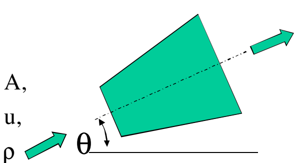
ONE DIMENSIONAL ANALYSIS

- In reactor thermal hydraulics one dimensional area averaged approach is most popular.
- While detailed averages can be taken from the three dimensional Navier-Stokes equations, the same can be better understood through the plug flow model approach

Conservation of Mass - I

A – Area (m^2)
u – Velocity (m/s)
ρ – Density (kg/m^3)
$\dot{m}$ – Mass flow rate (kg/s)
s – Coordinate along pipe (m)
Γ' – Mass source/sink (kg/s-m)

A, u, ρ, $\dot{m}$



$A + \frac{\partial A}{\partial s}\Delta s$

$u + \frac{\partial u}{\partial s}\Delta s$

$\rho + \frac{\partial \rho}{\partial s}\Delta s$

$\dot{m} + \frac{\partial \dot{m}}{\partial s}\Delta s$

Rate of accumulation of mass in CV = Mass flow rate into CV – Mass flow rate out of CV +/- Rate of Mass generated / destructed in CV

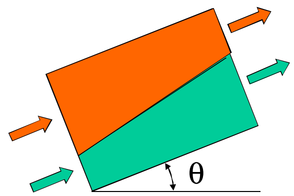
Conservation of Mass - II

$$\frac{\partial(\rho A \Delta s)}{\partial t} = \dot{m} - \left(\dot{m} + \frac{\partial(\dot{m})}{\partial s}\Delta s\right) \pm \Gamma'\Delta s$$

$$\frac{\partial(\rho A)}{\partial t} + \left(\frac{\partial(\dot{m})}{\partial s}\right) = \pm\Gamma' \quad (1)$$

The control volume need not extend over the entire cross section. An example is given in next slide

Conservation of Mass - III



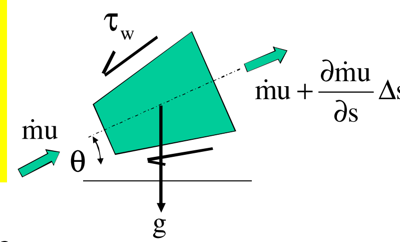
Gas $\frac{\partial(\rho_g A_g)}{\partial t} + \frac{\partial(\dot{m}_g)}{\partial s} = \Gamma'_g$

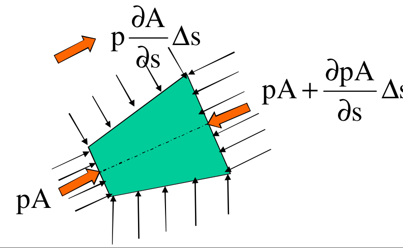
liquid $\frac{\partial(\rho_l A_l)}{\partial t} + \frac{\partial(\dot{m}_l)}{\partial s} = \Gamma'_l$

Note: In boiling flows $\Gamma'_g = -\Gamma'_l$

Conservation of Momentum - I

- τ_w – Wall shear stress (N/m^2)
- g – Gravitational acceleration (m^2/s)
- p – Pressure (N/m^2)
- P – Perimeter (m)
- H – Elevation (m)





Conservation of Momentum - II

Rate of accumulation of momentum in CV	=	Momentum rate into CV	−	Momentum rate out of CV	+	Sum of all forces in positive direction acting on CV
1		2		3		4

Term-1 $\frac{\partial(\rho A \Delta s u)}{\partial t}$ Term-2 $\dot{m}u$ Term-3 $\dot{m}u + \frac{\partial \dot{m}u}{\partial s}\Delta s$

Conservation of Momentum - III

Term-4 = Pressure force + Shear force + Gravity force

Pressure force $= pA - \left(pA + \frac{\partial pA}{\partial s}\Delta s\right) + p\frac{\partial A}{\partial s}\Delta s$

$= -A\frac{\partial p}{\partial s}\Delta s$

Shear force $= -\tau_w P \Delta s$

Gravity force $= -\rho A \Delta s g \mathrm{Sin}(\theta)$

Substitution of all the terms leads to

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Conservation of Momentum - IV

$$\frac{\partial(\rho A u \Delta s)}{\partial t} = \dot{m}u - \left(\dot{m}u + \frac{\partial(\dot{m}u)}{\partial s}\right) - A\frac{\partial p}{\partial s}\Delta s - \tau_w P\Delta s - \rho A \Delta s g \sin\theta$$

$$\Rightarrow \frac{\partial(\rho A u)}{\partial t} + \frac{\partial(\rho A u^2)}{\partial s} = -A\frac{\partial p}{\partial s} - \tau_w P - \rho A g \sin\theta \quad (2)$$

$$\Rightarrow \frac{\partial(\dot{m})}{\partial t} = -\frac{\partial(\dot{m}u)}{\partial s} - A\frac{\partial p}{\partial s} - \tau_w P - \rho A g \sin\theta \quad (3)$$

Eqs. (2) and (3) are the conservative forms of the momentum equation

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Conservation of Momentum - V

- We can get non-conservative form by expanding the LHS and using mass conservation equation

$$\rho A\frac{\partial u}{\partial t} + u\frac{\partial \rho A}{\partial t} + \rho A u\frac{\partial u}{\partial s} + u\frac{\partial \rho A u}{\partial s} = -A\frac{\partial p}{\partial s} - \tau_w P - \rho A g \sin\theta$$

$$\Rightarrow \rho A\frac{\partial u}{\partial t} + \rho A u\frac{\partial u}{\partial s} = -A\frac{\partial p}{\partial s} - \tau_w P - \rho A g \sin\theta \quad (4)$$

The above expression assumes source and sink of mass is zero.

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Mechanical Energy Equation - I

- Multiplication of momentum equation with velocity will give mechanical energy equation

$$\rho A u\frac{\partial u}{\partial t} + \rho A u^2\frac{\partial u}{\partial s} = -uA\frac{\partial p}{\partial s} - u\tau_w P - u\rho A g \sin\theta \quad (5)$$

$$\rho A\frac{\partial u^2/2}{\partial t} + \rho A u\frac{\partial u^2/2}{\partial s} = -uA\frac{\partial p}{\partial s} - u\tau_w P - u\rho A g\frac{\partial H}{\partial s} \quad (6)$$

For steady, inviscid and incompressible flow, we get

$$Au\frac{\partial \rho u^2/2}{\partial s} + Au\frac{\partial p}{\partial s} + uA\frac{\partial(\rho g H)}{\partial s} = 0$$

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Mechanical Energy Equation - I

$$\Rightarrow \frac{\partial\left(p + \rho u^2/2 + \rho g H\right)}{\partial s} = 0$$

$$\Rightarrow p + \rho u^2/2 + \rho g H = \text{Constant}$$

Bernoulli's Equation

I-Law of Thermodynamics

Definitions

h - Specific enthalpy (J/kg)

Specific Flow Energy

$$e = h + \frac{u^2}{2} + gH$$

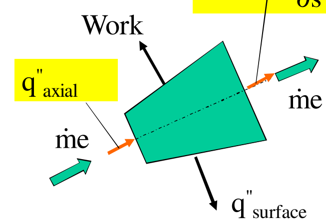
Specific Energy

$$i = h - \frac{p}{\rho} + \frac{u^2}{2} + gZ$$

Internal energy

I-Law - II

$$q''_{axial} + \frac{\partial q''_{axial}}{\partial s}\Delta s$$



$$\dot{m}e + \frac{\partial \dot{m}e}{\partial s}\Delta s$$

$$\dot{E}_{CV} = \dot{Q} - \dot{W} + \dot{m}_{inlet}e_{inlet} - \dot{m}_{exit}e_{exit}$$

$$q''_{axial} = -k\frac{\partial T}{\partial s}$$

$$q''_{surface} = U(T - T_\infty)$$

k – Thermal Conductivity (W/m-K)

U – Overall heat transfer coefficient (W/m²-K)

I-Law - III

$$\dot{E}_{CV} = \frac{\partial(A\Delta s\rho i)}{\partial t} = A\Delta s\frac{\partial\left(\rho\left[e - \frac{p}{\rho}\right]\right)}{\partial t}$$

- Using Taylor series, we can write

$$\dot{m}_{inlet}e_{inlet} - \dot{m}_{exit}e_{exit} = \dot{m}e - \left(\dot{m}e + \frac{\partial(\dot{m}e)}{\partial s}\Delta s\right)$$

$$= -\frac{\partial(\dot{m}e)}{\partial s}\Delta s$$

I-Law - IV

- We can write

$$\dot{W} = \tau_w P\Delta s u + W'_{CV}\Delta s$$

$$\dot{Q} = q''_{surface}P\Delta s + q''_{axial}A - \left(q''_{axial}A + \frac{\partial(q''_{axial}A)}{\partial s}\Delta s\right)$$

$$= q''_{surface}P\Delta s - \frac{\partial(q''_{axial}A)}{\partial s}\Delta s$$

Substitution of all the terms lead to

$$\frac{\partial(\rho Ae)}{\partial t} + \frac{\partial(\rho Aue)}{\partial s} = A\frac{\partial p}{\partial t} + q''_{surface}P - \tau_w Pu - \frac{\partial(q''_{axial}A)}{\partial s} - W'_{CV}$$

Conservative form

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I-Law - V

- Using mass conservation we can write

$$\rho A\frac{\partial e}{\partial t}+\rho Au\frac{\partial e}{\partial s}=A\frac{\partial p}{\partial t}+q''P-\tau_w Pu-\frac{\partial(q''_{axial}A)}{\partial s}-W'_{CV} \quad (8)$$

- Substituting the expression for e, we get

$$\rho A\frac{\partial(h+u^2/2+gH)}{\partial t}+\rho Au\frac{\partial(h+u^2/2+gH)}{\partial s}= \quad (9)$$

$$A\frac{\partial p}{\partial t}+q''_{surace}P-\tau_w Pu-\frac{\partial(q''_{axial}A)}{\partial s}-W'_{CV}$$

$$\rho A\frac{\partial u^2/2}{\partial t}+\rho Au\frac{\partial u^2/2}{\partial s}=-uA\frac{\partial p}{\partial s}-u\tau_w P-u\rho Ag\frac{\partial H}{\partial s} \quad (6)$$

I-Law - VI

- Subtracting both sides of mechanical energy equation Eq. (6) from I–Law Eq. (9) , we can write the thermal energy equation:

$$\rho A\frac{\partial(h)}{\partial t}+\rho Au\frac{\partial(h)}{\partial s}=uA\frac{\partial p}{\partial s}+A\frac{\partial p}{\partial t}+q''_{surface}P-\frac{\partial(q''_{axial}A)}{\partial s}-W' \quad (10)$$

- Here we have assumed that shear work is lost out to the surroundings. For insulated systems this gets ploughed back and hence we have to add $\tau_w Pu$ on RHS. If there is energy split, this has to be suitably handled

Final Set of Equations

Mass Balance

$$\frac{\partial(A\rho)}{\partial t}+\frac{\partial(\dot{m})}{\partial s}=0$$

Momentum Balance

$$-\frac{\partial p}{\partial s}=\frac{1}{A}\frac{\partial(\dot{m})}{\partial t}+\frac{1}{A}\frac{\partial(\rho Au^2)}{\partial s}+\tau_w\frac{P}{A}+\rho g\frac{\partial H}{\partial s}$$

Energy Balance

$$\rho A\frac{\partial(h)}{\partial t}+\rho Au\frac{\partial(h)}{\partial s}=uA\frac{\partial p}{\partial s}+A\frac{\partial p}{\partial t}+q''_{surface}P-\frac{\partial(q''_{axial}A)}{\partial s}-W'$$