Assignment 1

1. Consider open-channel flow of water in a circular pipe of diameter D. The water level is at a height h from the lowest point in the cross-section of the pipe. Compute the value of the ratio h/D using the bisection method, when (a) 20 % (b) 40 % (c) 60 %, and (d) 80 % of the cross-sectional area is occupied by air.
2. Consider the generalised equation of state given by Redlich-Kwong:

$$p = \frac{RT}{(v-b)} - \frac{a}{\sqrt{T}v(v+b)}$$

   where $a = 0.4275\ (R^2\ T_c^{2.5})/p_c$ and $b = 0.08664\ RT_c/p_c$, T_c and P_c being the critical temperature and pressure respectively, R is gas constant. Using this equation of state for water at a pressure of 1 bar, compute the value of the specific volume of water vapour using the Secant and the Newton's methods, at temperatures varying from 100 ^oC to 300 ^oC insteps of 50^oC. Compare your result with the values given in the steam table. Plot the error as a function iteration number for both the methods and comment.
3. Use Newton's method to find the roots of (a) $f(x) = x^2 - 2x + 1$ and (b) $f(x) = x^2 - 3x + 2$. Both the functions have a root x = 1. For both cases, start with an initial guess x(0) = 1.1. Use double precision variables in your program. Terminate your iterations when the absolute value of f(x) is less than 10^{-12}. Tabulate the values of x(k), e(k) = x(k)-1 and f(x(k)) for each iteration. Print out the number of iterations required for convergence for each case. What is the ratio e(k+1)/e(k) for the two cases? Comment on the rates of convergence for the two cases.
4. Consider the quadratic equation, $x^2 - 2.2x + 1.2$. Note that the roots of the equation are 1 and 1.2. You are asked to find the roots of the above equation using fixed point iteration with, $x = x + \omega\ g(x)$, where ω is a relaxation parameter. Perform the following steps and comment on the results with valid justifications
   (a) starting with the initial guess x=1.10, ω=1.00, perform 50 iterations
   (b) starting with the initial guess x=1.21, ω=1.00, perform 50 iterations
   (c) starting with the initial guess x=1.21, ω=-1.0, perform 50 iterations
   (d) starting with the initial guess x=1.21, ω=-5.0, perform 50 iterations
   (e) starting with the initial guess x=1.21, ω=-8.0, perform 50 iterations