Assignment 3

1. Consider a set of non-linear equation
   $3x_1 - \cos(x_2x_3) - ½ = 0$
   $x_1^2 - 81(x_2+0.1)^2 + \sin x_3 + 1.06 = 0$
   $\text{Exp}(-x_1x_2) + 20\, x_3 + (10\pi-3)/3 = 0$

   Solve the above by Newton's method as directed. The solution for the above is $x_1 = 0.5$, $x_2 = 0$, $x_3 = -\pi/6$. First take guesses near the solution and visualize its rapid convergence. Then take guesses far away and experience the divergence. Thirdly, take a far away guess and apply continuation method to obtain the solution. Document your experience.

2. Write a program to evaluate values of any assumed function to form a discrete set of data (y(i),x(i), i=1,10)). Now using second order Lagrange interpolation, write a routine to find the values of the function at a few intermediate x values (say 5). You can now compare this value with the exact value by substituting the value of x in the assumed function. Comment on your results.

   Any Lagrange interpolation routine should first have a search routine which will pass on three relevant values of y and x using the following logic. If the value of x lies in the last interval, then the last, last but one and last but two sets will be passed on. Otherwise, for the value of x lying between the interval i, i+1, then the values of the i, i+1 and i+2 will be passed on.Before solving the actual problem given above, first construct a table of y for the function y=a + b*x + c*x*x for suitable values of a, b and c with x varying from 0 to 1 in intervals of 0.1. Now write a numerical algorithm for Lagrangian interpolation as suggested by the procedure in class and estimate the values of y at 0.05, 0.25, 0.55, 0.75 and 0.95. Compare this with the actual values. If your algorithm is correct both values would match.
3. Using the algorithm given in the notes write a procedure for cubic spline algorithm. Verify its applicability by checking against the quadratic function data used in the previous problem. Having done that solve the following problem. Consider the table with 11 values of y for corresponding values of x

N	1	2	3	4	5	6	7	8	9	10	11
x	-1	-0.6	-0.2	0.2	0.6	1	0.6	0.2	-0.2	-0.6	-1
y	0.00	-1.60	-1.96	-1.96	-1.60	0.00	-1.60	-1.96	-1.96	-1.60	0.00

Note that neither x nor y is a single valued function of the other. This implies there can be 2 values of y for a given x and vice versa. Therefore to pass a smooth curve through the points (x,y) the following procedure is employed. Two splines are fitted (y with N and x with N, where N is just an arbitrary number that increases monotonically). Now the values of x and y for the intermediate points are generated using various values of N. For the given data, generate the values for (x,y) in increments of 0.2 for N. Plot this in a x-y plot and visualise the data. The data actually fits an ellipse $x^2/1+y^2/4 = 1$. Check whether the interpolated data actually lies on the exact curve?