Assignment 5

1. Consider a first order differential equation, $(dy/dx) = x-y$ with $y = 0$ at $x = 0$.
   The exact solution of the above equation is $y = x + \exp(-x) - 1$. Integrate the above equation numerically for $x = 0$ to $x = 12$ using a step size of 0.8 by (a) Euler's method, (b) modified Euler's method, (c) R-K fourth order method (classical), (d) Milne's Predictor Corrector method and (e) Adams-Moulton (AM-4) predictor corrector method. Compare the results with the analytical solution. Now repeat the above problem with decreasing step sizes to 0.4 and 0.2. Now compare the normalised errors at $x = 12$ as a function of step size and correlate it with the order of the method. For multistep methods, use analytical solution to get the first three points.
2. Derive the algorithm and error terms for Milne's method (students with odd roll numbers) and Adam-Moulton method (students with even roll numbers)
3. Consider the ODE, $(dy/dx) = -3y$, with $y(0)=1$. The exact solution for the ODE is $y = e^{-3x}$. Now use the programs developed for problem 1 as follows. (a) solve using Milne, AM-4 and RK-4 with a step size of 0.1 Comment on the solutions. (b) Increase the step size to 0.5 and then 1 and run RK-4. Discuss the results. (c) From the stability analysis carried out in class find out the value of the step size when the method will become unstable. Use values slightly below and above that number and obtain numerical results. Do you see the value of stability analysis? Comment?