Assignment 6

Solve 1, 2 and 4. If you have extra time, then solve 3. Bonus points will be given for solution of 3.

1. Consider the differential Equation y''+ 3.1 y' + 0.3y = 0. with the boundary conditions: y(x = 0) = 2, y'(x = 0) = -3.1. Note that the solution of the above equation is, y = exp(-3x)+exp(-0.1x).

(a) first express the above second order differential into two first order differential equations. Solve this as an initial value problem using RK-4 (classical method) for x = 0 to x = 20 with a constant step size. You are asked to decrease the step size such that the maximum normalised error in the domain is less than 10**(-5).

(b) Now repeat the above with an adaptive RK-4 method with the same tolerance. How many steps are required to solve the same problem

(c) Now transform the above problem as a BVP with BC y(x = 0) = 0, y(x = 20) = 0.1353352832. Now solve the above problem with finite difference method using central difference approximations of second order accuracy. Solve with decreasing values of step sizes till you get maximum normailsed error is within 10**(-5). Comment on your results

(d) Narrate your experience for solving the same problem as a IVP and a BVP.

2. Solve the Blasius Equation using Shooting method. Tabulate your solutions and compare them with that given your fluid mechanics textbook.
3. Now, solve the above problem with the boundary value method using finite differences. You may decide to use any of the treatment taught in the class for applying the boundary condition. Being a non-linear equation you would have to linearise the non-linear term and use underrelaxation. Just explore and summarise your observations.
4. Consider a heat generating slab of thickness L with a constant volumetric heat generation rate of q'''. The steady state governing equation for the case with constant thermal conductivity is given by the heat conduction equation $d^2T/dx^2 = -q'''/k$. The boundary surfaces are held at a given temperature T_s.

(a) Non-dimensionalise the governing equation and the boundary condition with the following definitions $T^* = (T-T_s)/(q'''L^2/k)$, $x^* = x/L$

(b) Obtain the analytical solution of the non-dimensional equation

(c) Solve the same by finite difference method. Perform sensitivity analysis and report your error results by comparing it with the analytical solution in terms of maximum error. Explain your findings.