Assignment 9

1. Consider a governing equation $\frac{\partial\phi}{\partial t}+(C-\mathrm{Sin}(\omega t))\frac{\partial\phi}{\partial s}=0$, subject to initial and boundary condition, $\phi(s,0)=\phi_{to}(s)$, and $\phi(0,t)=\phi_{so}(t)$. For a domain of 0 < s< 10, 0 < t < 50 and with, $\phi_{to}(s)=s/10$, and $\phi_{so}(t)=\sin 2t$, show that the analytical solution for this case can be expressed as $\phi(s,t)=\phi_{to}(s_0)$, for the condition $s\geq s^*$ and $\phi(s,t)=\phi_{so}(t_0)$, for the condition $s<s^*$, where $s_0=s-s^*$, $s^*=Ct+(\mathrm{Cos}(\omega t)-1)/\omega$, and $t_0$ is the root of the equation, $s-C(t-t_0)-(\mathrm{Cos}(\omega t)-\mathrm{Cos}(\omega t_0))/\omega=0$. (Note $s^*$ is the value of s(t) for the characterstic line starting from origin)
2. Consider convection equation $\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}=0$ where u = 0.05 cm/s, delx=0.05 cm, for the domain 0<x<2.5., t > 0. The initial condition, T(0,x) = F(x), where

| | | |
|---|---|---|
| F(x) | = 400(0.25x) | for 0<x<or=0.25 |
| | = 400 * (0.5-x) | for 0.25<x<or=0.5 |
| | = 0 | for x>0.5 |

The boundary condition T(t,0) = 0 The analytical solution for the above problem is T(t,x)=F(x-ut), for (x-ut)≥0. Note that the boundary condition will propagate and make F( x < 0) = 0. The above equation represents a translating triangle as discussed in the class.

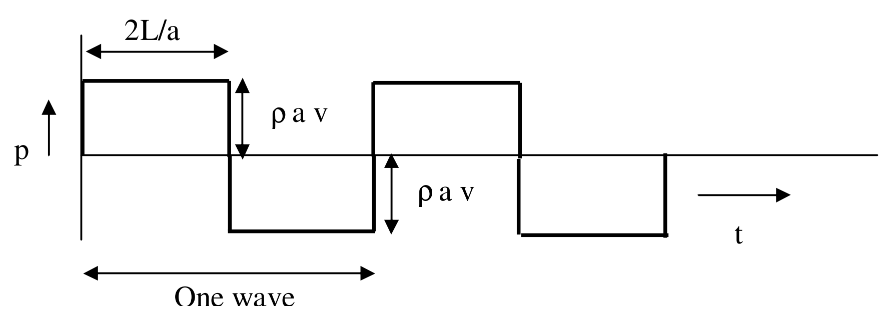
(i)Verify the results shown in the class, viz., that for upwind scheme,

(a) and for c=u*delt/delx=1, the numerical solution is exact.

(b) For c= 0.1, 0.5 and 0.9, there is progressive decrease in numerical diffusion

(ii)Repeat the same with Lax Wendroff Scheme and convince yourself the advantage of Lax Wendroff Scheme.

3. Consider that you are solving the water hammer equations for a horizontal duct of diameter 1m and length (L) 10 km. The initial velocity (moving from left to right) in the pipe can be assumed to be 2 m/s and the effective wave speed, a, to be 1000 m/s. For the case of no friction, the steady pressure distribution will be zero everywhere. Now consider the transient case where the left boundary is kept at p = 0 and the right boundary is suddenly brought to zero. Using MOC, solve for the pressure and velocity variation. Choose 10 nodes and choose $\Delta t$ to be such that $a\Delta t/\Delta x$ = 1. This will need no interpolations (note that we have ignored the velocity of fluid in our characteristic directions). Take $\rho$ = 1000 kg/m$^3$. Compare your solution with the analytical solution for this case at the right boundary. It is shown in the figure below. The pressure wave will keep oscillating with an amplitude of $\rho$ a v and a time period of = 4L/a.




Now let us try to analyse the case where friction also exists

(i) Calculate the frictional pressure drop in the duct using, $\Delta p=\frac{\rho f l v^2}{2d}$, where f is taken as constant = 0.02 for simplicity.

(ii) When water flows steadily, the pressure distribution will vary linearly with pressure at right boundary equal to 0 ( gauge pressure) and pressure at left boundary equal to 0 $+\Delta p$ arrived in part (i)

(iii) Now solve for the transient using Method of Characteristics. The following may be taken as boundary conditions (1) Pressure held constant in the left boundary (Assuming the presence of a reservoir), (2) The velocity suddenly reduced to zero and held at zero (valve closure) in the right boundary. The results for the above problem may be plotted in the form of (p,t) plot at right boundary and the mid pipe. Interpret your results.

4. Show for an ideal gas,

$$\frac{\rho h_\rho}{(\rho h_p - 1)} = -a^2$$