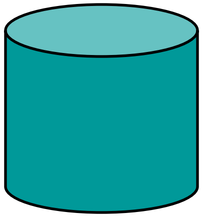
Forging Analysis – 2
Cylindrical Forging

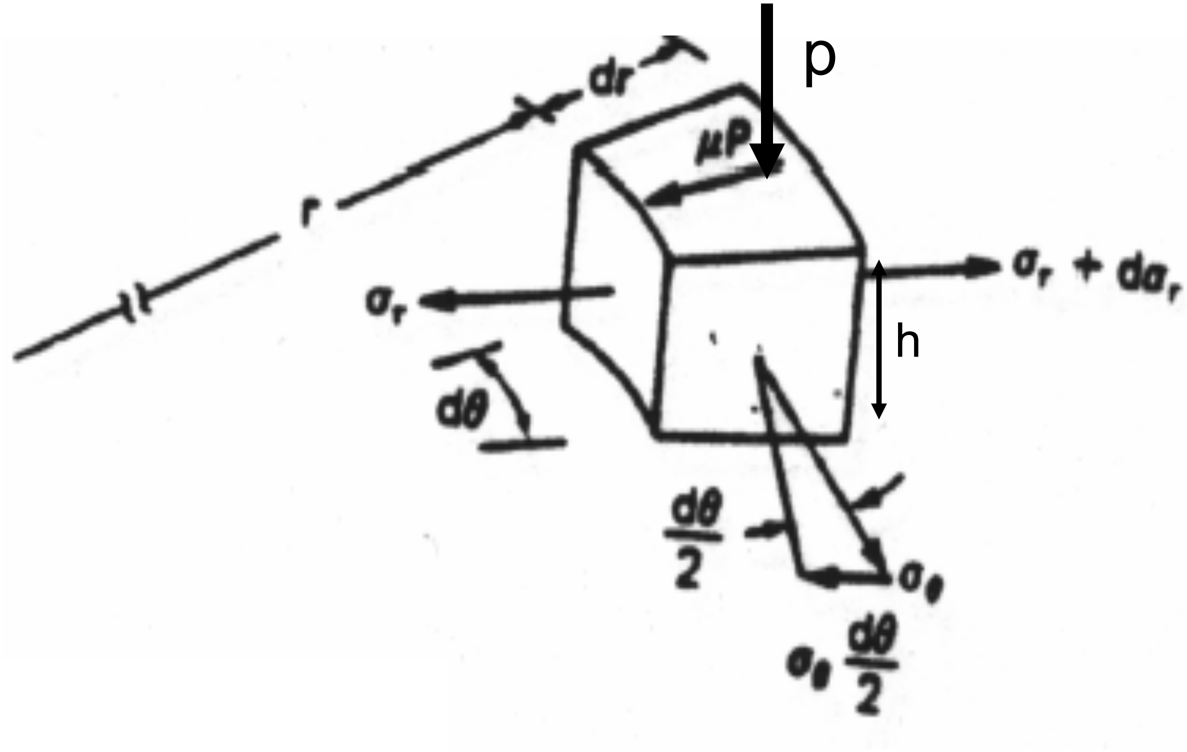
ver. 1

Overview

- Slab analysis
  - frictionless
  - with friction
  - Rectangular
  - Cylindrical
- Strain hardening and rate effects
- Flash
- Redundant work

Forging – cylindrical part sliding region





Equilibrium in r direction

$$\sum dF_r = 0 = -\sigma_r \cdot h \cdot r \cdot d\theta - 2 \cdot \mu \cdot p \cdot r \cdot d\theta \cdot dr$$

$$-2 \cdot \sigma_\theta \cdot h \cdot dr \cdot \frac{d\theta}{2} + \left(\sigma_r + d\sigma_r\right) \cdot \left(r + dr\right) \cdot h \cdot d\theta$$

N.B. $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$

neglecting HOTs

$$2\mu pr \cdot dr + h\sigma_\theta \cdot dr - h\sigma_r \cdot dr - hr \cdot d\sigma_r = 0$$

Axisymmetric flow and yield

For axisymmetric flow

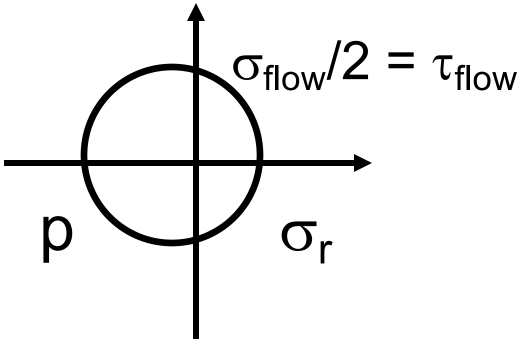
$$\varepsilon_r = \frac{dr}{r}; \quad \varepsilon_\theta = \frac{2\pi(r+dr) - 2\pi r}{2\pi r} = \frac{dr}{r}$$

$$\varepsilon_r = \varepsilon_\theta; \sigma_r = \sigma_\theta$$

By Tresca

$$\sigma_r + p = \sigma_{flow} = 2k = 2\tau_{flow}$$

$$d\sigma_r = -dp$$



Stress in z direction

substituting

$$2\mu pr \cdot dr + h\sigma_r \cdot dr - h\sigma_r \cdot dr + hr \cdot dp = 0$$

or

$$2\mu pr \cdot dr = -hr \cdot dp$$

rearranging

$$\frac{dp}{p} = -\frac{2\mu}{h} dr$$

Forging pressure - sliding

$$k = \tau_{flow};$$

Yield or flow stress, $\sigma_f = 2k = 2\tau_{flow}$

$$\int_{p_r}^{2\tau_{flow}} \frac{dp}{p} = -\int_{r}^{R} \frac{2\mu}{h} \cdot dr$$

for $r_k < r < R$

$$\frac{p_r}{2\tau_{flow}} = \exp\left[\frac{2\mu}{h}(R - r)\right]$$

Average forging pressure – sliding

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{1}{\pi\left(R^2 - r_k^2\right)}\int_{r_k}^{R}\frac{p_r}{2\tau_{flow}}\cdot 2\pi r\cdot dr = \frac{2}{\left(R^2 - r_k^2\right)}\int_{r_k}^{R}\exp\left[\frac{2\mu}{h}(R-r)\right]rdr$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{\left(R^2 - r_k^2\right)}\left(\frac{h}{2\mu}\right)^2\exp\left(\frac{2\mu R}{h}\right)\left\{\exp\left(\frac{-2\mu r}{h}\right)\cdot\left(\frac{-2\mu r}{h}-1\right)\right\}\Bigg|_{r_k}^{R}$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{\left(R^2 - r_k^2\right)}\left(\frac{h}{2\mu}\right)^2\exp\left(\frac{2\mu R}{h}\right)\left\{\left[\exp\left(\frac{-2\mu R}{h}\right)\cdot\left(\frac{-2\mu R}{h}-1\right)\right]-\left[\exp\left(\frac{-2\mu r_k}{h}\right)\cdot\left(\frac{-2\mu r_k}{h}-1\right)\right]\right\}$$

Average forging pressure – sliding

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{\left(R^2 - r_k^2\right)} \cdot \left(\frac{h}{2\mu}\right)^2 \left[\exp\left(\frac{2\mu\left(R - r_k\right)}{h}\right) \cdot \left(\frac{2\mu r_k}{h} + 1\right) - \left(\frac{2\mu R}{h}\right) - 1\right]$$

Average forging pressure – all sliding approximation ($r_k = 0$)

- Taking the first four terms of a Taylor's series expansion for the exponential about 0 for $|x| \le 1$

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} = \sum_{k=0}^{n} \frac{x^k}{k!}$$

yields

$$\frac{p_{ave}}{2\tau_{flow}} = \left[1 + \left(\frac{2\mu R}{3h}\right)\right]$$

Forging force – all sliding approximation

$$F_{forging} = p_{ave} \cdot A = p_{ave} \cdot \pi \cdot R^2$$

$$F_{forging} = 2\tau_{flow} \cdot \left[1 + \left(\frac{2\mu R}{3h} \right) \right] \cdot \pi R^2$$

Transition sticking / sliding

- Set $\tau_{flow} = \mu p$ and solve for r_k

$$\frac{p}{2\tau_{flow}} = \exp\left[2\mu\left(\frac{R - r_k}{h}\right)\right] \rightarrow \frac{p}{2\mu \cdot p} = \exp\left[2\mu\left(\frac{R - r_k}{h}\right)\right]$$

$$\ln\left(\frac{1}{2\mu}\right) = 2\mu\left(\frac{R - r_k}{h}\right) \longrightarrow r_k = R - \frac{h}{2\mu}\ln\left(\frac{1}{2\mu}\right)$$

Forging pressure - sticking region

- Use the same method as for sliding
- Substitute $\mu p = \tau_{flow}$,
- Assume Tresca yield criterion

$$2\mu pr \cdot dr = -hr \cdot dp$$

$$2\tau_{flow} r \cdot dr = -hr \cdot dp$$

$$dp = -\frac{2\tau_{flow}}{h} dr$$

Forging pressure - sticking region

$$\int_{p_{r_k}}^{p_r} dp = -\int_{r_k}^{r} \frac{2\tau_{flow}}{h} dr$$

$$p_r - p_{r_k} = -\frac{2\tau_{flow}}{h}\left(r - r_k\right)$$

$$\frac{p_r - p_{r_k}}{2\tau_{flow}} = \frac{\left(r_k - r\right)}{h}$$

Forging pressure - sticking region

p_{r_k} determined from sliding equation

$$\frac{p_{r_k}}{2\tau_{flow}} = \exp\left[\frac{2\mu}{h}(R - r_k)\right]$$

for $0 < r < r_k$

$$\frac{p_r}{2\tau_{flow}} = \exp\left[\frac{2\mu}{h}(R - r_k)\right] + \frac{(r_k - r)}{h}$$

Average forging pressure - sticking

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{1}{\pi r_k^2}\int_0^{r_k} p_r \cdot 2\pi r \cdot dr = \frac{2}{r_k^2}\int_0^{r_k}\left(\exp\left[\frac{2\mu}{h}\left(R-r_k\right)\right]+\frac{r_k-r}{h}\right)\cdot rdr$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2}\int_0^{r_k}\left(r\cdot\exp\left[\frac{2\mu}{h}\left(R-r_k\right)\right]+\frac{r_k\cdot r}{h}-\frac{1}{h}r^2\right)\cdot dr$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2}\left(\frac{r^2}{2}\cdot\exp\left[\frac{2\mu}{h}\left(R-r_k\right)\right]+\frac{r_k\cdot r^2}{2h}-\frac{r^3}{3h}\right)\Bigg|_0^{r_k}$$

Average forging pressure - sticking

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2}\left(\frac{r^2}{2}\cdot\exp\left[\frac{2\mu}{h}\left(R-r_k\right)\right]+\frac{r_k\cdot r^2}{2h}-\frac{r^3}{3h}\right)\Bigg|_0^{r_k}$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2}\left(\frac{r_k^2}{2}\cdot\exp\left[\frac{2\mu}{h}\left(R-r_k\right)\right]+\frac{r_k^3}{2h}-\frac{r_k^3}{3h}\right)$$

$$\frac{p_{ave}}{2\tau_{flow}} = \left(\exp\left[\frac{2\mu}{h}\left(R-r_k\right)\right]+\frac{r_k}{3h}\right)$$

Forging force – sticking region

$$F_{forging} = p_{ave} \cdot A = p_{ave} \cdot \pi \cdot r_k^2$$

$$F_{forging} = 2\tau_{flow} \cdot \left(\exp\left[\frac{2\mu}{h}(R - r_k) \right] + \frac{r_k}{3h} \right) \cdot \pi \cdot r_k^2$$

Sticking and sliding

- If you have both sticking and sliding, and you can't approximate by one or the other,
- Then you need to include both in your pressure and average pressure calculations.

$$F_{forging} = F_{sliding} + F_{sticking}$$

$$F_{forging} = \left(p_{ave} \cdot A\right)_{sliding} + \left(p_{ave} \cdot A\right)_{sticking}$$

Strain hardening (cold - below recrystallization point)

Tresca

$$2\tau_{flow} = Y = K\varepsilon^{n}$$

Strain rate effect
(hot – above recrystallization point)

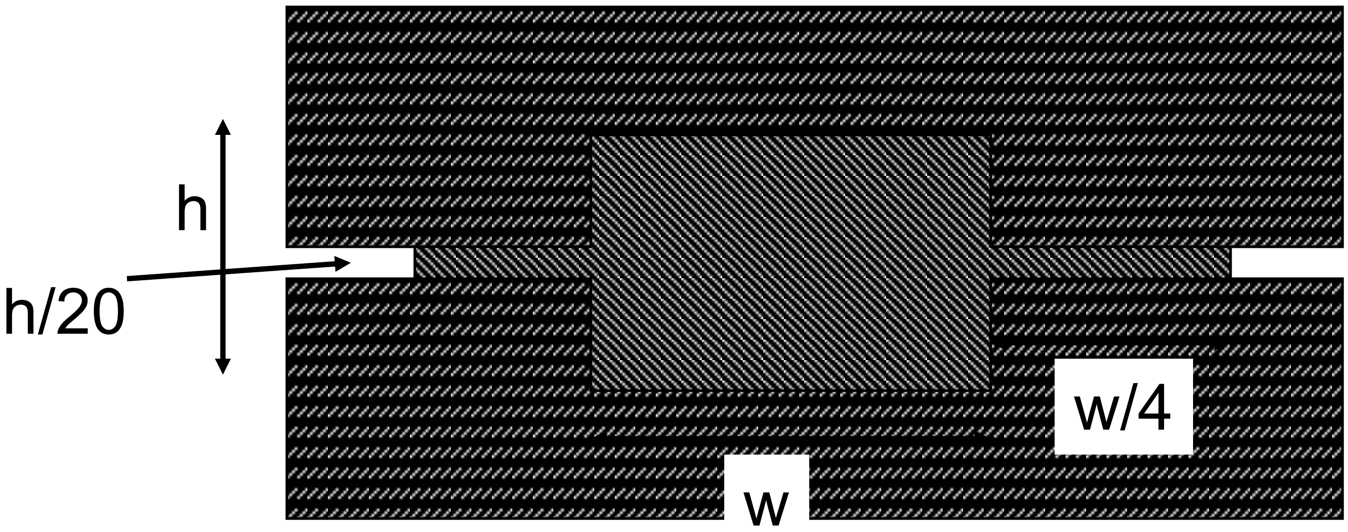
$$\dot{\varepsilon} = \frac{1}{h}\frac{dh}{dt} = \frac{v}{h} = \frac{\text{platen velocity}}{\text{instantaneous height}}$$

Tresca

$$2\tau_{flow} = Y = C\left(\dot{\varepsilon}\right)^{m}$$

Flash for closed die forging
(plane strain)

- Say we have a typical flash with thickness h/20 and length w/4



Average forging pressure

- in forging (Tresca)

$$p_{ave} = 2\tau_{flow} \cdot \left(1 + \frac{\mu w}{2h}\right)$$

- in flash (Tresca)

$$p_{ave} = 2\tau_{flow} \cdot \left(1 + \frac{5\mu w}{h}\right)$$

Flash

- Flash's high deformation resistance results in filled mold
- Process wouldn't work without friction