Fatigue Failure in Variable Loading

ME 423: Machine Design
Instructor: Ramesh Singh

Outline

- Fatigue Introduction
- Fatigue Life Methods
  - Stress-based Approach
  - Strain-based Approach
  - Linear Elastic Fracture Mechanics
- Endurance Limit and Fatigue Strength
- Characterizing Fluctuating Stresses
- Fatigue Failure Criteria for Fluctuating Stresses
- Combination of Loading Modes

Introduction to Fatigue

- Cyclic loading produces stresses that are variable, repeated, alternating, or fluctuating
- Fracture at maximum stresses well below yield strength (S_Y)
- Failure occurs after many stress cycles (100, 000 cycles)
- Failure is by sudden ultimate fracture
- No visible warning in advance of failure

History

In May 1842, a train to Paris crashed in Meudon because the leading locomotive broke an axle. The picture tells it all; at least 55 passengers were killed.

A first explanation of what might have happened came from William John Macquorn Rankine one of the many famous Scottish physicists. He had investigated broken axles, highlighting the importance of stress concentration, and the mechanism of crack growth with repeated loading or "vibrations". This means that he was the first to suggest that the basic mechanisms of fatigue is tied to vibrations.

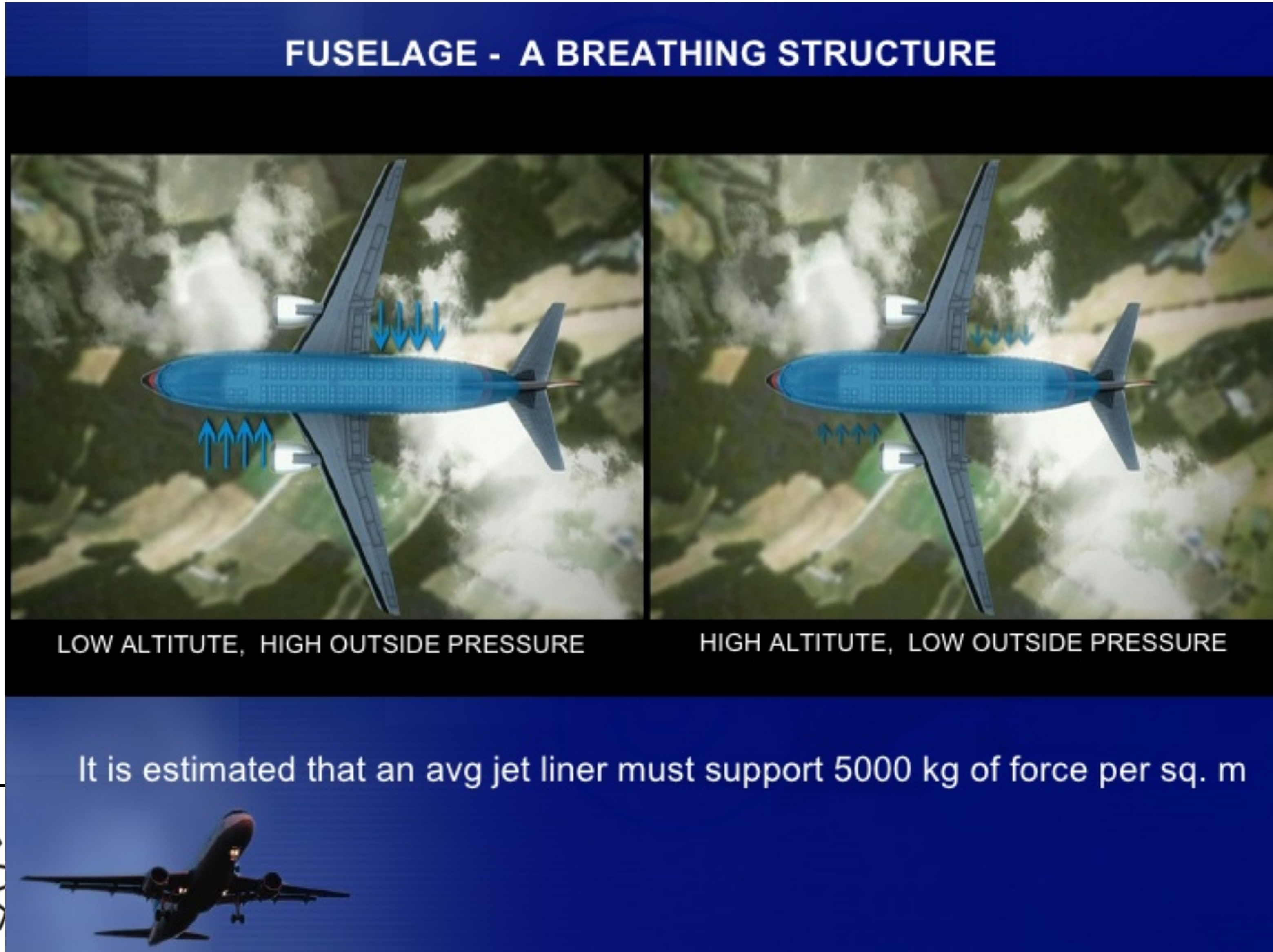
Haviland Crash

Two de Havilland Comet passenger jets (the first commercial jet planes!) broke up in mid-air and crashed within a few months of each other in 1954. The crashes were a result of metal fatigue, caused by the repeated pressurization and de-pressurization of the aircraft cabin

Aloha Airline Accident

Damage due to Fatigue

Cyclic Loading on Fuselage



Failure Analysis

Why did the fuselage rip apart?

•According to boeing engineers if disbonding occurred,the hoop load trasferred through the joint would be borne by 3 rows of countersunk rivets

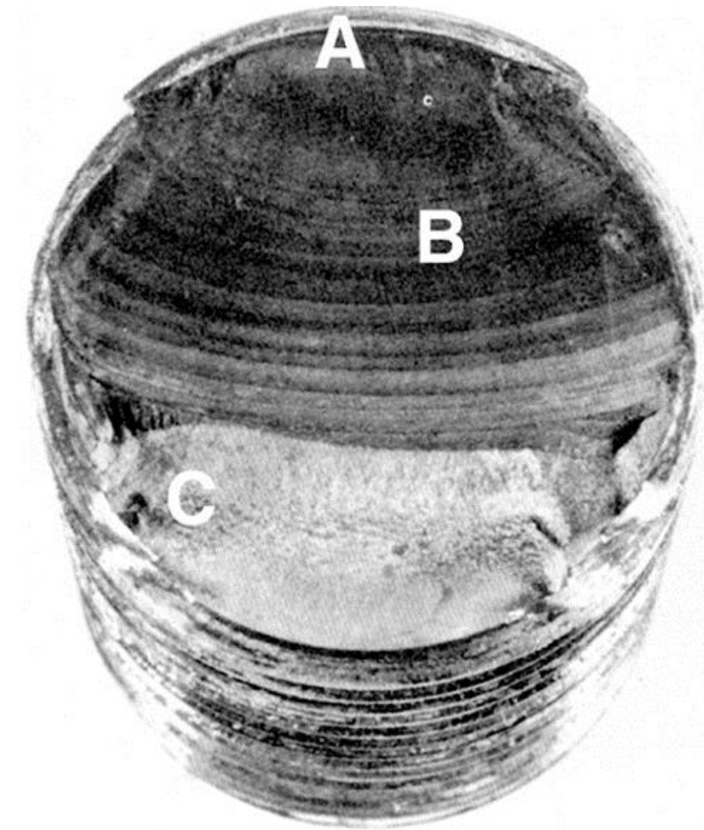
•The countersink for the entire rivet heads extended through the entire thickness of the fuselage,creating a knife edge at the bottom of the hole which concentrated stresses.

•Stresses were cyclic with pressurization loads and fatigue cracking ultimately occurred at the site.

ICE Accident due to Fatigue of Wheel

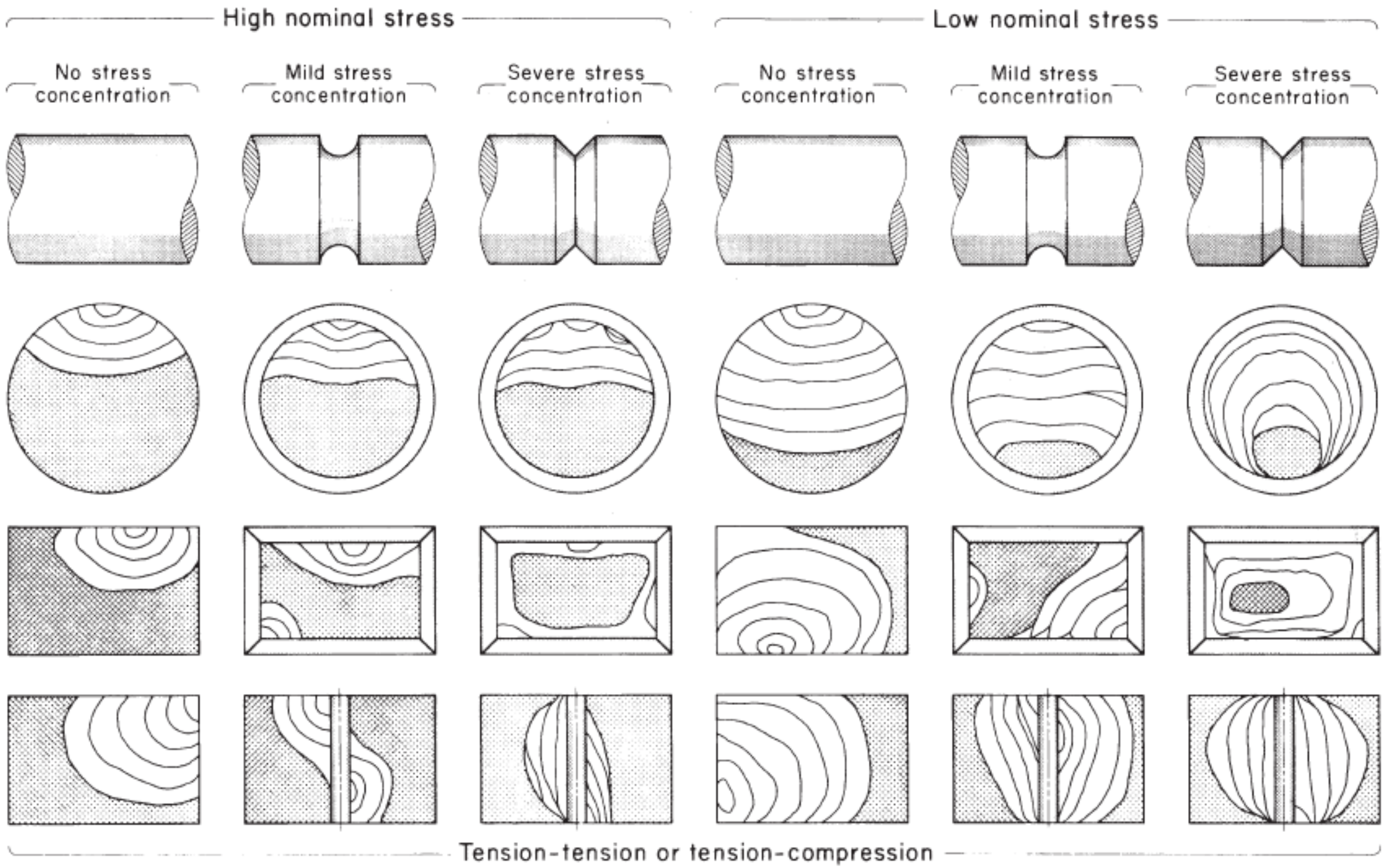
Failure Stages

- Stage I – Initiation of micro- crack due to cyclic plastic deformation
- Stage II–Progresses to macro-crack which forms plateau-like surface that repeatedly opens and closes, creating bands called beach marks
- Stage III–Crack has propagated far enough that remaining material is insufficient to carry the load, and fails by simple ultimate failure

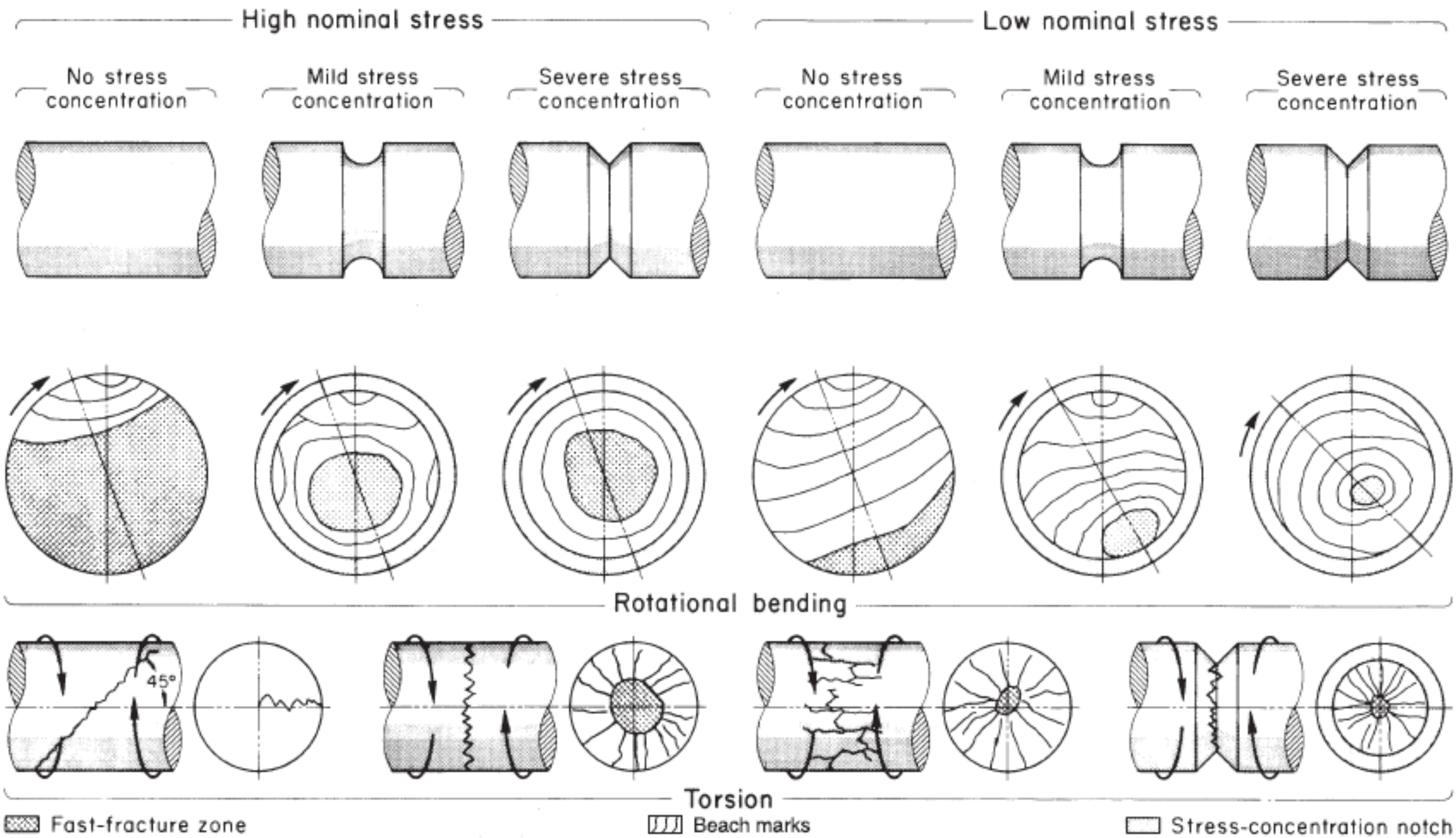


Started at A propagated towards B and finally sudden failure at C

Fatigue Fractured Surfaces

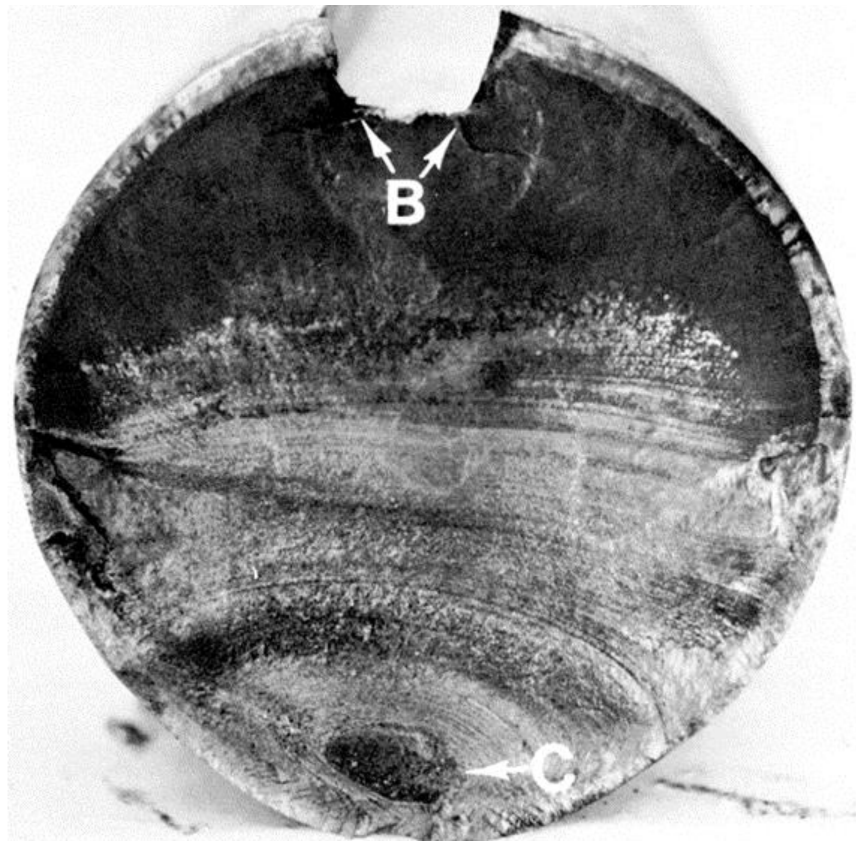


Fractured Surface in Torsion



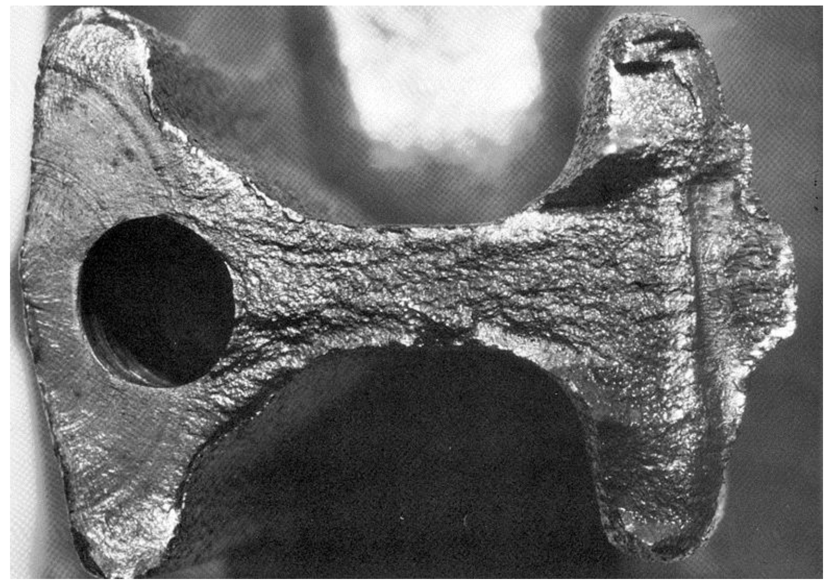
Fatigue Fracture – Examples

- AISI 4320 drive shaft
- B– crack initiation at stress concentration in keyway
- C– Final brittle failure



Fatigue Fracture – Examples

- Fatigue failure of forged connecting rod
- Crack initiated at flash line of the forging at the left edge of picture
- Beach marks show crack propagation halfway around the hole before ultimate fracture



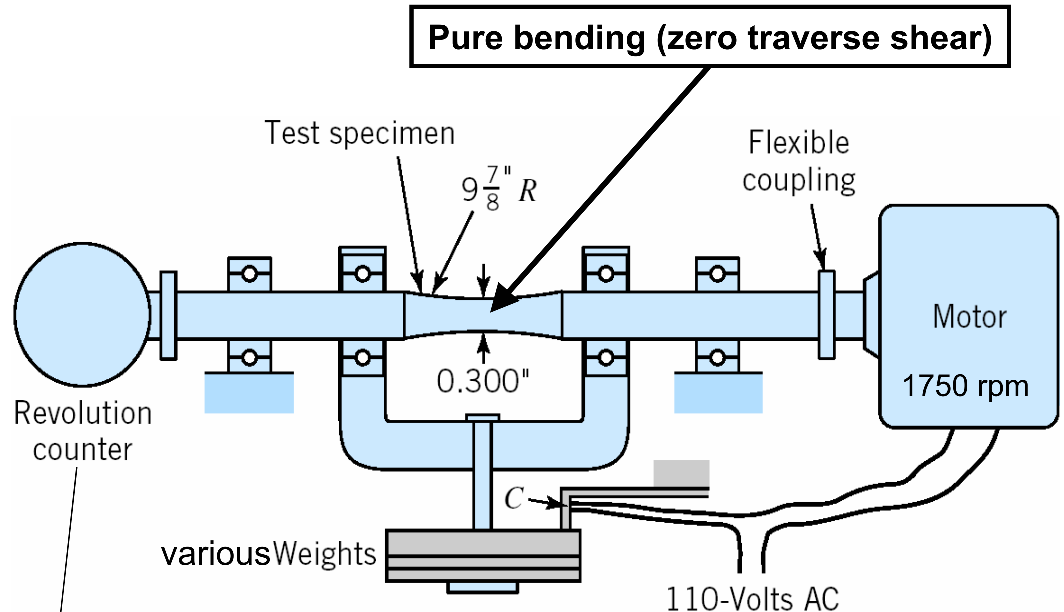
Fatigue Life Methods

- Three major fatigue life models
- Methods predict life in number of cycles to failure, N, for a specific level of loading
- Stress-life method (used in most designs)
  - Least accurate, particularly for low cycle applications
  - Most traditional, easiest to implement
- Strain-life method
  - Detailed analysis of plastic deformation at localized regions
  - Several idealizations are compounded, leading to uncertainties in results
- Linear-elastic fracture mechanics method
  - Assumes crack exists
  - Predicts crack growth with respect to stress intensity

R. R. Moore Fatigue Testing Machine

Courtesy: Instron

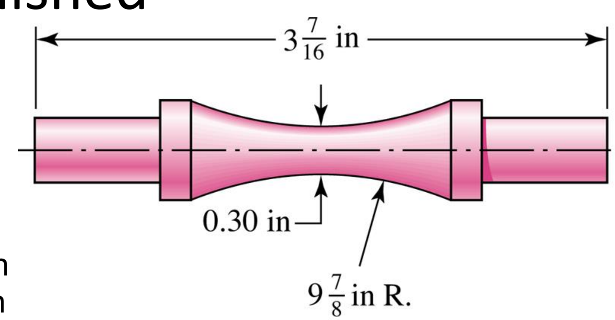
Rotating-beam fatigue-testing machine



N cycles of tension-to-compression-to-tension

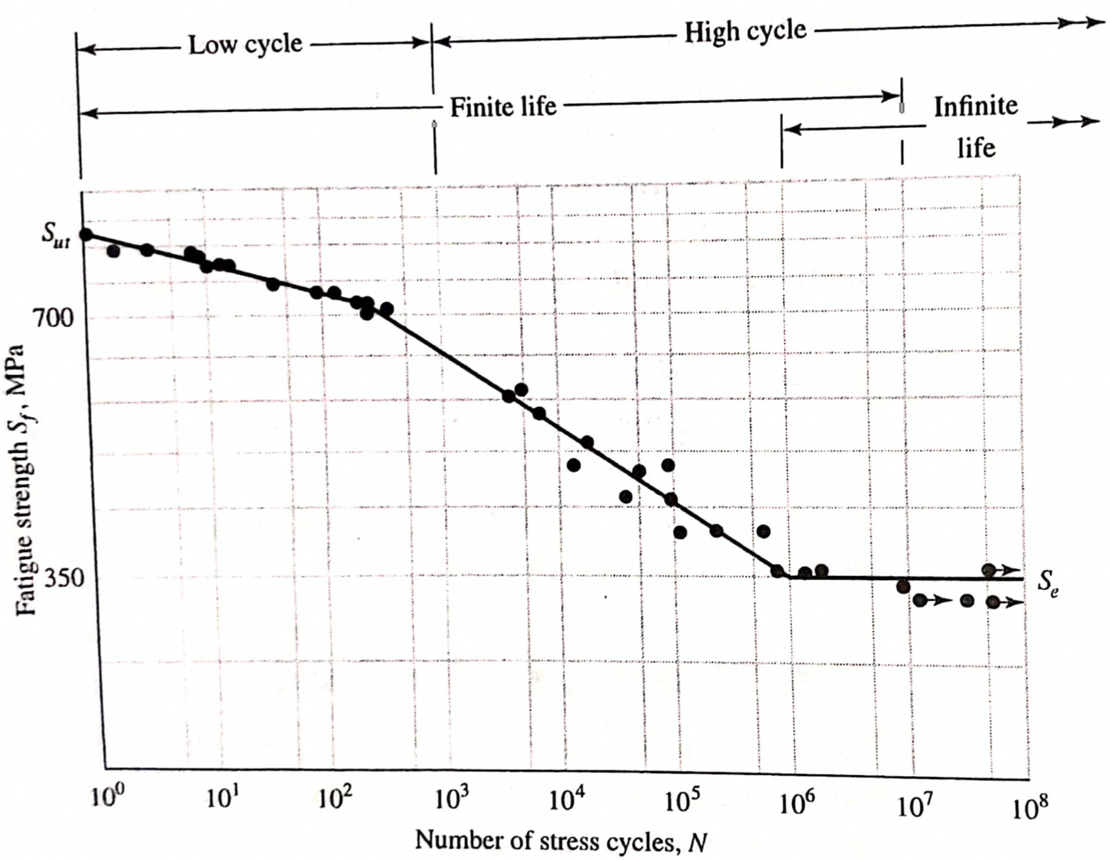
Stress-Life Method

- Test specimens are subjected to repeated stress while counting cycles to failure
- Most common test machine is R. R. Moore high-speed rotating-beam machine
- Subjects specimen to pure bending with no transverse shear
- As specimen rotates, stress fluctuates between equal magnitudes of
- Tension and compression, known as completely reversed stress cycling
- Specimen is carefully machined and polished



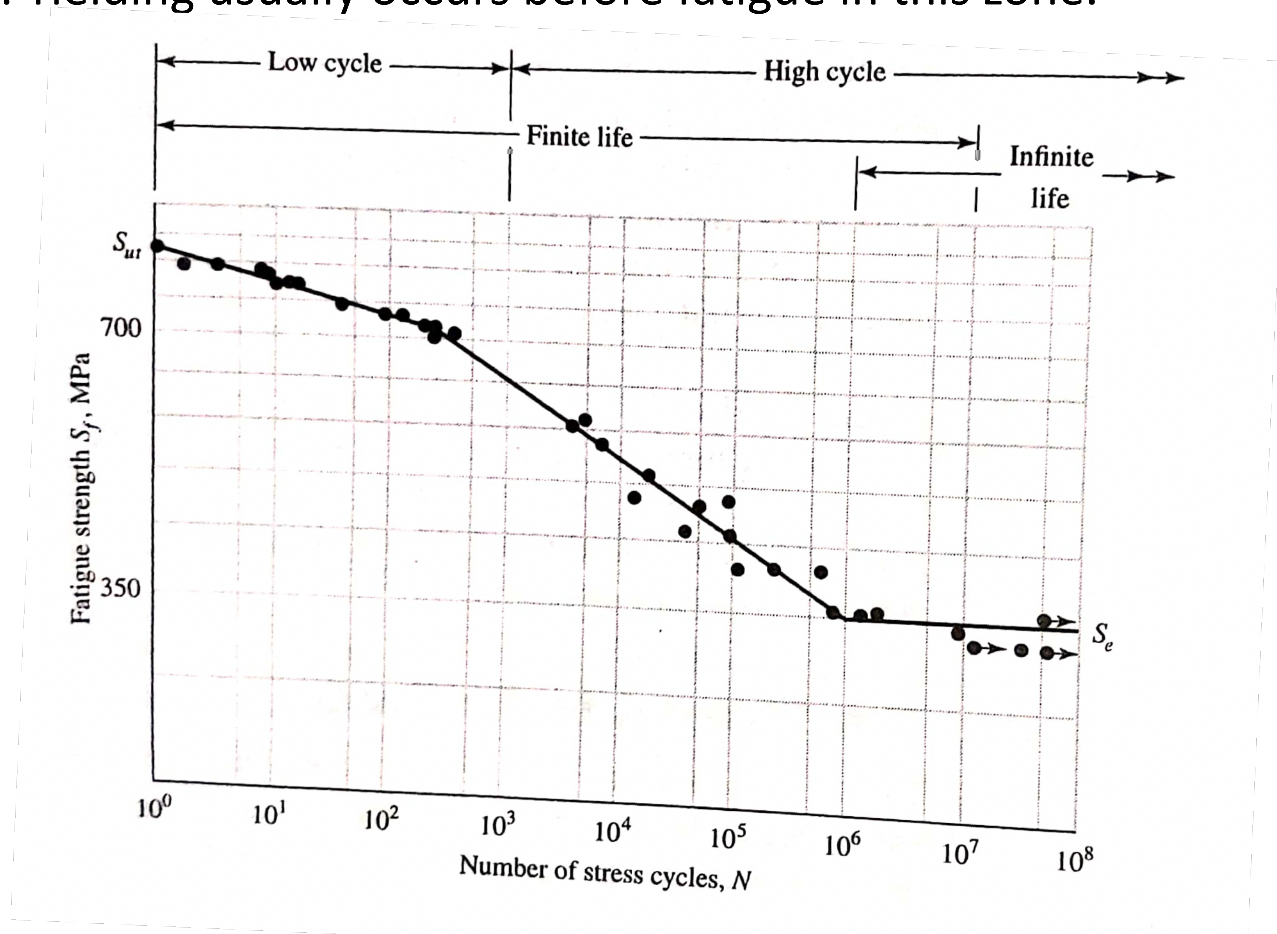
S-N Diagram

- Number of cycles to failure at varying stress levels is plotted on log-log scale
- For steels, a knee occurs near 10^6 cycles
- Strength corresponding to the knee is called endurance limit Se

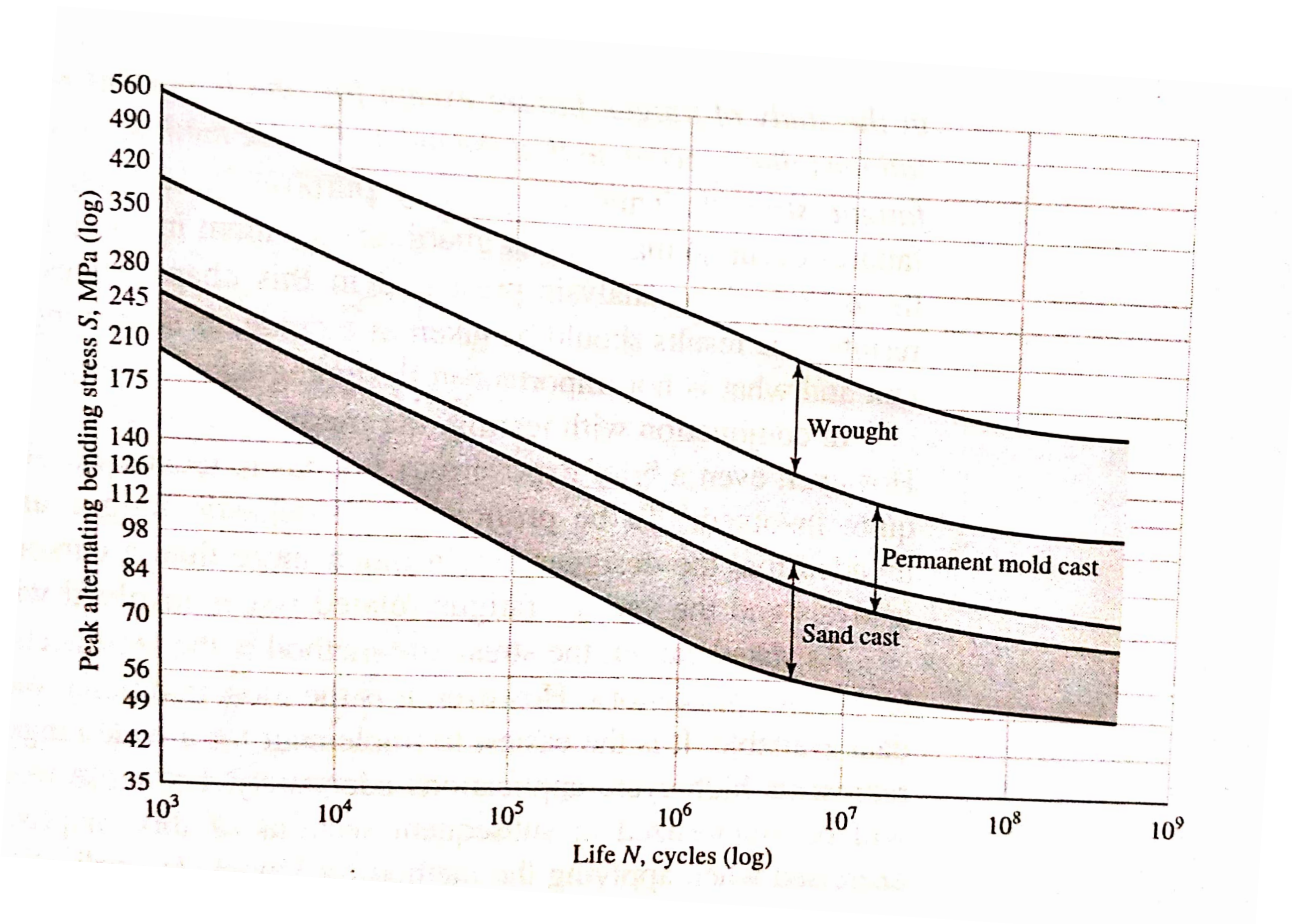


S-N Diagram

- Stress levels below Se (Endurance Strength) predict infinite life
- Between 10^3 and 10^6 cycles, finite life is predicted
- Below 10^3 cycles is known as low cycle, and is often considered quasi-static. Yielding usually occurs before fatigue in this zone.



S-N Diagram for Non-ferrous Materials

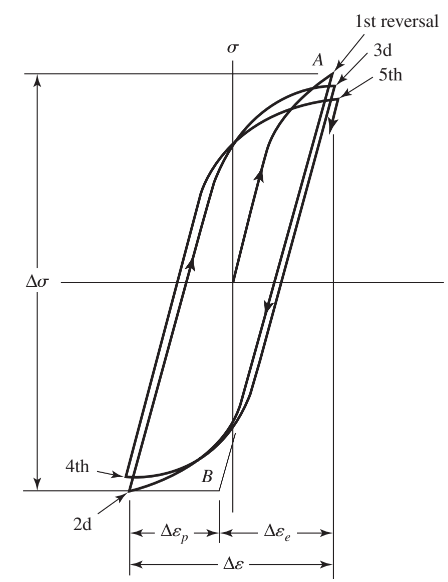


Strain-Life Method

- Strain-life method uses a detailed analysis of plastic deformation at localized regions
- Compounding of several idealizations leads to significant uncertainties in numerical results
- Useful for explaining nature of fatigue
- Not so useful in Machine Design

Figure 6–12

True stress–true strain hysteresis loops showing the first five stress reversals of a cyclic-softening material. The graph is slightly exaggerated for clarity. Note that the slope of the line AB is the modulus of elasticity E. The stress range is $\Delta\sigma$, $\Delta\varepsilon_p$ is the plastic-strain range, and $\Delta\varepsilon_e$ is the elastic strain range. The total-strain range is $\Delta\varepsilon = \Delta\varepsilon_p + \Delta\varepsilon_e$.



Strain-Life Method

- Fatigue ductility coefficient ε′F is the true strain corresponding to fracture in one reversal (point A in Fig. 6–12). The plastic-strain line begins at this point in Fig. 6–13.
- Fatigue strength coefficient σF′ is the true stress corresponding to fracture in one reversal (point A in Fig. 6–12). Note in Fig. 6–13 that the elastic-strain line begins at σ F′ / E .
- Fatigue ductility exponent c is the slope of the plastic-strain line in Fig. 6–13 and is the power to which the life 2N must be raised to be proportional to the true plastic- strain amplitude. If the number of stress reversals is 2N, then N is the number of cycles.

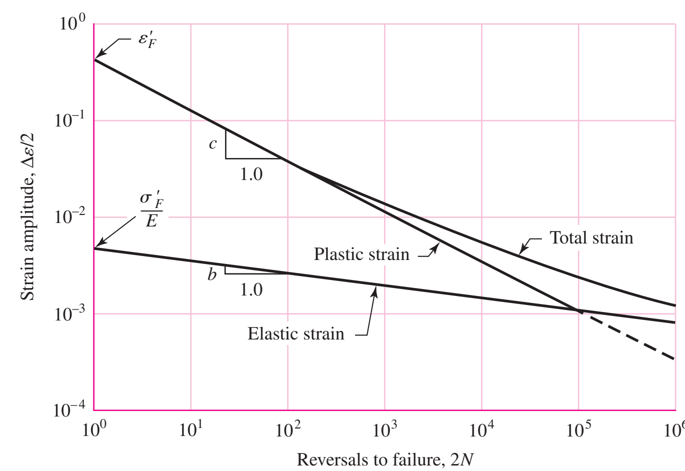


Fig 6.13

Strain-Life

The total strain is given by,

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2}$$

The equation of plastic strain line

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_f'(2N)^c$$

The equation of elastic strain line

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma_f'}{E}(2N)^b$$

$$\frac{\Delta\varepsilon}{2} = \varepsilon_f'(2N)^c + \frac{\sigma_f'}{E}\varepsilon_f'(2N)^b$$

Linear Elastic Fracture Mechanics

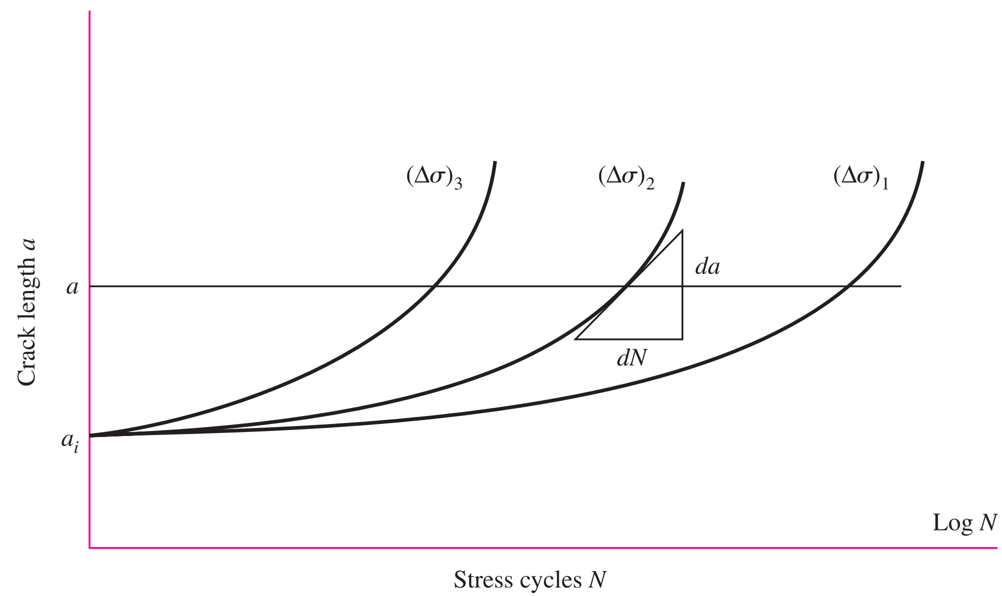
- Fatigue cracks nucleate and grow when the stresses vary. Let the stress variation be

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

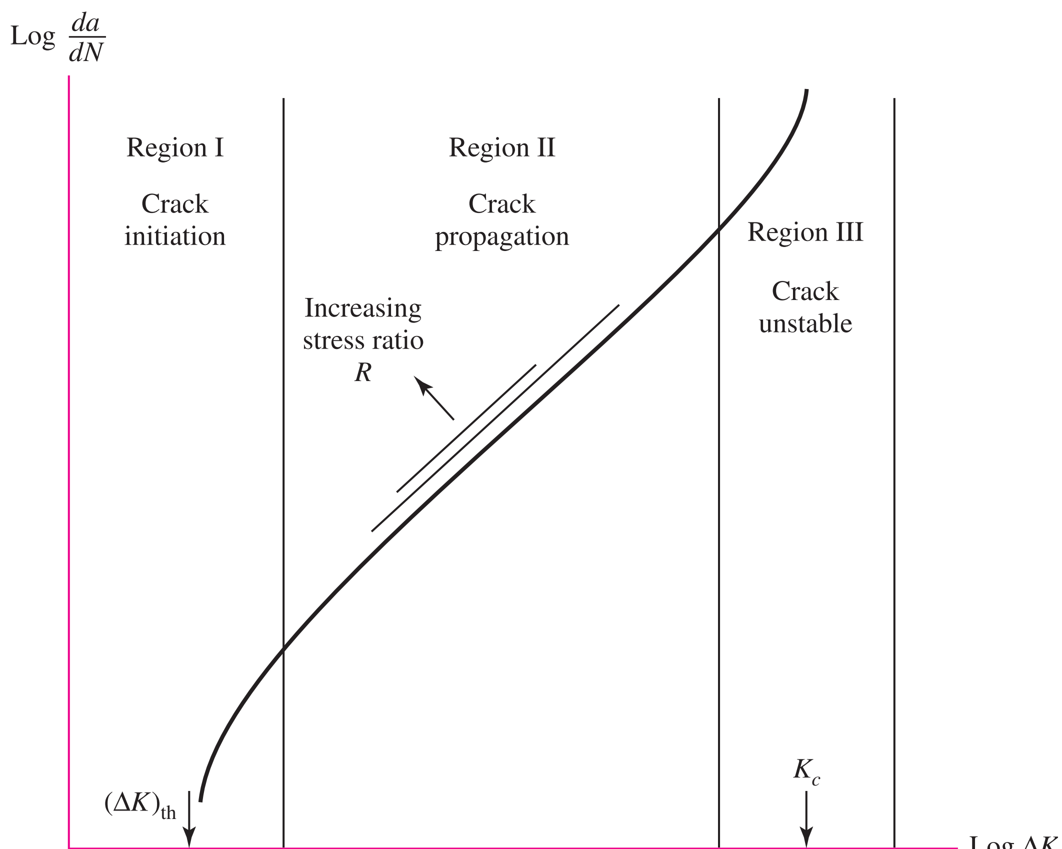
- The stress intensity change is given by,

$$\Delta K_{Ic} = \beta\Delta\sigma\sqrt{\pi a}$$

$$\Delta\sigma_1 < \Delta\sigma_2 < \Delta\sigma_3$$



Regions of Crack Growth



Crack propagation in steady state

- In the steady state region the crack growth can be estimated by,

$$\left(\frac{da}{dN}\right) = C(\Delta K_I)^m$$

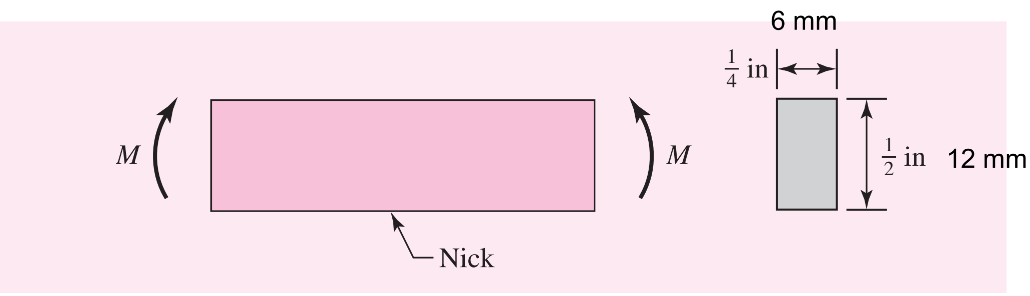
where C and m are material constants

$$\int_0^{N_f} d\,N = \frac{1}{C}\int_{a_i}^{a_f} \frac{da}{(\beta\Delta\sigma\sqrt{\pi a})^m}$$

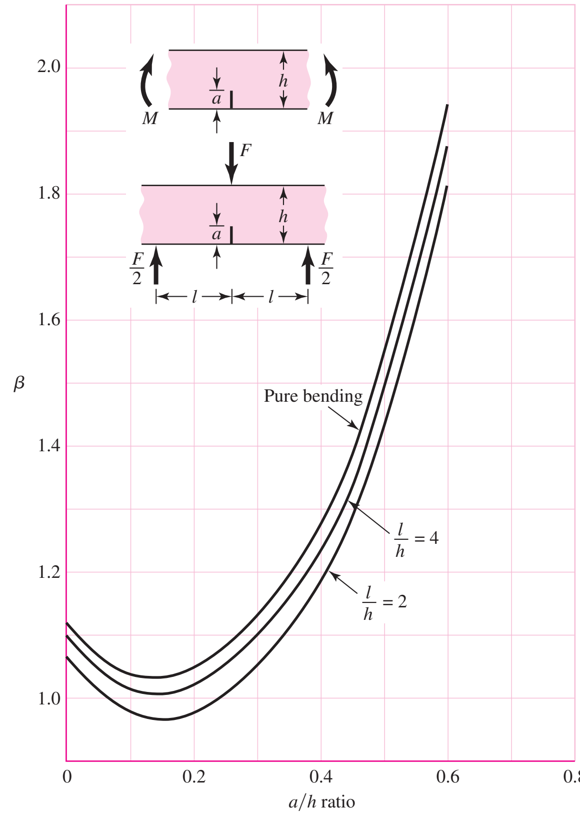
Material	$C, \frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	$C, \frac{\text{in/cycle}}{(\text{kpsi}\sqrt{\text{in}})^m}$	m
Ferritic-pearlitic steels	$6.89(10^{-12})$	$3.60(10^{-10})$	3.00
Martensitic steels	$1.36(10^{-10})$	$6.60(10^{-9})$	2.25
Austenitic stainless steels	$5.61(10^{-12})$	$3.00(10^{-10})$	3.25

Example of Crack Propagation

- The bar shown in Figure below is subjected to a repeated moment 0 ≤ M ≤ 135 Nm. The bar is AISI 4430 steel with Sut = 1.28 GPa, Sy = 1.17 GPa, and KIc = 81 Mpa$\sqrt{m}$. Material tests on various specimens of this material with identical heat treatment indicate worst-case constants of C = 114x(10^{-15}) (m/cycle)/(Mpa$\sqrt{m}$)^{m} and m = 3.0. As shown, a nick of size 0.1 mm has been discovered on the bottom of the bar. Estimate the number of cycles of life remaining.

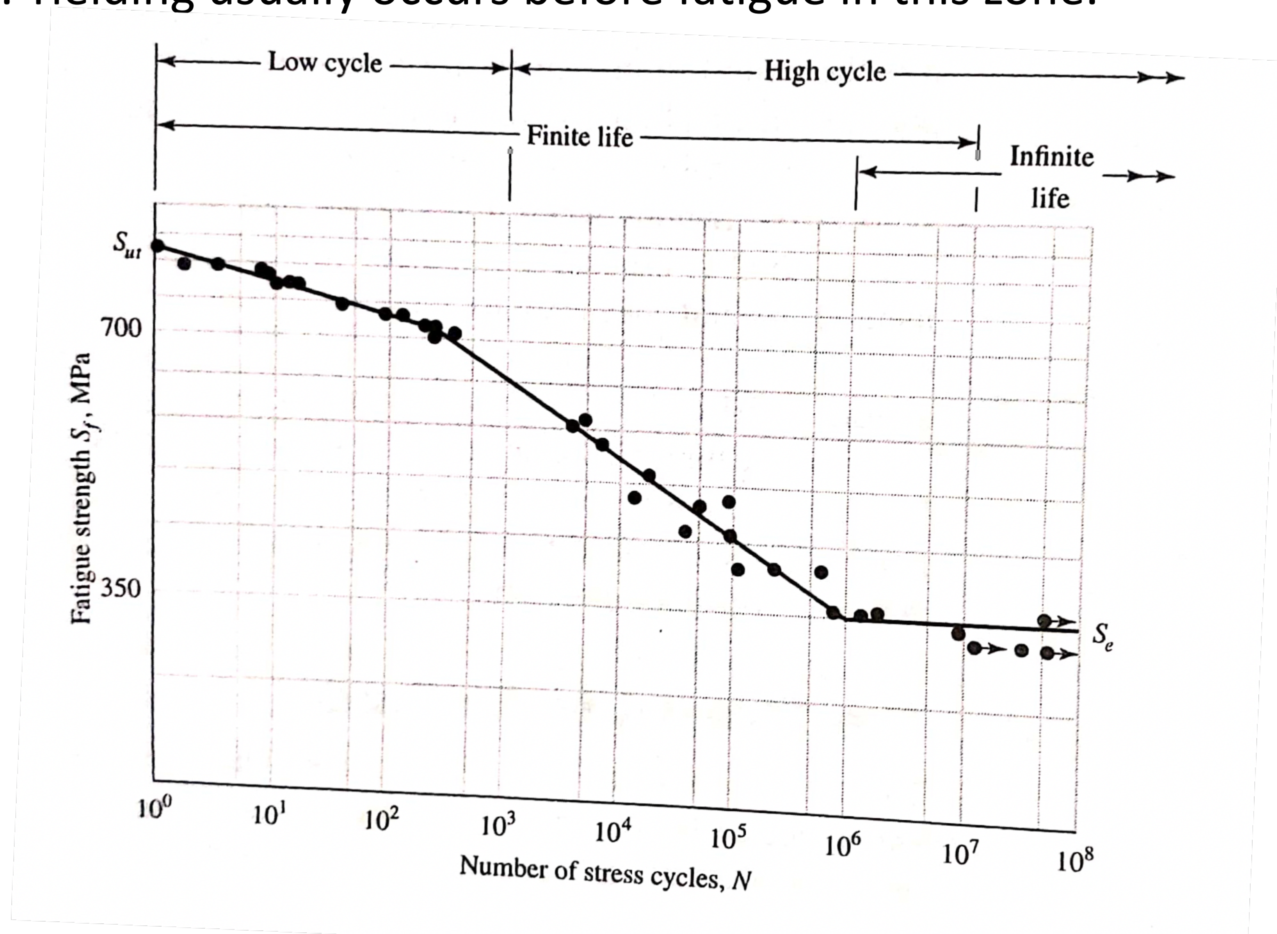


Solved in class



S-N Diagram

- Stress levels below Se (Endurance Strength) predict infinite life
- Between 10^3 and 10^6 cycles, finite life is predicted
- Below 10^3 cycles is known as low cycle, and is often considered quasi-static. Yielding usually occurs before fatigue in this zone.



Modeling of Fatigue Strength

- For design, an approximation of the idealized S-N diagram is desirable.
- To estimate the fatigue strength at 10^3 cycles, start with Eq. from Strain-Life

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma_f'}{E}(2N)^b$$

The fatigue strength at specific number of cycles is approximated as

$$\left(S_f'\right)_N = E\,\frac{\Delta\varepsilon_e}{2}$$

- The fatigue strength at a specific number of cycles can be expressed as,

$$\left(S_f'\right)_N = \sigma_f'(2N)^b$$

Modeling of Fatigue Strength

- The fatigue strength at 10^3 cycles,

$$\left(S'_f\right)_{10^3} = \sigma'_f(2\times10^3)^b = fS_{ut}$$

$$f = \frac{\sigma'_f}{S_{ut}}(2\times10^3)^b$$

- The SAE approximation for steels with HB ≤ 500 may be used or use strain hardening relationship

$$\sigma'_f = S_{ut} + 345\ MPa$$

$$\left(S'_f\right)_N = \sigma'_f(2N)^b$$

At endurance values of S'_f and N are known,

$$(S'_e) = \left(\sigma'_f\right)(2N_e)^b$$

$$b = -\frac{\log\left(\frac{\sigma'_f}{S'_e}\right)}{\log(2N_e)}$$

Fatigue Modeling

For a component the fatigue will be of the form,

$S_f = a(N)^b$

Write equation for S-N line from 10^3 to 10^6 cycles

Two known points

At N $=10^3$ cycles, Sf =f.S_{ut}

At N $=10^6$ cycles, Sf = Se

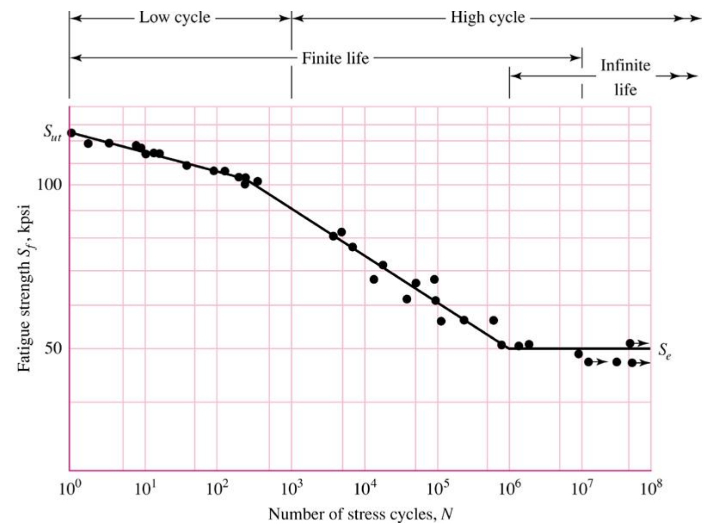
Equations for line:

$S_f = a(N)^b$

$$\mathrm{b} = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right)$$

$$a = \frac{(fS_{ut})^2}{S_e}$$

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{\frac{1}{b}}$$



Fatigue Modeling

- If a completely reversed stress σ_{rev} is given, setting Sf = σ_{rev} solving for N (cycles to failure) gives,
- $N = \left(\frac{\sigma_{rev}}{a}\right)^{\frac{1}{b}}$
- Note that the typical S-N diagram is only applicable for completely reversed stresses
- For other stress situations, a completely reversed stress with the same life expectancy must be used on the S-N diagram

Low Cycle Fatigue

- Low-cycle fatigue is defined for fatigue failures in the range $1 \le N \le 10^3$
- On the idealized S-N diagram on a log-log scale, failure is predicted by a straight line between two points (10^3, f S_{ut}) and (1, S_{ut})

$$S_f \ge S_{ut} N^{(\log f)/3} \qquad 1 \le N \le 10^3$$

Worked Example

EXAMPLE 6–2 Given a 1050 HR steel, *estimate*

(*a*) the rotating-beam endurance limit at 10^6 cycles.

(*b*) the endurance strength of a polished rotating-beam specimen corresponding to 10^4 cycles to failure

(*c*) the expected life of a polished rotating-beam specimen under a completely reversed stress of 55 kpsi.

Solution (*a*) From Table A–20, $S_{ut} = 90$ kpsi. From Eq. (6–8),

Answer

$$S'_e = 0.5(90) = 45\,\text{kpsi}$$

(*b*) From Fig. 6–18, for $S_{ut} = 90$ kpsi, $f \doteq 0.86$. From Eq. (6–14),

$$a = \frac{[0.86(90)]^2}{45} = 133.1 \text{ kpsi}$$

From Eq. (6–15),

$$b = -\frac{1}{3}\log\left[\frac{0.86(90)}{45}\right] = -0.0785$$

Thus, Eq. (6–13) is

$$S'_f = 133.1\ N^{-0.0785}$$

Answer For 10^4 cycles to failure, $S'_f = 133.1(10^4)^{-0.0785} = 64.6$ kpsi

(*c*) From Eq. (6–16), with $\sigma_{\text{rev}} = 55$ kpsi,

Answer

$$N = \left(\frac{55}{133.1}\right)^{1/-0.0785} = 77\,500 = 7.75(10^4) \text{ cycles}$$

Keep in mind that these are only *estimates.* So expressing the answers using three-place accuracy is a little misleading.

Modifiers

Experimental results are used to obtain modifiers

$$S_e = \left(k_a\ k_b\ k_c\ k_d\ k_e\ k_f\right) S'_e$$

Where:

- k_a = Surface condition modification factor
- k_b = Size modification factor
- k_c = Load modification factor
- k_d = Temperature modification factor
- k_e = Reliability modification factor
- k_f = Others…

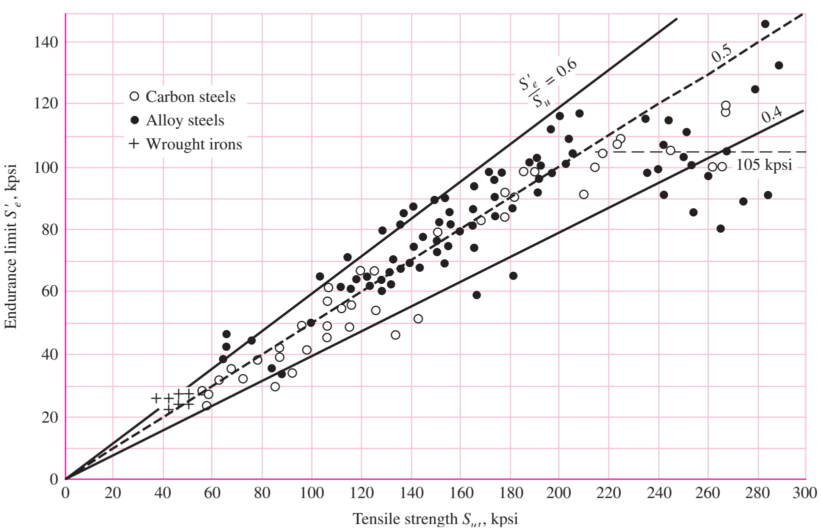
- S'_e = Rotary-beam test endurance limit
- S_e = Predicted endurance limit for your part

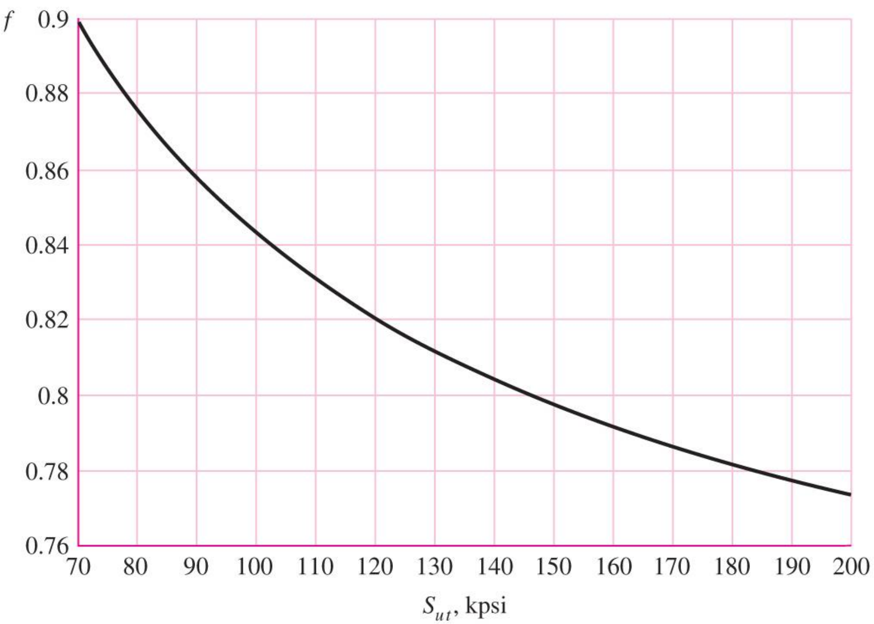
Determining fraction f

Use f for different S_{ut} from the plot

$S'_f = f\, S_{ut}$ at 10^3 cycles

$S_e = S'_e = 0.5 S_{ut}$ at 10^6 cycles



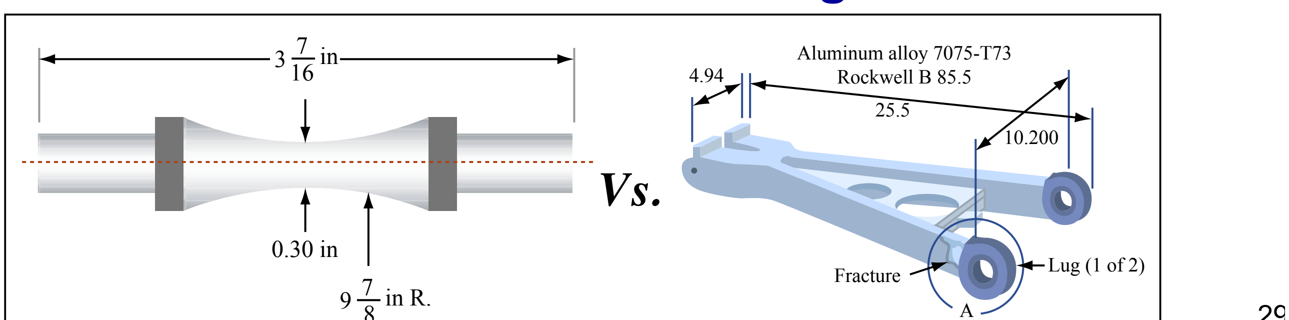


Estimate for Endurance

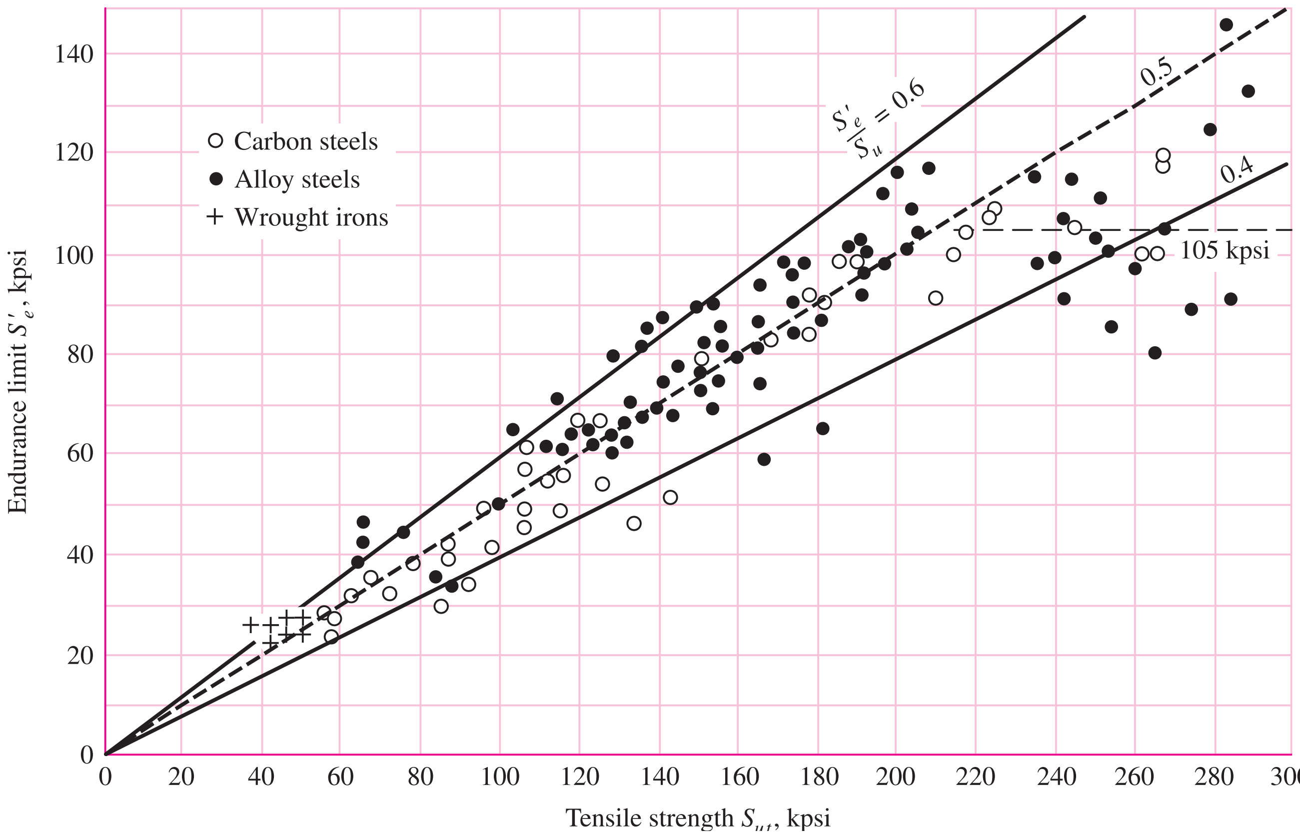
For ferrous materials, the following approximations may be used for first pass design

$$S'_e = \begin{cases} 0.5\, S_{ut} & S_{ut} \le 200 kpsi \\ 100 kpsi & S_{ut} > 200 kpsi \\ 700 MPa & S_{ut} > 1400 MPa \end{cases}$$

This is for ideal conditions… but designs are never ideal



Plots



ME 423: Machine Design
Instructor: Ramesh Singh

Surface Modifier

Experimental results are used to obtain modifiers

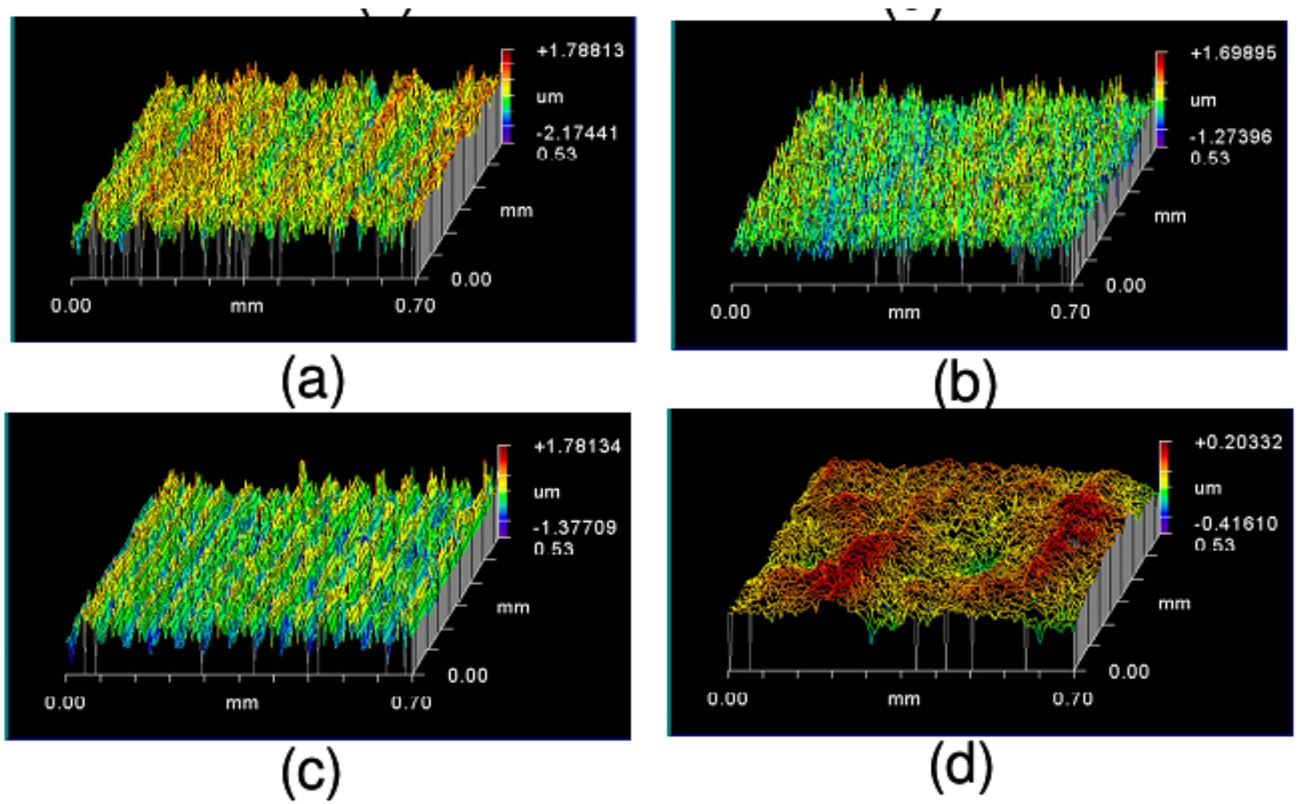
$$k_a = a\, S^{b}_{ut}$$

Where:

- a = function of fabrication process
- b = function of fabrication process
  - *Why does finish matter?*

Surface Finish	Factor a S_{ut}, kpsi	S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

Cylindrical Bearing Surface



(a) GD (ground), (b) HN (honed), (c) HT (hard turned) and (d) IF (isotropic finished) surfaces

Example of the variation of k_a

- For a steel with a S_{ut} of 400 Mpa k_a for different processes are as follows:
  - Grinding: 0.95
  - Machining: 0.92
  - Hot Rolled: 0.78
  - As Forged: 0.70

Size Factor, k_b

- Larger diameter parts have high stress level in bending and torsion as $\varepsilon = -k\,y;\ \gamma = \frac{\rho\phi}{L}$
- Larger areas are subjected to higher stresses so likelihood of crack initiation is higher
- Size factor is obtained from experimental data with wide scatter
- For bending and torsion loads, the trend of the size factor data is given by

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases}$$

- Applies only for round, rotating diameter
- For axial load, there is no size effect, so $k_b = 1$

Load factor, k_c

- Accounts for changes in endurance limit for different types of fatigue loading
- Only to be used for single load types. Use Combination Loading method (Sec. 6–14, Shigley) when more than one load type is present

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases}$$

Temperature Factor, k_d

- At lower temperature chances of brittle facture is high whereas at higher temperature
- This relation is summarized in Table 6–4 in Shigley's book

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

*Data source: Fig. 2–9.

Reliability Factor, k_e

- From Fig. 6–17, $S'_e = 0.5\ S_{ut}$ is typical of the data and represents 50% reliability.
- Reliability factor can help develop a conservative estimate of endurance limit

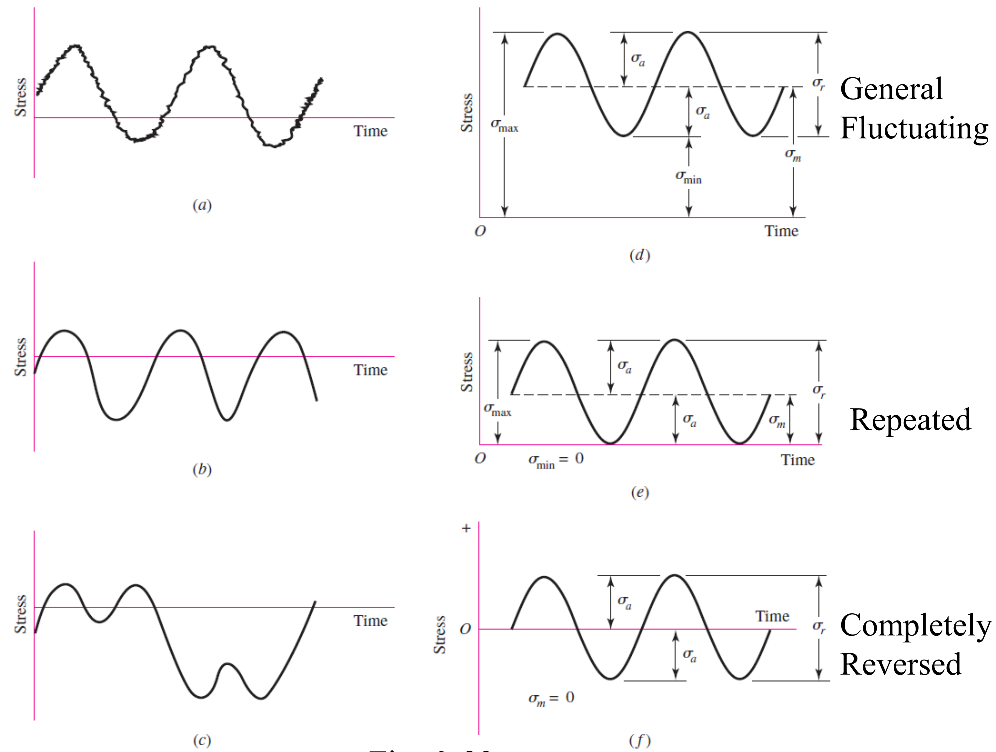
Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Miscellaneous Effects

- Reminder to consider other possible factors
  - Residual stresses
  - Directional characteristics from cold working
  - Case hardening
  - Corrosion
  - Surface conditioning, e.g. electrolytic plating and metal spraying
  - Cyclic Frequency
  - Frettage Corrosion
- Limited data is available.
- May require research or testing.

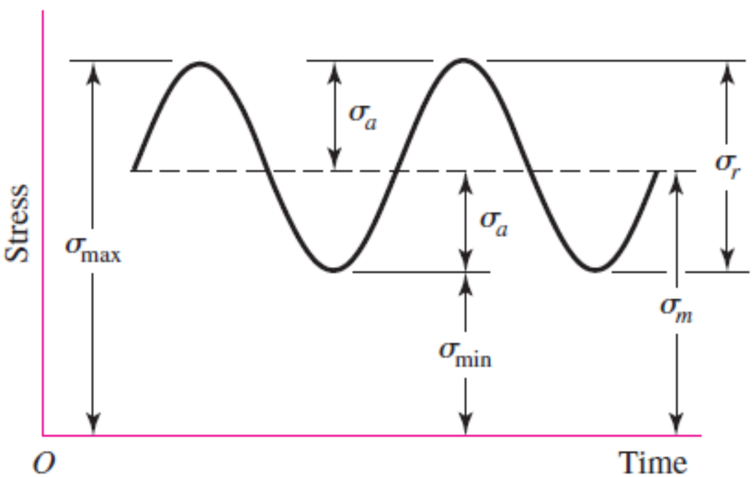
Characterizing Fluctuating Stresses

- All the previous analysis for fatigue life has been presented for complete stress reversal just as in the R-R Moore Tests where the stresses reverse from compression to tension
- However, there can be fluctuating stresses as well
  - Sinusoidal or Non-sinusoidal



Characterizing Fluctuating Stresses

- Important Stress Definitions



σ_{min}	Minimum stress
σ_{max}	Maximum stress
σ_a	Amplitude component = $(\sigma_{max} - \sigma_{min})/2$
σ_m	Midrange component = $(\sigma_{max} + \sigma_{min})/2$
σ_s	Steady component
R	Stress ratio = $\sigma_{min} / \sigma_{max}$
A	Amplitude ratio = σ_a / σ_m

Stress Intensity Factor K_f for Fluctuating Stresses

- For fluctuating loads at points with stress concentration, the best approach is to design to avoid all localized plastic strain
- K_f should be applied to both alternating and midrange stress components
- When localized strain does occur, some methods (e.g. nominal mean stress method and residual stress method) recommend only applying K_f to the alternating stress
- The Dowling method recommends applying K_f to the alternating stress and K_{fm} to the mid-range stress, where K_{fm} is

$$K_{fm} = K_f \qquad K_f|\sigma_{\max,o}| < S_y$$

$$K_{fm} = \frac{S_y - K_f\sigma_{ao}}{|\sigma_{mo}|} \qquad K_f|\sigma_{\max,o}| > S_y$$

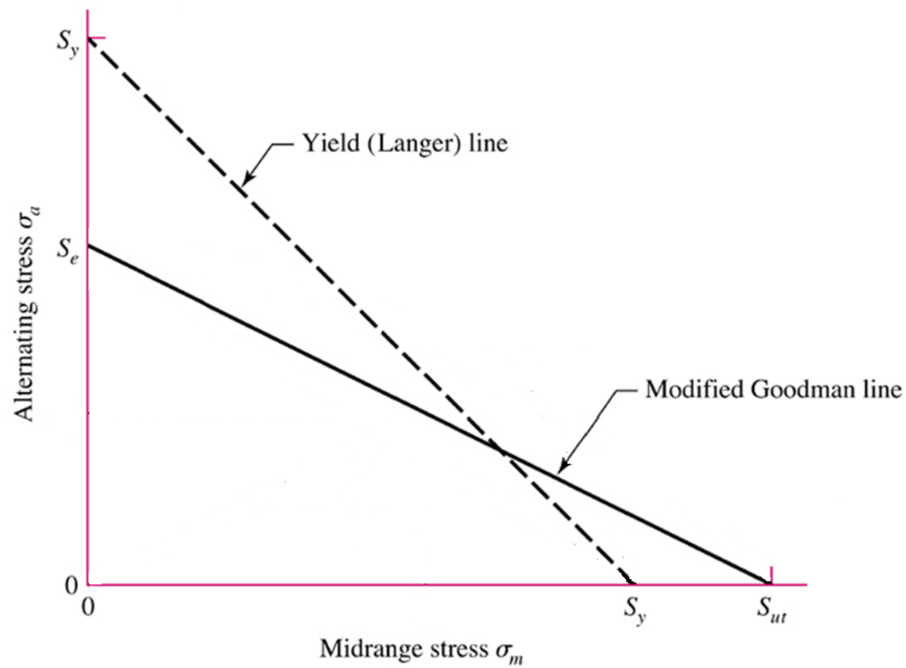
$$K_{fm} = 0 \qquad K_f|\sigma_{\max,o} - \sigma_{\min,o}| > 2S_y$$

Methods of plotting data

- Vary the σ_m and σ_a to learn about the fatigue resistance under fluctuating loading
- Three common methods of plotting results are as follows:
  - Plotting mid-range stress on the abscissa and other stress components on ordinate
  - The abscissa represents the ratio of the midrange strength S_m to the ultimate strength, with tension plotted to the right and compression to the left. The ordinate is the ratio of the alternating strength to the endurance limit
  - Another way is to four of the stress components as well as the two stress ratios

Plot of Alternating vs Midrange Stress

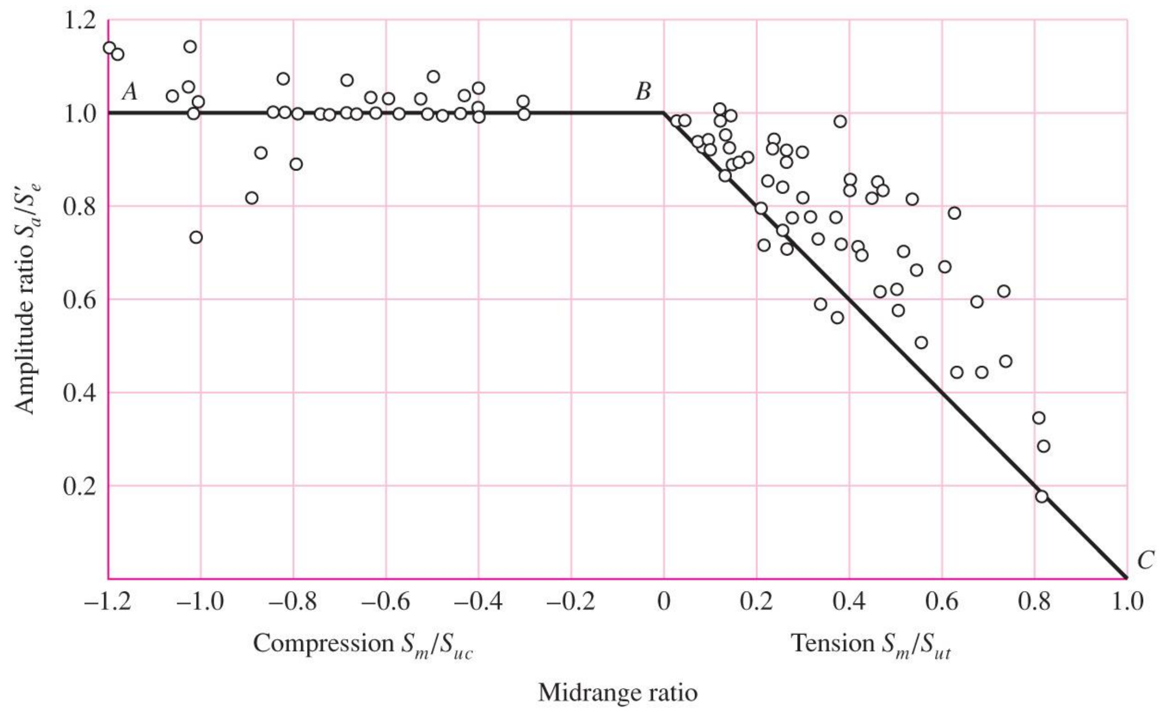
- Most common and simple to use is the plot of σ_a vs σ_m
- Known as Goodman or Modified Goodman diagram
- Modified Goodman line from S_e to S_{ut} is one simple representation of the limiting boundary for infinite life



Experimental Plot of Normalized Alternating vs Midrange Stress

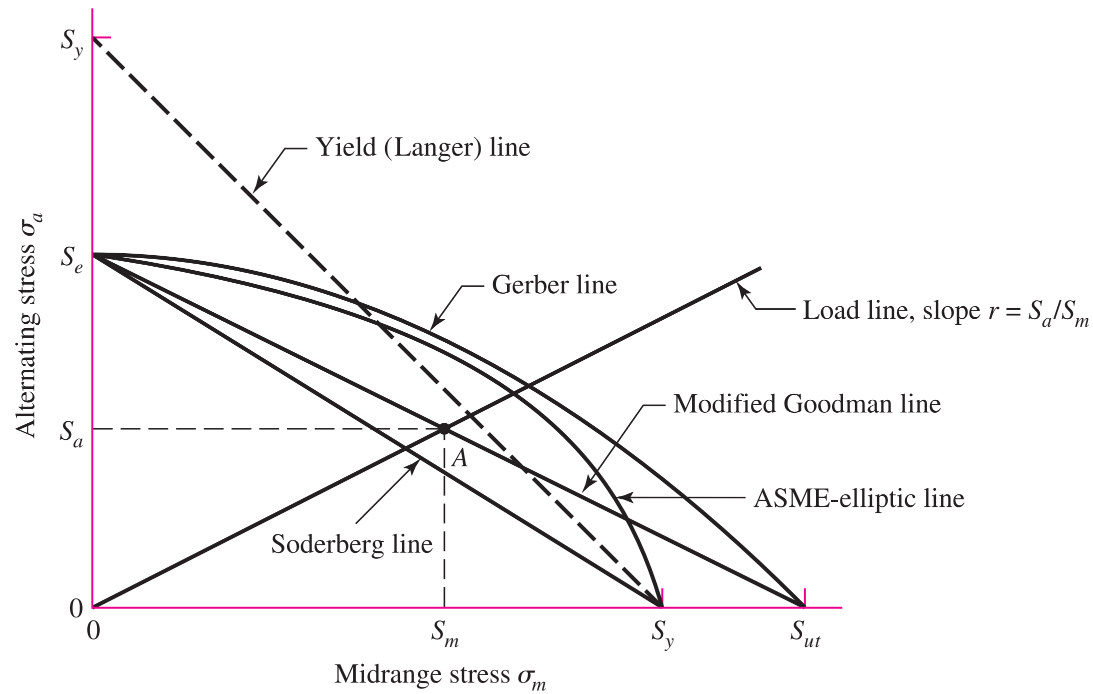
- The abscissa represents the ratio of the midrange strength S_m to the ultimate strength, with tension plotted to the right and compression to the left
- The ordinate is the ratio of the alternating strength to the endurance limit

Demonstrates little effect of negative midrange stress and BC represents modified Goodman



Commonly Used Failure Criteria

- Five commonly used failure criteria are shown
- Gerber passes through the data (best Fit)
- ASME-elliptic passes through data and incorporates rough yielding check



Intersection Points with Gerber and ASME Ellipitical with Langer

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{r S_{ut}}\right)^2}\right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2S_e}\left[1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2\left(1 - \frac{S_y}{S_e}\right)}\right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{\text{crit}} = S_a/S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

Intersecting Equations	Intersection Coordinates
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$
Load line $r = S_a/S_m$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = S_a/S_m$	$S_m = \frac{S_y}{1 + r}$
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = 0,\ \frac{2S_y S_e^2}{S_e^2 + S_y^2}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_m = S_y - S_a,\ r_{\text{crit}} = S_a/S_m$

Fatigue factor of safety

$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

Commonly Used Failure Criteria

- Modified Goodman is linear, so simple to use for design. It is more conservative than Gerber
- Soderberg provides a very conservative single check of both fatigue and yielding
- Langer line represents standard yield check and it is equivalent to comparing maximum stress to yield strength.

Commonly Used Failure Criteria

- A fatigue criterion, rather than being a "fence," is more like a zone or band wherein the probability of failure could be estimated.
- Failure criterion of Goodman has certain advantages
  - It is a straight line and the algebra is linear and easy
  - It is easily graphed, every time for every problem
  - It reveals subtleties of insight into fatigue problems
  - Answers can be scaled from the diagrams as a check on the algebra
- Note that this model is deterministic but the phenomenon is not deterministic

Commonly Used Failure Criteria

- Intersecting a constant slope load line with each failure criteria produces design equations
- n is the design factor or factor of safety for infinite fatigue life

$$\textbf{Soderberg} \quad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

$$\textbf{mod-Goodman} \quad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

$$\textbf{Gerber} \quad \frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$\textbf{ASME-elliptic} \quad \left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$$

Factor of Safety

- Modified Goodman and Langer Failure criteria

$$n_f = \frac{1}{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}}$$

- Gerber and Langer Failure criteria

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

- ASME Elliptic and Langer Failure criteria

$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

Problem: Solved in class

- A 40 mm-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 70 kN. Because of the ends, and the fillet radius, a fatigue stress-concentration factor Kf is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (a) the Gerber fatigue line and (b) the ASME-elliptic fatigue line.

Solution Steps

- Find S_y and S_{ut} from data
- Find the modifiers for the endurance limit
- Determine the stress components and correct them for stress concentration K_f
- Determine the factor of Safety for Gerber line and Yield line
- Find Sa (intersection with load line r= σ_a/σ_m occurring at B). Also find Sm. You can check the factor of safety
- Sm (intersection with yield line occurring at D) for Gerber Line. Find Sa
- Find the critical slope.

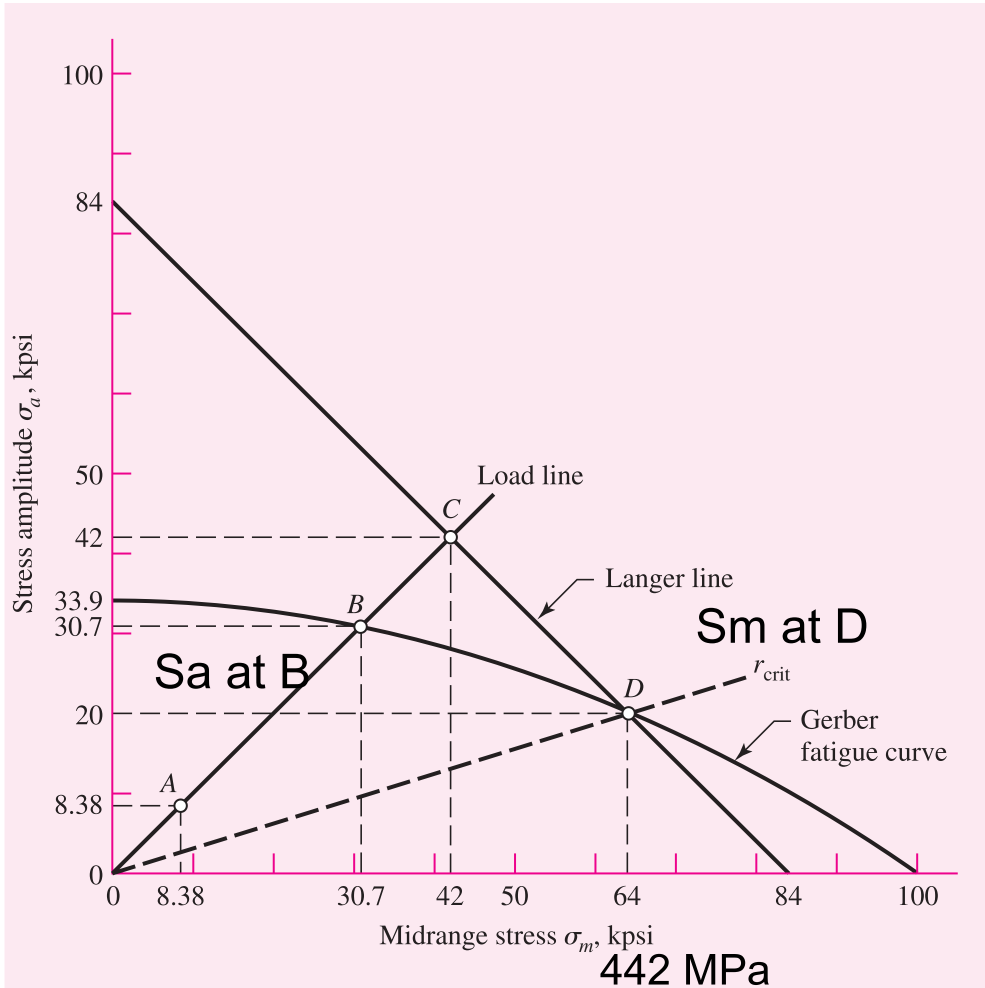
Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

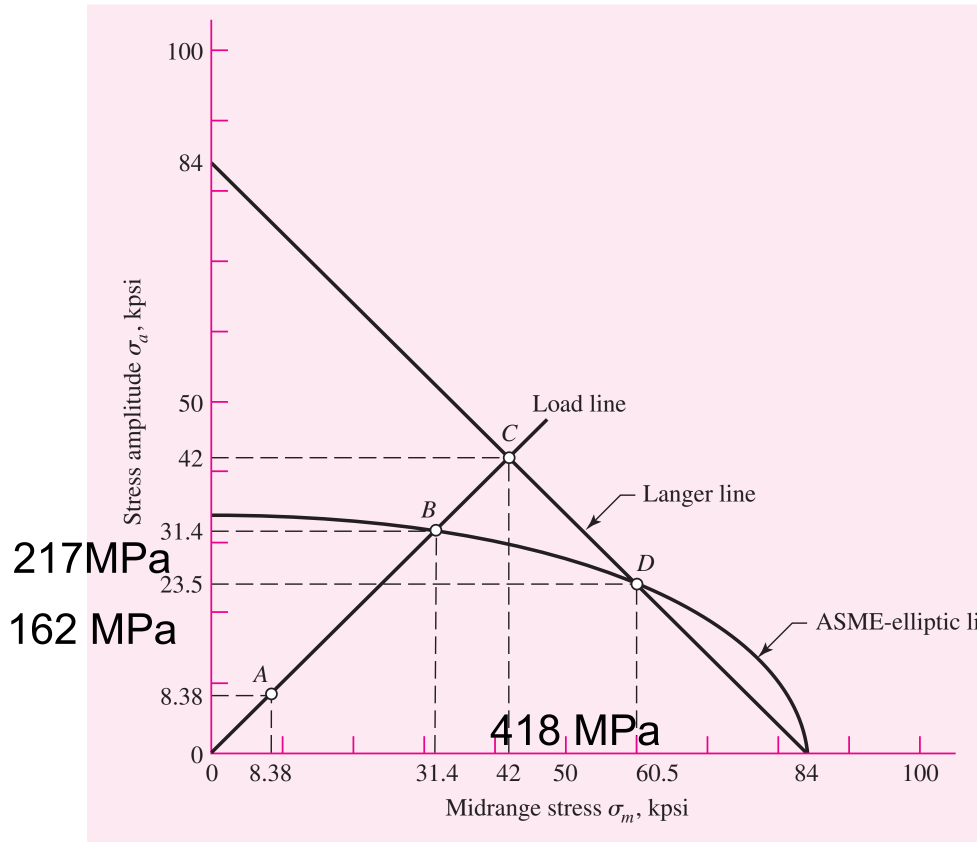
Gerber Plot

211.9 MPa



442 MPa

ASME Elliptic Plot



ME 423: Machine Design
Instructor: Ramesh Singh

Problem

A steel bar undergoes cyclic loading such that $\sigma max = 420$ MPa and $\sigma min = -140$ Mpa. For the material, Sut = 560 MPa, Sy = 455 MPa, a fully corrected endurance limit of Se = 280 MPa, and f = 0.9. Estimate the number of cycles to a fatigue failure using:

(a) Modified Goodman criterion.

(b) Gerber criterion.

Solved in Class