ME 423
Instructor: Ramesh Singh
HW#2

Assigned: September 10, 2023
Due: September 21, 2023

1. The composite properties are a combination of those of the components. The ***rule of mixtures*** predicts that there exists an upper limit of the elastic modulus of the composite given in terms of the elastic moduli of the matrix (E_m) and the particulate (E_p) phases. Please list all the assumptions and derive the value of elastic modulus of the composite (E_c) for the upper limit as given in the following relationship.

   $$E_c = E_m V_m + E_p V_p$$

   where V_m and V_p are the volume fraction of the two phases.

   Also prove that the lower bound of the elastic modulus is given by:

   $$E_c = \frac{E_m E_p}{E_p V_m + E_m V_p}$$

2. Derive the material indices that give highest strength and stiffness at the lowest mass for tie rods and beams.

3. You have to design a mirror for a large telescope. the mirror is a circular disc, of diameter *2a* and mean thickness ***t*****,** simply supported at its periphery (Figure 1).

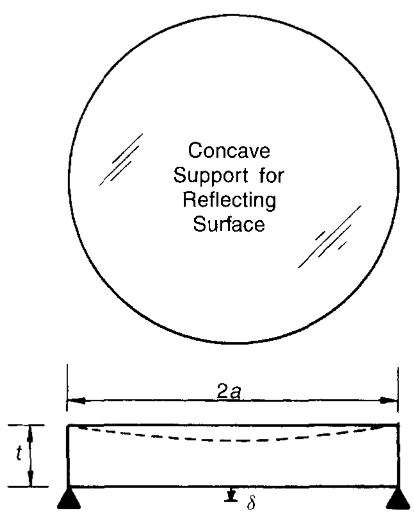

Figure 1. The telescope geometry

When horizontal, it will deflect under its own weight *m;* when vertical it will not deflect significantly. This distortion (which changes the focal length and introduces aberrations into the mirror) must be small enough that it does not interfere with performance; in practice, this means that the deflection δ of the midpoint of the mirror must be less than the wavelength of light. Additional requirements are: high dimensional stability (no creep), and low thermal expansion. ρ is the density of the material of the disc. The elastic deflection, δ, of the centre of a horizontal disc due to its own weight for a material with Poisson's ratio of 0.3 can be looked from simply supported disc deflection solutions.

The objective is to minimize the mass. The constraints are as follows:

- Radius *a* specified
- Must not distort more than δ under its own weight
- High dimensional stability: no creep, no moisture take-up, low thermal expansion

We can treat the remaining constraints as property limits, requiring a melting point greater than 1000K *to* avoid creep, zero moisture take **up,** and a low thermal expansion coefficient *(*$\alpha < 20 \times 10^{-6}$/K).

Identify the material index for this condition and make a table with following columns: 1. Material; 2. Material index; mass in tons given the radius is 6m and provide your comments and identify some right candidates from the appropriate property chart. Provide a rationale behind the recommendation.

4. You have to design an elastic spring. The constraints are: No failure ($\sigma < \sigma_f$ at every location in the spring). The objective functions are: (a) maximize the stored elastic energy per unit volume and (b) maximize the stored elastic energy per unit weight. The only free variable is the choice of material.

   Identify the material indices separately for both these objectives and make two tables (one for each objective) with following columns: 1. Material; 2. Material index and identify some right candidates from the appropriate property chart and provide a rationale behind the selection.

5. Derive the closed form solution for crack propagation from a_i to a_f after N_f cycles based on fracture toughness and Paris Law.

6. A steel rotating-beam test specimen has an ultimate strength of 1600 MPa. Estimate the life of the specimen if it is tested at a completely reversed stress amplitude of 900 MPa.

7. A solid round bar with diameter of 50 mm has a groove cut to a diameter of 45 mm, with a radius of 2.5 mm. The bar is not rotating. The bar is loaded with a repeated bending load that causes the bending moment at the groove to fluctuate between 0 and 2825 Nm. The bar is hot-rolled AISI 1095, but the groove has been machined. Determine the factor of safety for fatigue based on infinite life and the factor of safety for yielding (a) Using Modified Goodman; (b) ASME Elliptic.

8. The cold-drawn AISI 1040 steel bar shown in Figure 1 is subjected to a completely reversed axial load fluctuating between 28 kN in compression to 28 kN in tension. Estimate the fatigue factor of safety based on achieving infinite life, and the yielding factor of safety. If

infinite life is not predicted, estimate the number of cycles to failure.

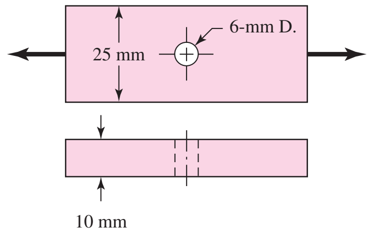

Fig. 1. Cold drawn steel bar under reversed axial loading

9. In Figure 2, shaft A, made of AISI 1020 hot-rolled steel, is welded to a fixed support and is subjected to loading by equal and opposite forces F via shaft B. A theoretical stress-concentration factor K_{ts} of 1.6 is induced by the 3 mm fillet. The length of shaft A from the fixed support to the connection at shaft B is 1 m. The load F cycles from 0.5 to 2 kN. Find the factor of safety for infinite life using the modified Goodman and Gerber fatigue failure criteria.