Mechanics Review

- Concept of stress and strain, True and engineering
- Stresses in 2D/3D, Mohr's circle (stress and strain) for 2D/3D
- Elements of Plasticity
- Material Models
- Yielding criteria, Tresca and Von Mises
- Invariants of stress and strain
- Levy-Mises equations

Outline

- Stress and strain
  - Engineering stresses/strain
  - True stresses/strain
- Stress tensors and strain tensors
  - Stresses/strains in 3D
  - Plane stress and plane strain
- Principal stresses
- Mohr’s circle in 2D/3D

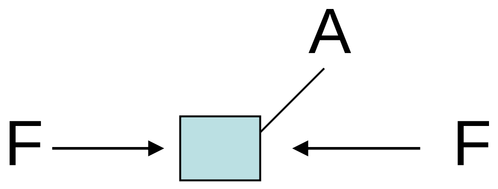
#1 - Match

a) Force equilibrium
b) Compatibility of deformation
c) Constitutive equation

1) $\delta_T = \delta_1 + \delta_2$
2) $\Sigma F=0; \Sigma M=0$
3) $\sigma = E\ e$

Steps of a Mechanics Problem

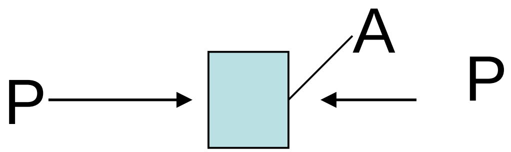
- O Read and understand problem
- 1 Free body diagram



- 2 Equilibrium of forces
  - e.g. $\Sigma F=0$, $\Sigma M=0$.
- 3 Compatibility of deformations
  - e.g. $\delta_T = \delta_1 + \delta_2$
- 4 Constitutive equations
  - e.g. $\sigma = E\, e$
- 5 Solve

Key Concepts

Load (Force), P, acting over area, A, gives rise to stress, σ.



Engineering stress: $\sigma = P/A_o$

(A_o = original area)

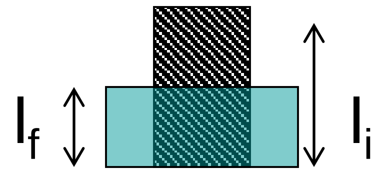
True stress: $\sigma_t = P/A$

(A = actual area)

Deformation

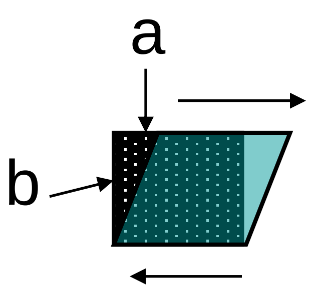
- Quantified by strain, e or ε

- Engineering strain: $e = (l_f - l_i)/l_i$



- True Strain: $\varepsilon = \ln(l_f/l_i)$

- Shear strain: $\gamma = a/b$



True and Engineering strains

$e = (l_f - l_i)/l_i$

$e = (l_f/l_i) - 1$

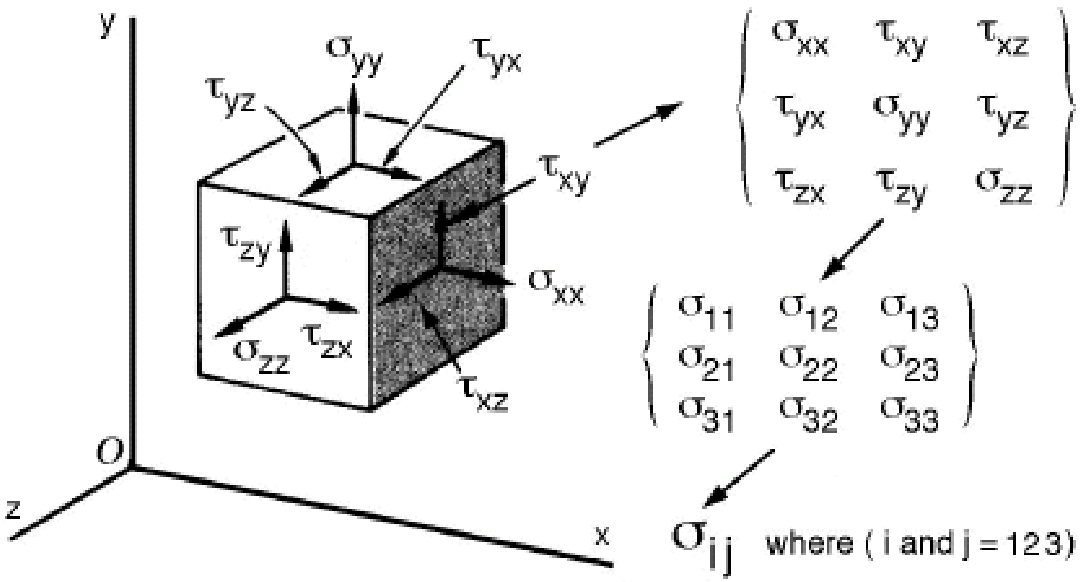
$(l_f/l_i) = e + 1$

$\ln(l_f/l_i) = \ln(e + 1)$

$\varepsilon = \ln(e + 1)$

3-D Stress State in Cartesian Plane

Courtesy: http://www.jwave.vt.edu/crcd/kriz/lectures/Anisotropy.html



There are two subscripts in any stress component:

Direction of normal vector of the plane (first subscript)

σ_{xx} and τ_{xy}

Direction of action (second subscript)

Stress Tensor

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

Mechanics Notation

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Expanded Tensorial Notation

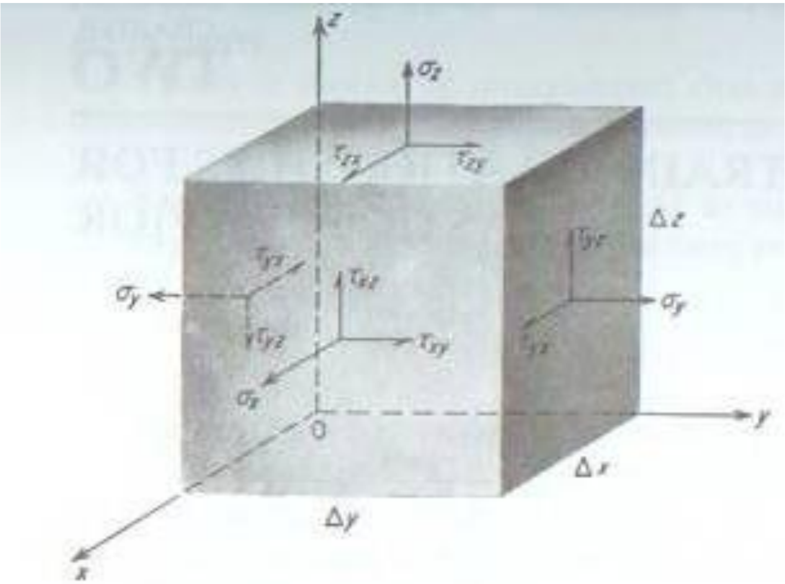
It can also be written as σ_{ij} in condensed form and is a second order tensor, where i and j are indices

The number of components to specify a tensor

- 3^n , where n is the order of matrix

Symmetry in Shear Stress

- Ideally, there has to be nine components
- For small faces with no change in stresses
- Moment about z-axis,



$$(\tau_{xy}\Delta y\Delta z)\Delta x = (\tau_{yx}\Delta x\Delta z)\Delta y$$

$$\tau_{xy} = \tau_{yx}$$

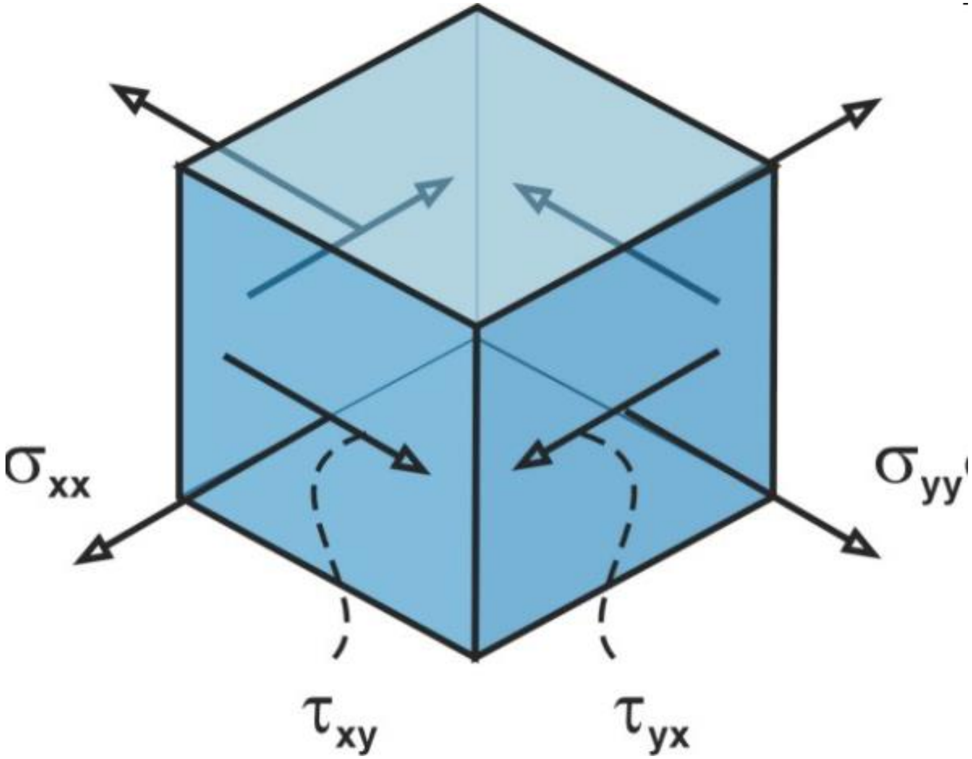
$$Similarly,$$

$$\tau_{yz} = \tau_{zy}$$

$$\tau_{zx} = \tau_{xz}$$

- Tensor becomes symmetric and have only six components, 3 normal and 3 shear stresses

Plane Stress



$$\sigma_{zz} = 0$$
$$\tau_{xz} = \tau_{yz} = 0$$

Only three components of stress

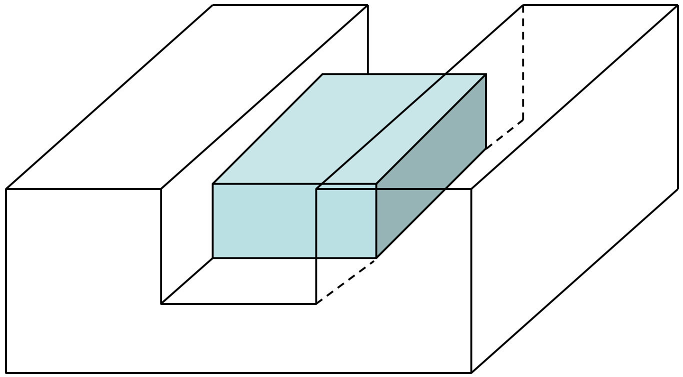
http://www.shodor.org/~jingersoll/weave4/tutorial/Figures/sc.jpg

Plane Strain (1)

One pair of faces has NO strain

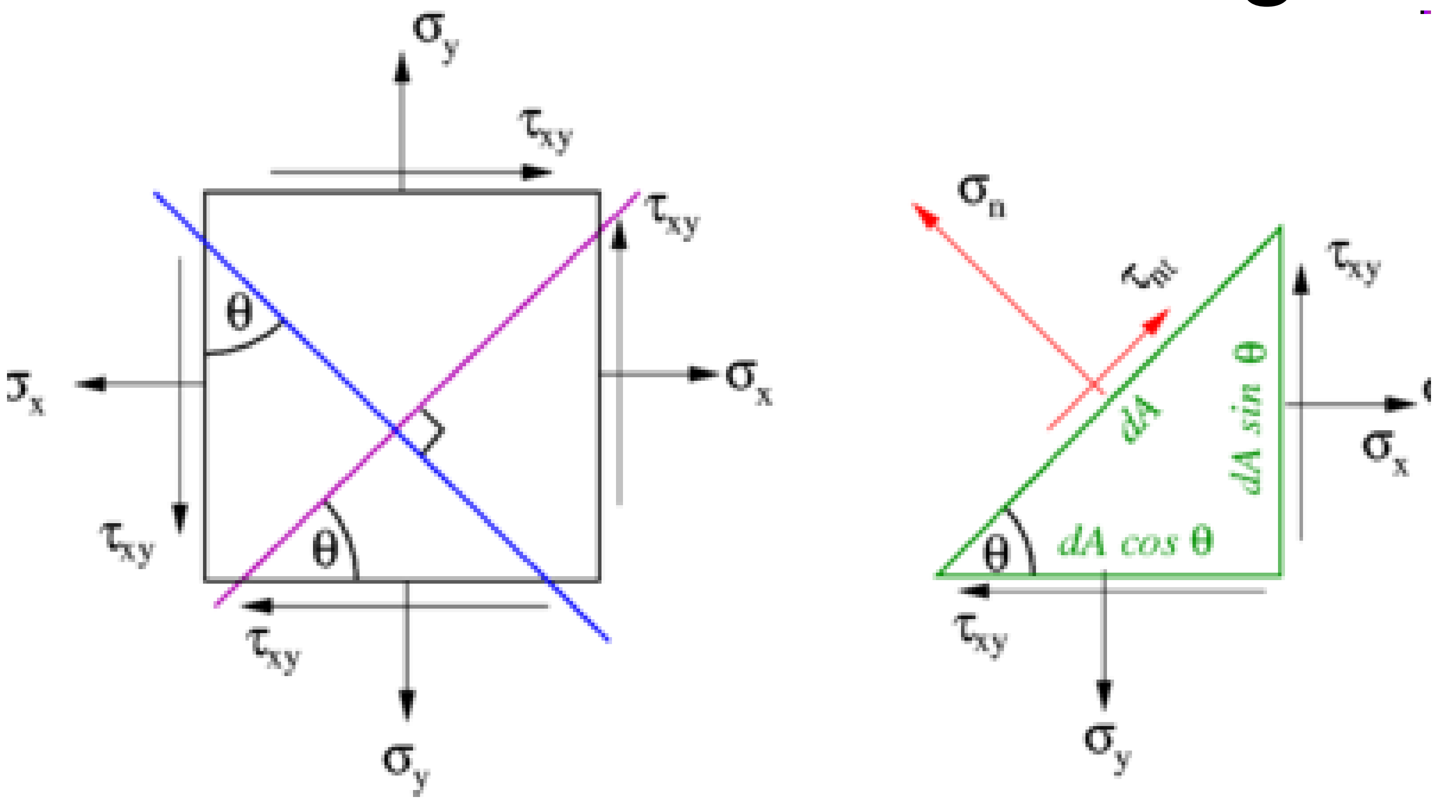
– each cross-section has the same strain

$$\varepsilon_{zz} = 0$$
$$\tau_{xz} = \tau_{yz} = 0$$



Material in a groove

2-D Stresses at an Angle

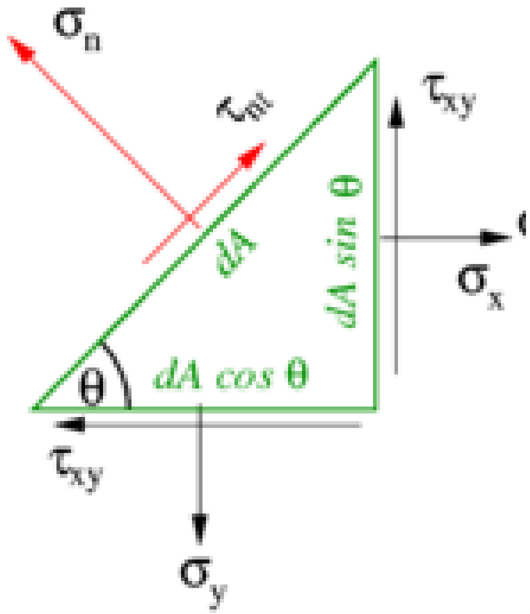


$$\sum f_n = \sigma_n\, dA - (\sigma_y\, dA\, \cos\theta)\, \cos\theta + (\tau_{xy}\, dA\, \cos\theta)\, \sin\theta - (\sigma_x\, dA\, \sin\theta)\, \sin\theta + (\tau_{xy}\, dA\, \sin\theta)\, \cos\theta = 0$$

$$\sigma_n = \sigma_x\ \sin^2\theta + \sigma_y\ \cos^2\theta - 2\ \tau_{xy}\ \sin\theta\cos\theta$$

$$\sigma_n = \frac{1}{2}\ (\sigma_x + \sigma_y) - \frac{1}{2}\ (\sigma_x - \sigma_y)\ \cos 2\theta - \tau_{xy}\ \sin 2\theta$$

Shear Stress on Inclined Plane

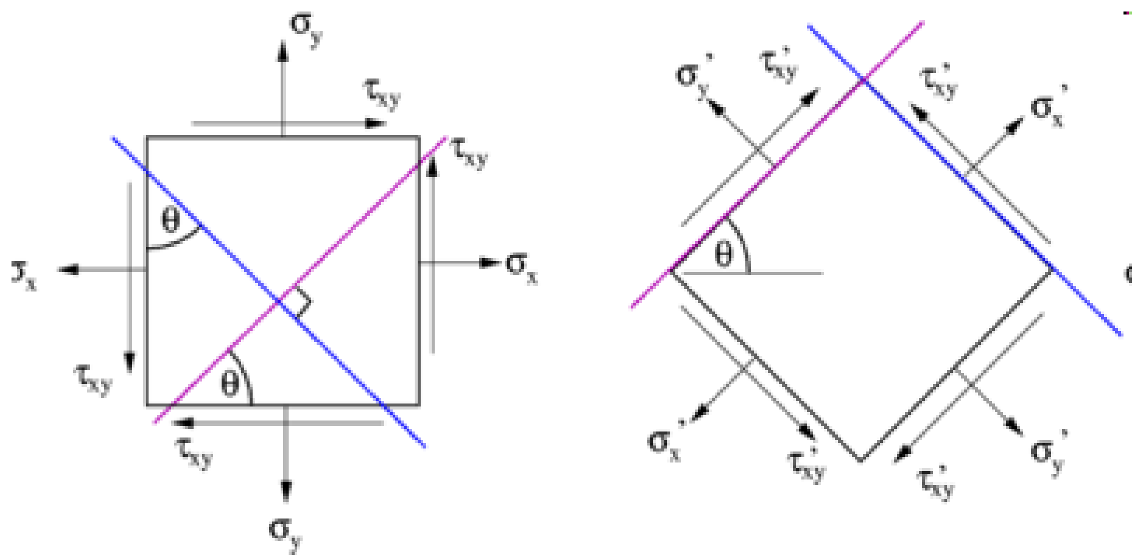


$$\sum f_t = \tau_{nt}\, dA - (\sigma_y\, dA\, \cos\theta)\, \sin\theta - (\tau_{xy}\, dA\, \cos\theta)\, \cos\theta + (\sigma_x\, dA\, \sin\theta)\, \cos\theta + (\tau_{xy}\, dA\, \sin\theta)\, \sin\theta = 0$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\, \sin\theta\cos\theta + \tau_x y\, (\cos^2\theta - \sin^2\theta)$$

$$\tau_{nt} = -\frac{1}{2}(\sigma_x - \sigma_y)\, \sin 2\theta + \tau_{xy}\, \cos 2\theta$$

Transformation Equations



Use θ and 90 +θ for x and y directions, respectively

$$\sigma_x' = \frac{1}{2}\,(\sigma_x + \sigma_y) + \frac{1}{2}\,(\sigma_x - \sigma_y)\,\cos 2\theta + \tau_{xy}\,\sin 2\theta$$

$$\sigma_y' = \frac{1}{2}\,(\sigma_x + \sigma_y) - \frac{1}{2}\,(\sigma_x - \sigma_y)\,\cos 2\theta - \tau_{xy}\,\sin 2\theta$$

$$\tau_{xy}' = -\frac{1}{2}\,(\sigma_x - \sigma_y)\,\sin 2\theta + \tau_{xy}\,\cos 2\theta$$

Principal Stresses

For principal stresses $\tau_{xy}=0$

$$-\frac{1}{2}\left(\sigma_x-\sigma_y\right)\sin 2\theta+\tau_{xy}\cos 2\theta = 0$$

$$\tan 2\theta_p=\frac{\tau_{xy}}{\left(\frac{\sigma_x-\sigma_y}{2}\right)}$$

$$\sin 2\theta_p=\pm\frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

$$\cos 2\theta_p=\pm\frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

Principal Stresses

- Substituting values of $\sin 2\theta_p$ and $\cos 2\theta_p$ in transformation equation we get principal stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Angle of maximum shear stress, θ_s

$$\frac{d\tau'_{xy}}{d\vartheta} = \frac{d}{d\vartheta}\left(\frac{\sigma_y - \sigma_x}{2}\sin 2\vartheta + \tau_{xy}\cos 2\theta\right) = 0$$

$$\tan 2\theta_s = -\frac{\frac{\sigma_x - \sigma_y}{2}}{\tau_{xy}}$$

The maximum principal stress plane,

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

Maximum Shear Stresses

$$\tan 2\theta_s = -\cot 2\theta_p$$

$$2\theta_s = 2\theta_p \pm 90$$

$$\theta_s = \theta_p \pm 45$$

substituting value of θ_s in τ'_{xy}

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Equations of Mohr's Circle

$$\sigma_x' = \frac{1}{2}\,(\sigma_x + \sigma_y) + \frac{1}{2}\,(\sigma_x - \sigma_y)\ \cos 2\theta + \tau_{xy}\ \sin 2\theta$$

$$\sigma_y' = \frac{1}{2}\,(\sigma_x + \sigma_y) - \frac{1}{2}\,(\sigma_x - \sigma_y)\ \cos 2\theta - \tau_{xy}\ \sin 2\theta$$

$$\tau_{xy}' = -\frac{1}{2}\,(\sigma_x - \sigma_y)\ \sin 2\theta + \tau_{xy}\ \cos 2\theta$$

Mohr's Circle

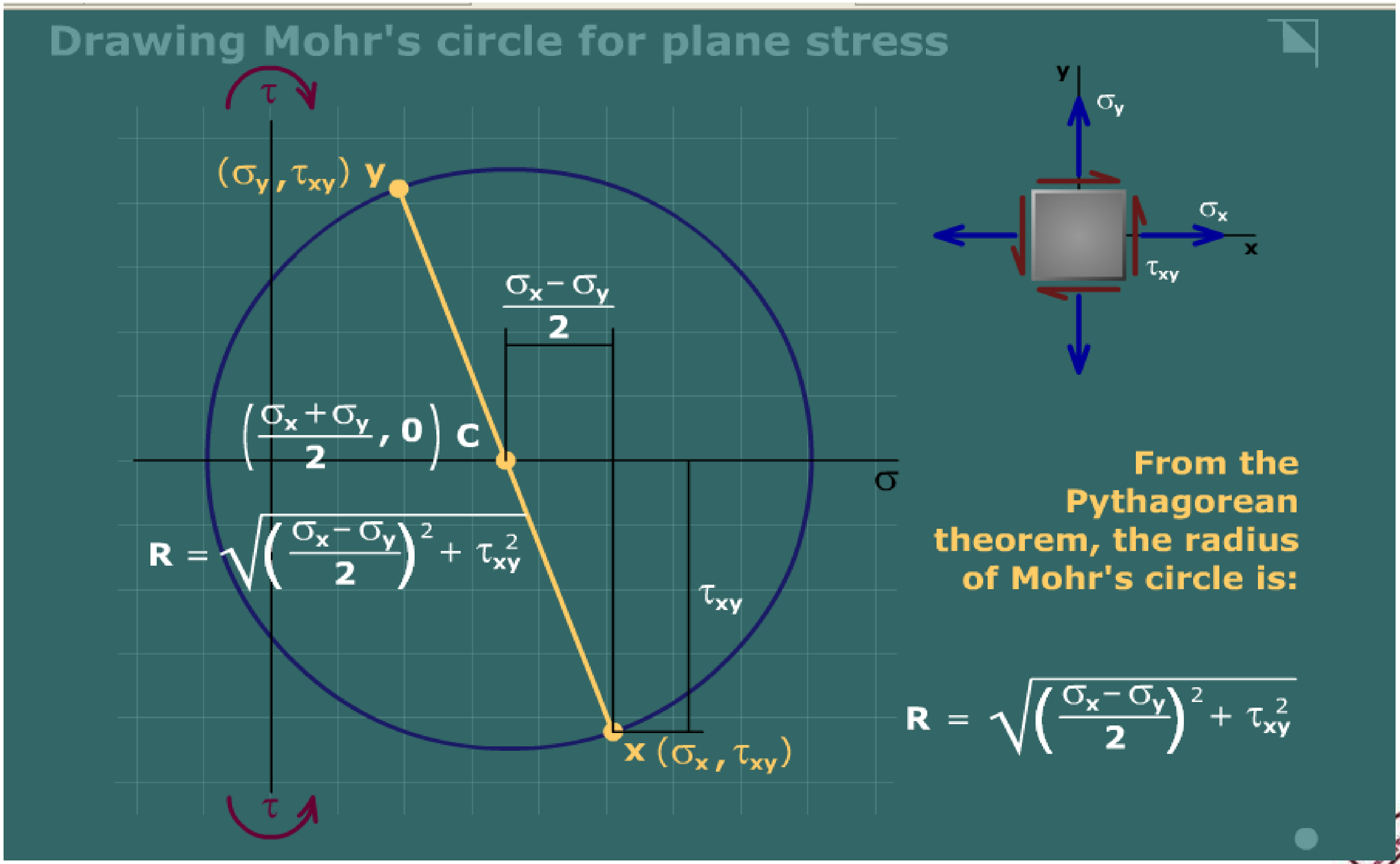


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Figure 6.14 Failure of a 15 mm diameter copper water pipe due to excess pressure from freezing. In the cross section on the right, note that failure occurred on a plane inclined 45° to the tube surface, which is the plane of the maximum shear stress. (Photos by R. A. Simonds.)

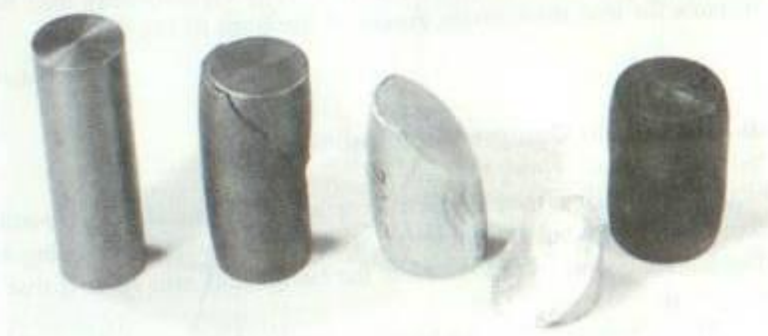

Figure 4.23 Compression specimens of metals (left to right): untested specimen, and tested specimens of gray cast iron, aluminum alloy 7075-T651, and hot-rolled AISI 1020 steel. Diameters before testing were approximately 25 mm, and lengths were 76 mm. (Photo by R. A. Simonds.)

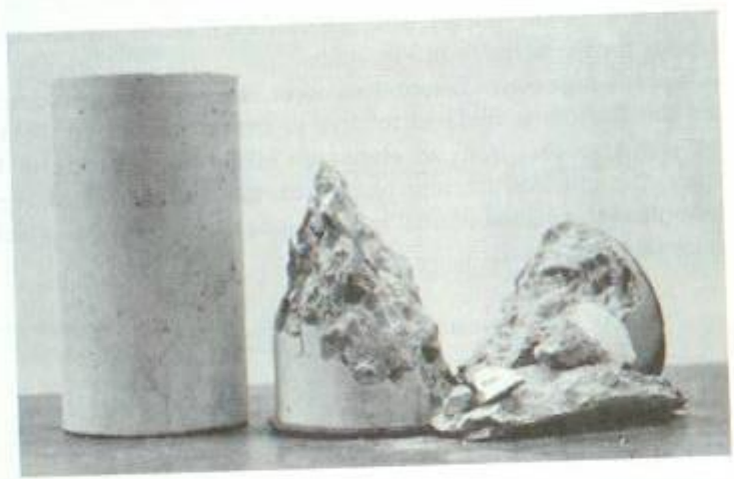

Figure 4.24 Untested and tested 150 mm diameter compression specimens of concrete with Hokie limestone aggregate. (Photo by R. A. Simonds.)

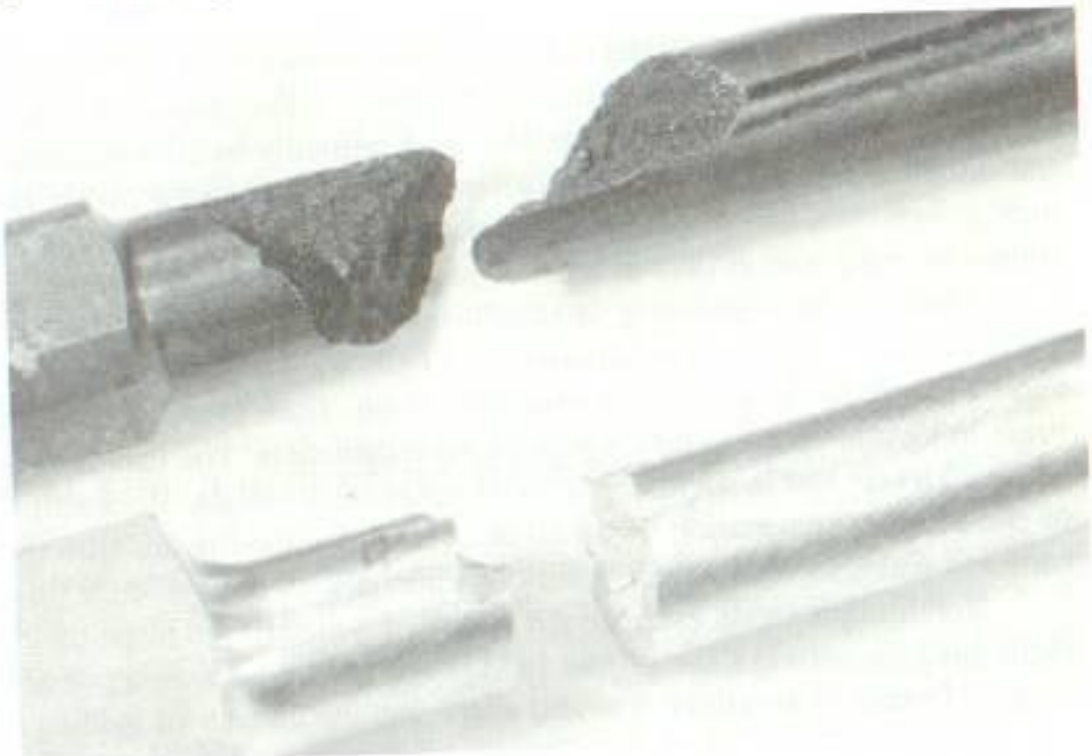

Figure 4.40 Typical torsion failures showing brittle behavior (above) in gray cast iron, and ductile behavior (below) in aluminum alloy 2024-T351. (Photo by R. A. Simonds.)