Forging Analysis - 1

ver. 1

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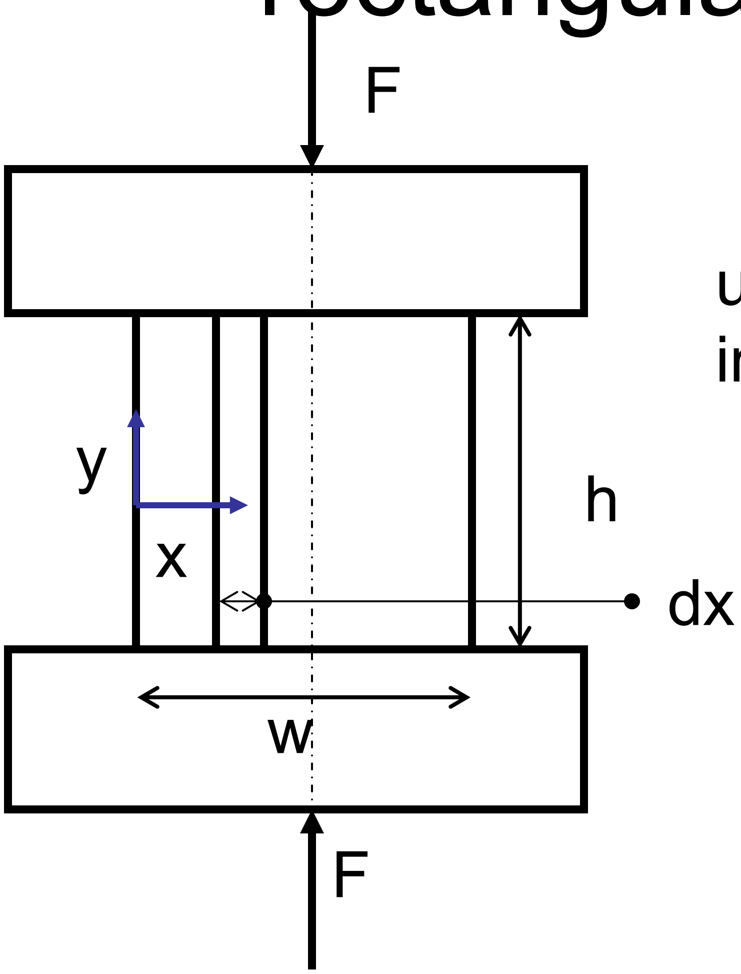
Overview

- Slab analysis
  - frictionless
  - with friction
  - Rectangular
  - Cylindrical
- Strain hardening and rate effects
- Flash
- Redundant work

Slab analysis assumptions

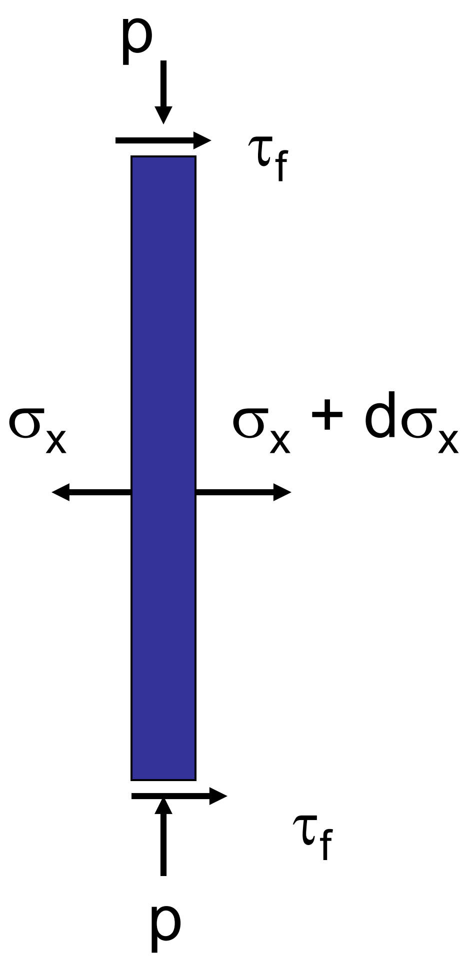
- Entire forging is plastic
  - no elasticity
- Material is perfectly plastic
  - strain hardening and strain rate effects later
- Friction coefficient (μ) is constant
  - all sliding, to start
- Plane strain
  - no z-direction deformation
- In any thin slab, stresses are uniform

Open die forging analysis – rectangular part



unit depth into figure

Expanding the dx slice on LHS



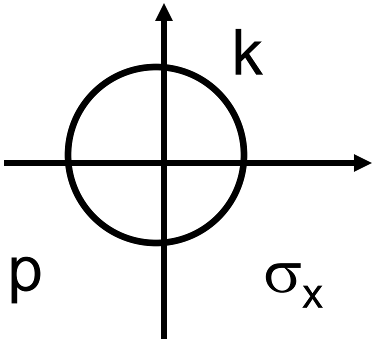
- p = die pressure
- σ_x, $d\sigma_x$ from material on side
- $\tau_{friction}$ = friction force = μp

Force balance in x-direction

$$hd\sigma_x + 2\tau_{friction}dx = 0$$

$$d\sigma_x = -\frac{2\tau_{friction}}{h}dx$$

Mohr's circle



$$\sigma_x + p = 2k = \frac{2}{\sqrt{3}}\sigma_{flow} = 1.15 \cdot \sigma_{flow}$$

(distortion energy (von Mises) criterion, plane strain)

N.B. all done on a per unit depth basis

Force balance

Differentiating, and substituting into Mohr's circle equation

$$d(2k) = d(\sigma_x + p) \quad \therefore\ dp = -d\sigma_x$$

$$d\sigma_x = -\frac{2\tau_{friction}}{h}dx \qquad \therefore\ dp = \left(\frac{2\tau_{friction}}{h}\right)dx$$

noting: $\tau_{friction} = \mu p$

$$dp = \frac{2\mu}{h}p\,dx \longrightarrow \frac{dp}{p} = \frac{2\mu}{h}dx$$

Sliding region

$$\int_{2k}^{p_x} \frac{dp}{p} = \int_0^x \frac{2\mu}{h} dx$$

- Noting: @ x = 0, $\sigma_x = 2k = 1.15\ \sigma_{flow}$

Forging pressure – sliding region

$$\ln p_x - \ln(2k) = 2\mu \frac{x}{h}$$

Sliding region result ($0 < x < x_k$)

$$\frac{p_x}{2k} = \exp\left(\frac{2\mu x}{h}\right)$$

$$p_x = 1.15 \cdot \sigma_{flow} \cdot \exp\left(\frac{2\mu x}{h}\right)$$

N.B done on a per unit depth basis

Forging pressure – approximation

- Taking the first two terms of a Taylor's series expansion for the exponential about 0, for $|x| \leq 1$

$$\exp(x) = 1 + x + \cancelto{0}{\frac{x^2}{2!}} + \cancelto{0}{\frac{x^3}{3!}} + \cdots + \frac{x^n}{n!} = \sum_{k=0}^{n} \frac{x^k}{k!}$$

yields

$$\frac{p_x}{2k} = \left(1 + \frac{2\mu x}{h}\right) \qquad p_x = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{2\mu x}{h}\right)$$

Average forging pressure – all sliding approximation

- using the Taylor's series approximation

$$\frac{p_{ave}}{2k}=\frac{\int_0^{\frac{w}{2}}\frac{p_x}{2k}dx}{\frac{w}{2}}=\frac{\int_0^{\frac{w}{2}}\left(1+\frac{2\mu x}{h}\right)dx}{\frac{w}{2}}=\frac{\left(x+\frac{2\mu x^2}{2h}\right)\Big|_0^{\frac{w}{2}}}{\frac{w}{2}}$$

$$\frac{p_{ave}}{2k}=\left(1+\frac{\mu w}{2h}\right)$$

$$p_{ave}=1.15\cdot\sigma_{flow}\cdot\left(1+\frac{\mu w}{2h}\right)$$

N.B done on a per unit depth basis

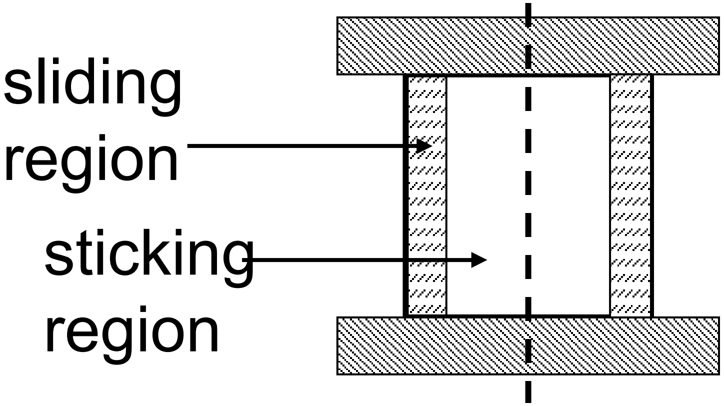
Forging force – all sliding approximation

$$F_{forging} = p_{ave} \cdot width \cdot depth$$

$$F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{\mu w}{2h}\right) \cdot w \cdot depth$$

Slab - die interface

- Sliding if $\tau_f < \tau_{flow}$
- Sticking if $\tau_f \geq \tau_{flow}$
  - can't have a force on a material greater than its flow (yield) stress
  - deformation occurs in a sub-layer just within the material with stress τ_{flow}



Sliding / sticking transition

- Transition will occur at x_k
- using $k = \mu p$, in:

$$\frac{p_x}{2k} = \exp\left(\frac{2\mu x}{h}\right) \qquad \frac{k}{2\mu k} = \exp\left(\frac{2\mu x_k}{h}\right)$$

- hence:

$$\frac{x_k}{h} = \frac{1}{2\mu} \ln \frac{1}{2\mu}$$

Sticking region

$$dp = \frac{2\mu}{h} p dx$$

- Using p = k/ μ

$$dp = \frac{2\mu}{h}\frac{k}{\mu} dx$$

$$\int_{p_{x_k}}^{p_x} dp = \int_{x_k}^{x} \frac{2k}{h} dx$$

$$p_x - p_{x_k} = \frac{2k}{h}\left(x - x_k\right)$$

Sticking region

We know that

- at $x = x_k$, $p_{x_k} = k/\mu$
- and $$\frac{x_k}{h} = \frac{1}{2\mu}\ln\frac{1}{2\mu}$$

Forging pressure - sticking region

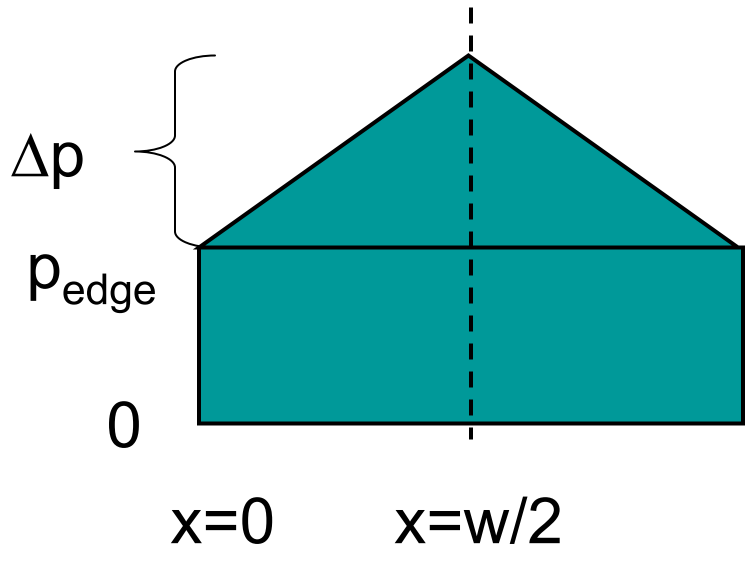
Combining (for $x_k < x < w/2$)

$$\frac{p_x}{2k} = \frac{1}{2\mu}\left(1 - \ln\left(\frac{1}{2\mu}\right)\right) + \frac{x}{h}$$

$$p_x = 1.15 \cdot \sigma_{flow} \cdot \left[\frac{1}{2\mu}\left(1 - \ln\left(\frac{1}{2\mu}\right)\right) + \frac{x}{h}\right]$$

Forging pressure – all sticking approximation

- If $x_k << w$, we can assume all sticking, and approximate the total forging force per unit depth (into the figure) by:



Forging pressure – all sticking approximation

$$p_{edge} = 2k$$

$$\int_{2k}^{p_x} dp = \int_0^x \frac{2k}{h} dx \qquad p_x - 2k = \frac{2k}{h}(x)$$

$$\therefore \frac{p_x}{2k} = \left(1 + \frac{x}{h}\right)$$

$$p_x = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{x}{h}\right)$$

Average forging pressure – all sticking approximation

$$\frac{p_{ave}}{2k} = \frac{\int_0^{\frac{w}{2}} \frac{p_x}{2k} dx}{w/2} = \frac{\int_0^{\frac{w}{2}} \left(1+\frac{x}{h}\right) dx}{w/2} = \frac{\left(x+\frac{x^2}{2h}\right)\Big|_0^{\frac{w}{2}}}{w/2}$$

$$\frac{p_{ave}}{2k} = \left(1+\frac{w}{4h}\right)$$

$$p_{ave} = 1.15 \cdot \sigma_{flow} \cdot \left(1+\frac{w}{4h}\right)$$

Forging force – all sticking approximation

$$F_{forging} = p_{ave} \cdot width \cdot depth$$

$$F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{w}{4h}\right) \cdot w \cdot depth$$

Sticking and sliding

- If you have both sticking and sliding, and you can't approximate by one or the other,
- Then you need to include both in your pressure and average pressure calculations.

$$F_{forging} = F_{sliding} + F_{sticking}$$

$$F_{forging} = \left(p_{ave} \cdot A\right)_{sliding} + \left(p_{ave} \cdot A\right)_{sticking}$$

Material Models

Strain hardening (cold – below recrystallization point)

$$\sigma_{flow} = Y = K\varepsilon^n$$

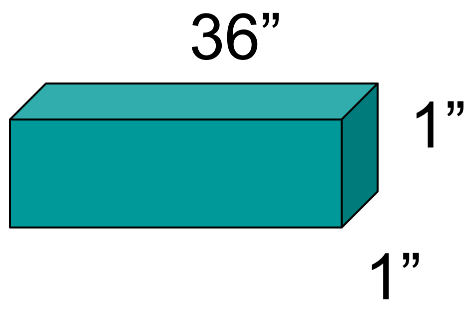
Strain rate effect (hot – above recrystallization point)

$$\sigma_{flow} = Y = C\left(\dot{\varepsilon}\right)^m$$

$$\dot{\varepsilon} = \frac{1}{h}\frac{dh}{dt} = \frac{v}{h} = \frac{\text{platen velocity}}{\text{instantaneous height}}$$

Forging - Ex. 1-1

- Lead 1" x 1" x 36"
- σ_y = 1,000 psi
- h_f = 0.25", μ = 0.25
- Show effect of friction on total forging force.
- Use the slab method.
- Assume it doesn't get wider in 36" direction.
- Assume cold forging.



Forging - Ex. 1-2

- At the end of forging:

h_f = 0.25", w_f = 4" (conservation of mass)

- Sliding / sticking transition

$$\frac{x_k}{h} = \frac{1}{2\mu} \ln \frac{1}{2\mu}$$

$$x_k = \frac{0.25}{2 \times 0.25} \ln \frac{1}{2 \times 0.25} = 0.347''$$

Forging - Ex. 1-3

- Sliding region:

$$p_x = 1.15 \cdot \sigma_{flow} \cdot \exp\left(\frac{2\mu x}{h_f}\right)$$

$$= 1150 \cdot \exp(2x)$$

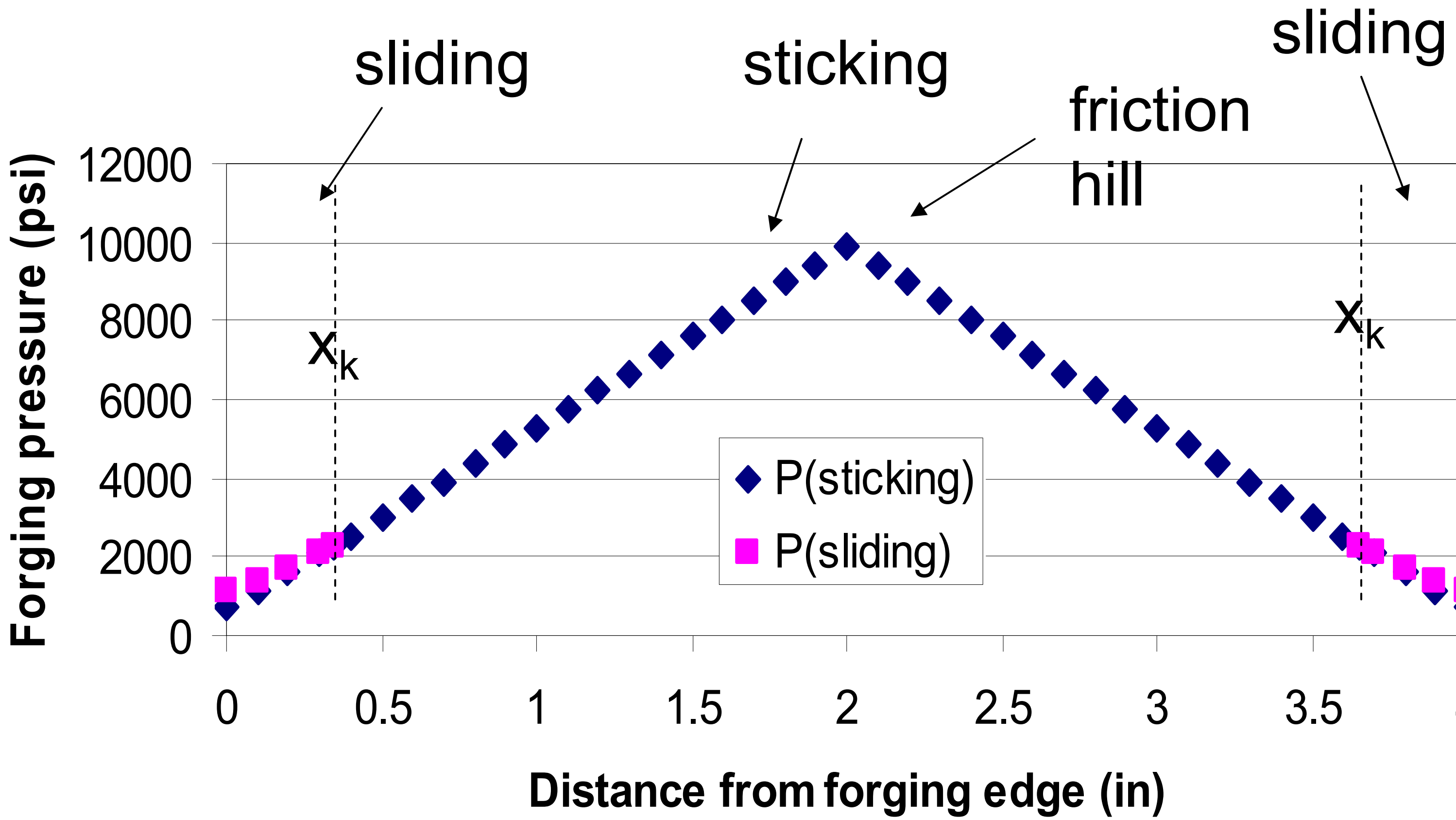
Forging - Ex. 1-4

- Sticking region

$$p_x = 1.15 \cdot \sigma_{flow} \cdot \left\{ \frac{1}{2\mu} \left(1 - \ln\left(\frac{1}{2\mu} \right) \right) + \frac{x}{h_f} \right\}$$

$$= 1150 \cdot (0.6 + 4x)$$

Forging - Ex. 1-5



Forging - Ex. 1-6

- Friction hill
  - forging pressure must be large (8.7x) near the center of the forging to "push" the outer material away against friction

Forging - Ex. 1-7

- Determine the forging force from:

$$Force = \iint p \cdot dA$$

- since we have plane strain

$$\frac{F}{unit\ depth} = \int_0^x p_x dx$$

Forging - Ex. 1-8

- We must solve separately for the sliding and sticking regions

$$F_{forging} = 2\left(\left(\int_0^{x_k} p_x dx\right).depth\right)_{sliding} + 2\left(\left(\int_{x_k}^{w/2} p_x dx\right).depth\right)_{sticking}$$

Forging - Ex. 1-9

Sliding first

$$\frac{\dfrac{p_{ave}}{1.15\sigma_{flow}}}{unit\, depth} = \frac{\int_0^{x_k} \exp\left(\dfrac{2\mu x}{h}\right) dx}{x_k - 0} = \frac{h}{2\mu} \exp\left(\frac{2\mu x}{h}\right)\Bigg|_0^{x_k}$$

$$= \frac{\dfrac{h}{2\mu}\left[\exp\left(\dfrac{2\mu x_k}{h}\right) - 1\right]}{\left(x_k - 0\right)}$$

Forging - Ex. 1-12

Substituting values

sliding

$$\frac{\frac{p_{ave}}{1.15\sigma_{flow}}}{unit\ depth} = \frac{\frac{0.25}{2\times 0.25}\left[\exp\left(\frac{2\times 0.25\times 0.347}{0.25}\right)-1\right]}{(0.347-0)} = 1.44$$

sticking

$$\frac{\frac{p_{ave}}{1.15\sigma_{flow}}}{unit\ depth} = \frac{\frac{1}{2\times 0.25}\left(1-\ln\frac{1}{2\times 0.25}\right)\left(\frac{4}{2}-0.347\right)+\frac{1}{2\times 0.25}\left(\frac{4^2}{4}-0.347^2\right)}{\frac{4}{2}-0.347}$$

$$= 5.3$$

Forging - Ex. 1-10

Sticking next

$$\frac{\dfrac{p_{ave}}{1.15\sigma_{flow}}}{unit\, depth} = \frac{\displaystyle\int_{x_k}^{w/2} \left(\frac{1}{2\mu}\left(1 - \ln\frac{1}{2\mu}\right) + \frac{x}{h} \right) dx}{w/2 - x_k}$$

$$= \left[\frac{\dfrac{1}{2\mu}\left(1 - \ln\dfrac{1}{2\mu}\right)\cdot x + \dfrac{x^2}{2h}}{w/2 - x_k} \right]\Bigg|_{x_k}^{w/2}$$

Forging - Ex. 1-11

$$\frac{\dfrac{p_{ave}}{1.15\sigma_{flow}}}{unit\,depth} = \frac{\dfrac{1}{2\mu}\left(1-\ln\dfrac{1}{2\mu}\right)\left(\frac{w}{2}-x_k\right)+\dfrac{1}{2h}\left(\dfrac{w^2}{4}-x_k^2\right)}{\frac{w}{2}-x_k}$$

Forging - Ex. 1-13

Now calculate the area/unit depth

$$A_{sliding} = 0.347 \times 2 = 0.69$$

$$A_{sticking} = 4 - 2 \times 0.347 = 3.31$$

Forging - Ex. 1-14

Now calculate the forces

$$\frac{F}{unit\ \ depth} = 1.15\sigma_{flow}\left(\left(p_{ave}\cdot A\right)_{sliding} + \left(p_{ave}\cdot A\right)_{sticking}\right)$$

$$\frac{F}{unit\ depth} = 1150\times\left(\left(1.44\times 0.69\right)+\left(5.3\times 3.31\right)\right)$$

$$= 21{,}110\ lb/inch$$

Forging - Ex. 1-15

- Now, assume all sticking, so:

$$\frac{F}{unit\ length} = 1.15\sigma_{flow} \cdot w_f \cdot \left(1 + \frac{w_f}{4h_f}\right)$$

$$= 1150 \cdot 4 \cdot \left(1 + \frac{4}{4 \cdot 0.25}\right)$$

$$= 23{,}000\ lb / inch\ depth$$

Forging - Ex. 1-16

or since the part is 36" deep:

F(both) = 759,960 lbs = 380 tons

$$F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{w}{4h}\right) \cdot w \cdot depth$$

F(all sticking) = 828,000 lbs = 414 tons

$$F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{\mu w}{2h}\right) \cdot w \cdot depth$$

F(all sliding) = 496,800 lbs = 225 tons

All sticking over-estimates actual value.

Forging – Effect of friction

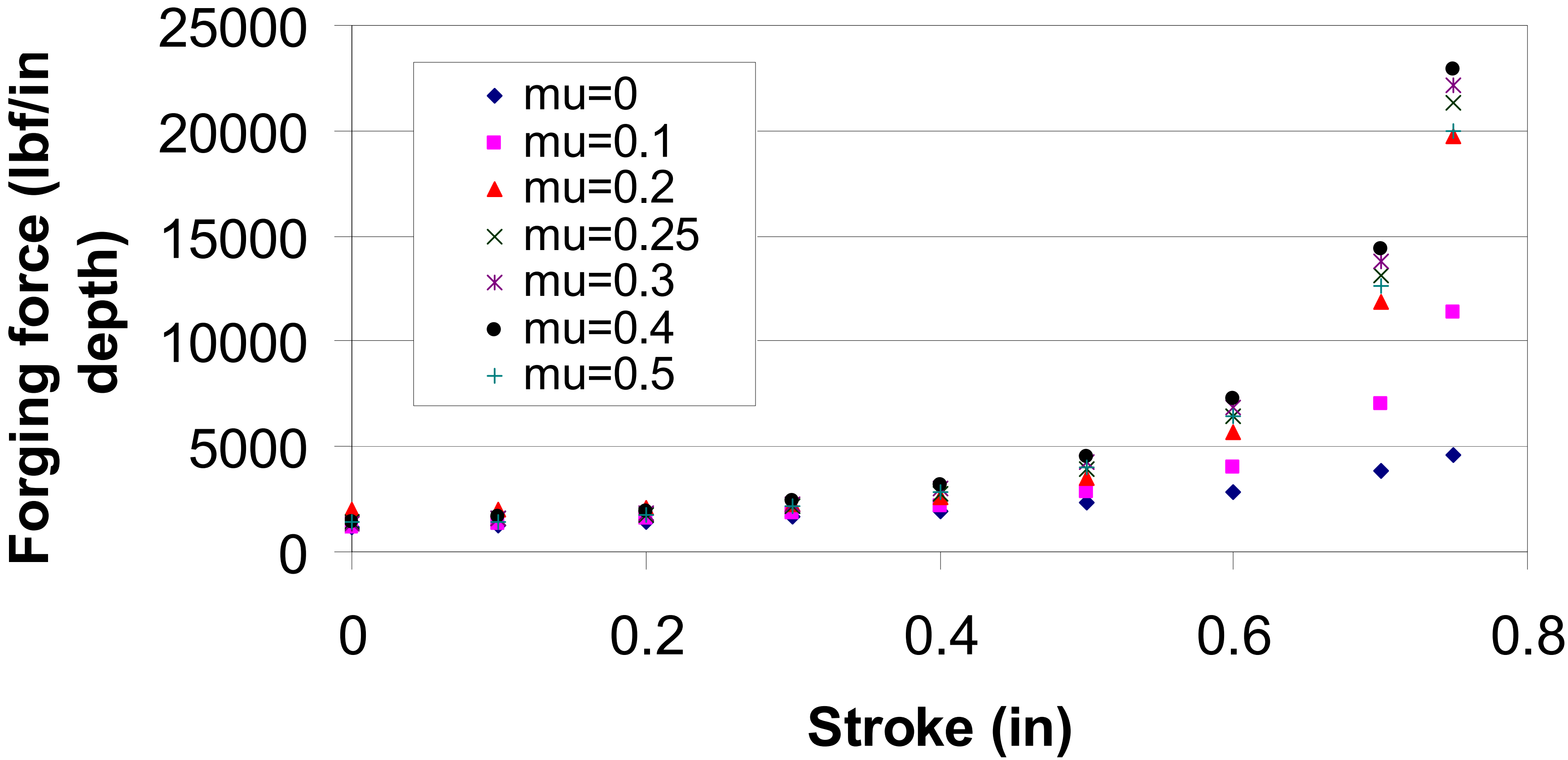
- Effect of friction coefficient (μ) – all sticking

Friction coefficient	Fmax (lbf/in depth)	xk	Stick/slide
0	4600	2	slide
0.1	11365	2	slide
0.2	19735	0.573	both
0.25	21331	0.347	both
0.3	22182	0.213	both
0.4	22868	0.070	both
0.5	23000	0	stick

- Friction is <u>very</u> important

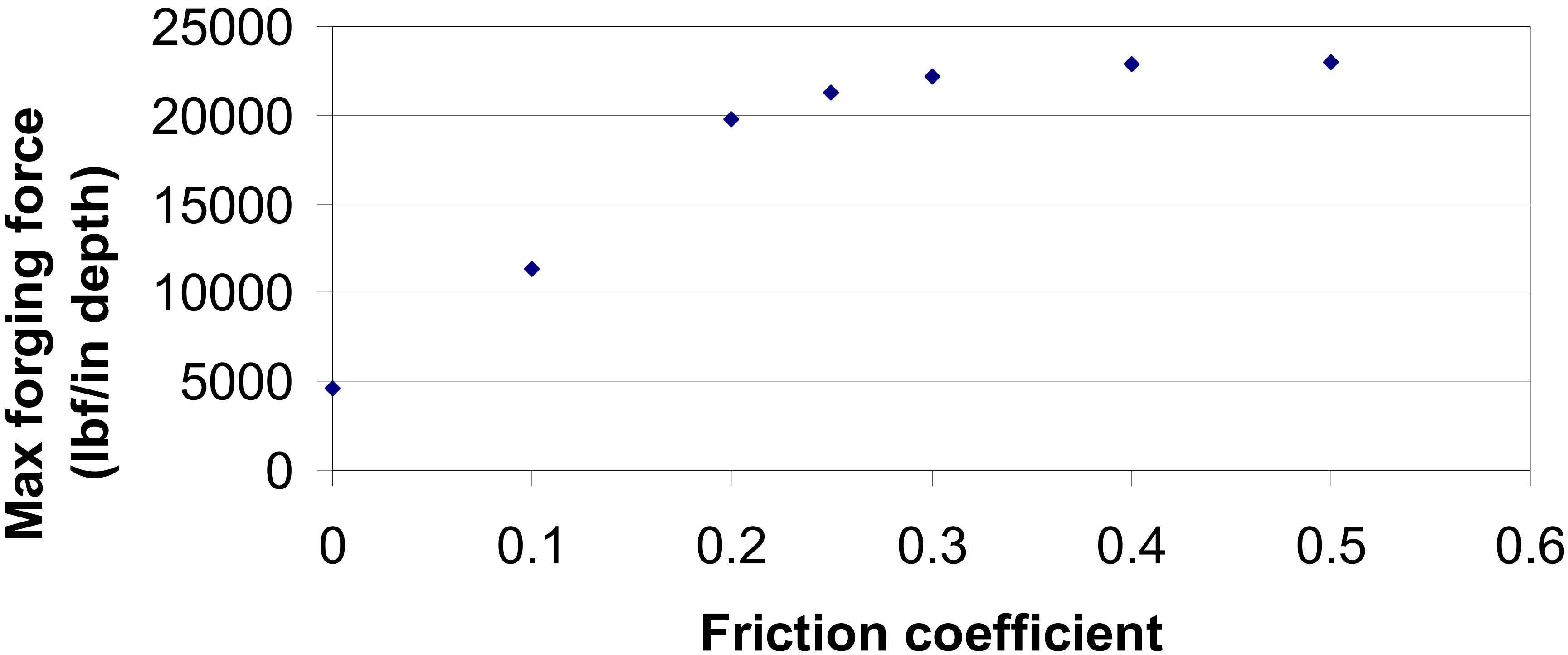
Forging - Ex. 1-17

Forging force vs. stroke – all sticking



Forging - Ex. 1-19

Maximum forging force vs. friction coefficient (μ) all sticking



Summary

- Slab analysis
  - frictionless
  - with friction
  - Rectangular
  - Cylindrical
- Strain hardening and rate effects
- Flash
- Redundant work