Forging Analysis - 2

ver. 1

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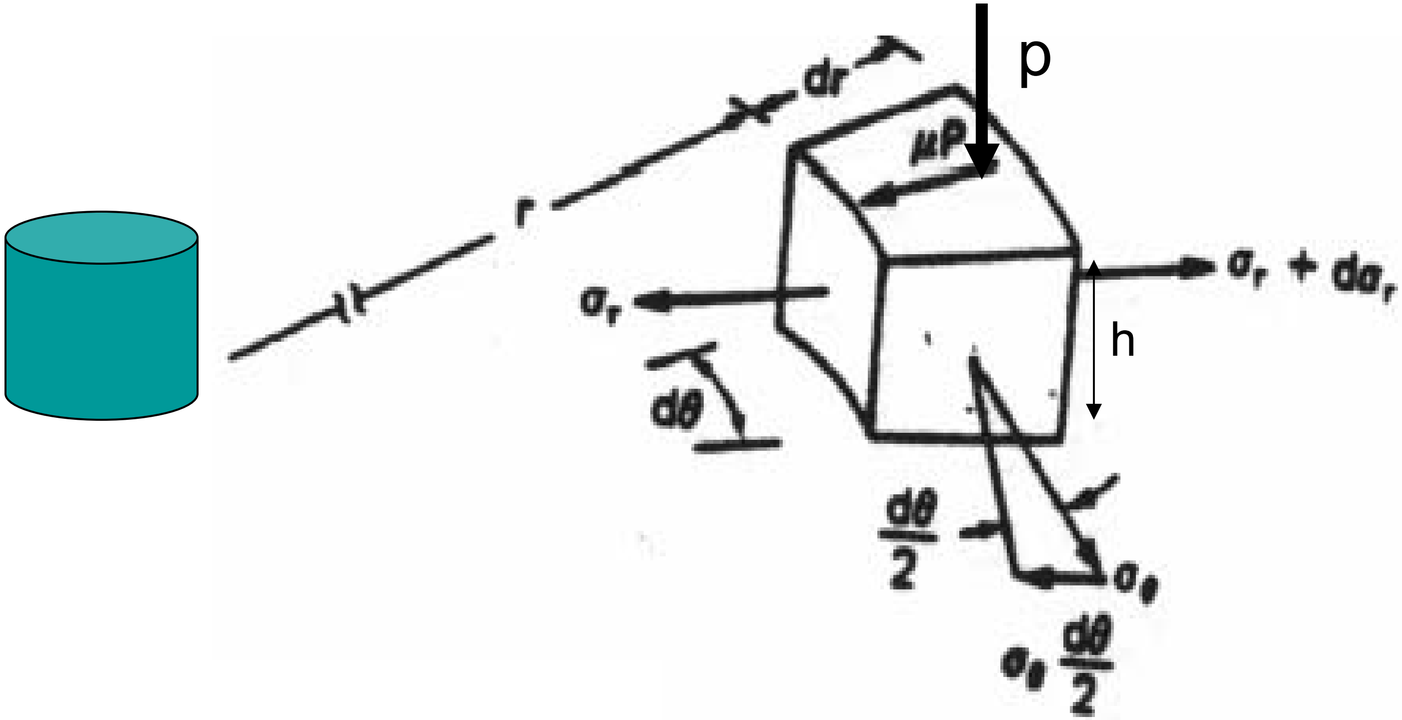
Overview

- Slab analysis
  - frictionless
  - with friction
  - Rectangular
  - Cylindrical
- Strain hardening and rate effects
- Flash
- Redundant work

Forging – cylindrical part sliding region



Equilibrium in r direction

$$\sum dF_r = 0 = -\sigma_r \cdot h \cdot r \cdot d\theta - 2 \cdot \mu \cdot p \cdot r \cdot d\theta \cdot dr$$

$$-2 \cdot \sigma_\theta \cdot h \cdot dr \cdot \frac{d\theta}{2} + \left(\sigma_r + d\sigma_r\right) \cdot \left(r + dr\right) \cdot h \cdot d\theta$$

N.B. $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$

neglecting HOTs

$$2\mu pr \cdot dr + h\sigma_\theta \cdot dr - h\sigma_r \cdot dr - hr \cdot d\sigma_r = 0$$

Axisymmetric flow and yield

For axisymmetric flow

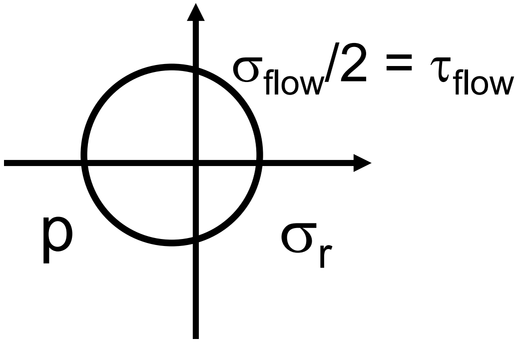
$$\varepsilon_r = \frac{dr}{r}; \quad \varepsilon_\theta = \frac{2\pi(r+dr) - 2\pi r}{2\pi r} = \frac{dr}{r}$$

$$\varepsilon_r = \varepsilon_\theta; \sigma_r = \sigma_\theta$$

By Tresca

$$\sigma_r + p = \sigma_{flow} = 2k = 2\tau_{flow}$$

$$d\sigma_r = -dp$$



Stress in z direction

substituting

$$2\mu pr \cdot dr + h\sigma_r \cdot dr - h\sigma_r \cdot dr + hr \cdot dp = 0$$

or

$$2\mu pr \cdot dr = -hr \cdot dp$$

rearranging

$$\frac{dp}{p} = -\frac{2\mu}{h} dr$$

Forging pressure - sliding

$$\int_{p_r}^{2\tau_{flow}} \frac{dp}{p} = -\int_{r}^{R} \frac{2\mu}{h} \cdot dr$$

for $r_k < r < R$

$$\frac{p_r}{2\tau_{flow}} = \exp\left[\frac{2\mu}{h}(R-r)\right]$$

Average forging pressure – sliding

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{1}{\pi\left(R^2 - r_k^2\right)}\int_{r_k}^{R}\frac{p_r}{2\tau_{flow}}\cdot 2\pi r\cdot dr = \frac{2}{\left(R^2 - r_k^2\right)}\int_{r_k}^{R}\exp\left[\frac{2\mu}{h}(R-r)\right]rdr$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{\left(R^2 - r_k^2\right)}\left(\frac{h}{2\mu}\right)^2\exp\left(\frac{2\mu R}{h}\right)\left\{\exp\left(\frac{-2\mu r}{h}\right)\cdot\left(\frac{-2\mu r}{h}-1\right)\right\}\Bigg|_{r_k}^{R}$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{\left(R^2 - r_k^2\right)}\left(\frac{h}{2\mu}\right)^2\exp\left(\frac{2\mu R}{h}\right)\left\{\left[\exp\left(\frac{-2\mu R}{h}\right)\cdot\left(\frac{-2\mu R}{h}-1\right)\right]-\left[\exp\left(\frac{-2\mu r_k}{h}\right)\cdot\left(\frac{-2\mu r_k}{h}-1\right)\right]\right\}$$

Average forging pressure – sliding

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{\left(R^2 - r_k^2\right)} \cdot \left(\frac{h}{2\mu}\right)^2 \left[\exp\left(\frac{2\mu\left(R - r_k\right)}{h}\right) \cdot \left(\frac{2\mu r_k}{h} + 1\right) - \left(\frac{2\mu R}{h}\right) - 1\right]$$

Forging force – sliding

$$F_{forging} = p_{ave} \cdot A = p_{ave} \cdot \pi \cdot \left(R^2 - r_k^2\right)$$

$$F_{forging} = 4\tau_{flow} \cdot \left(\frac{h}{2\mu R}\right)^2 \left[\exp\left(\frac{2\mu\left(R - r_k\right)}{h}\right) \cdot \left(\frac{2\mu r_k}{h} + 1\right) - \left(\frac{2\mu R}{h}\right) - 1\right] \cdot \pi \cdot \left(R^2 - r_k^2\right)$$

Average forging pressure – all sliding approximation ($r_k = 0$)

- Taking the first four terms of a Taylor's series expansion for the exponential about 0 for $|x| \le 1$

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} = \sum_{k=0}^{n} \frac{x^k}{k!}$$

yields

$$\frac{p_{ave}}{2\tau_{flow}} = \left[1 + \left(\frac{2\mu R}{3h}\right)\right]$$

Forging force – all sliding approximation

$$F_{forging} = p_{ave} \cdot A = p_{ave} \cdot \pi \cdot R^2$$

$$F_{forging} = 2\tau_{flow} \cdot \left[1 + \left(\frac{2\mu R}{3h}\right)\right] \cdot \pi R^2$$

Transition sticking / sliding

- Set $\tau_{flow} = \mu p$ and solve for r_k

$$\frac{p}{2\tau_{flow}} = \exp\left[2\mu\left(\frac{R-r_k}{h}\right)\right] \rightarrow \frac{p}{2\mu \cdot p} = \exp\left[2\mu\left(\frac{R-r_k}{h}\right)\right]$$

$$\ln\left(\frac{1}{2\mu}\right) = 2\mu\left(\frac{R-r_k}{h}\right) \longrightarrow r_k = R - \frac{h}{2\mu}\ln\left(\frac{1}{2\mu}\right)$$

Forging pressure - sticking region

- Use the same method as for sliding
- Substitute $\mu p = \tau_{flow}$,
- Assume Tresca yield criterion

$$2\mu pr \cdot dr = -hr \cdot dp$$

$$2\tau_{flow} r \cdot dr = -hr \cdot dp$$

$$dp = -\frac{2\tau_{flow}}{h} dr$$

Forging pressure - sticking region

$$\int_{p_{r_k}}^{p_r} dp = -\int_{r_k}^{r} \frac{2\tau_{flow}}{h} dr$$

$$p_r - p_{r_k} = -\frac{2\tau_{flow}}{h}\left(r - r_k\right)$$

$$\frac{p_r - p_{r_k}}{2\tau_{flow}} = \frac{\left(r_k - r\right)}{h}$$

Forging pressure - sticking region

p_{r_k} determined from sliding equation

$$\frac{p_{r_k}}{2\tau_{flow}} = \exp\left[\frac{2\mu}{h}\left(R - r_k\right)\right]$$

for $0 < r < r_k$

$$\frac{p_r}{2\tau_{flow}} = \exp\left[\frac{2\mu}{h}\left(R - r_k\right)\right] + \frac{\left(r_k - r\right)}{h}$$

Average forging pressure - sticking

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{1}{\pi r_k^2}\int_0^{r_k} p_r \cdot 2\pi r \cdot dr = \frac{2}{r_k^2}\int_0^{r_k}\left(\exp\left[\frac{2\mu}{h}(R - r_k)\right] + \frac{r_k - r}{h}\right)\cdot r dr$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2}\int_0^{r_k}\left(r\cdot\exp\left[\frac{2\mu}{h}(R - r_k)\right] + \frac{r_k \cdot r}{h} - \frac{1}{h}r^2\right)\cdot dr$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2}\left(\frac{r^2}{2}\cdot\exp\left[\frac{2\mu}{h}(R - r_k)\right] + \frac{r_k \cdot r^2}{2h} - \frac{r^3}{3h}\right)\Bigg|_0^{r_k}$$

Average forging pressure - sticking

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2}\left(\frac{r^2}{2}\cdot\exp\left[\frac{2\mu}{h}\left(R-r_k\right)\right]+\frac{r_k\cdot r^2}{2h}-\frac{r^3}{3h}\right)\Bigg|_0^{r_k}$$

$$\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2}\left(\frac{r_k^2}{2}\cdot\exp\left[\frac{2\mu}{h}\left(R-r_k\right)\right]+\frac{r_k^3}{2h}-\frac{r_k^3}{3h}\right)$$

$$\frac{p_{ave}}{2\tau_{flow}} = \left(\exp\left[\frac{2\mu}{h}\left(R-r_k\right)\right]+\frac{r_k}{3h}\right)$$

Forging force – sticking region

$$F_{forging} = p_{ave} \cdot A = p_{ave} \cdot \pi \cdot r_k^2$$

$$F_{forging} = 2\tau_{flow} \cdot \left(\exp\left[\frac{2\mu}{h}(R - r_k) \right] + \frac{r_k}{3h} \right) \cdot \pi \cdot r_k^2$$

Sticking and sliding

- If you have both sticking and sliding, and you can't approximate by one or the other,
- Then you need to include both in your pressure and average pressure calculations.

$$F_{forging} = F_{sliding} + F_{sticking}$$

$$F_{forging} = \left(p_{ave} \cdot A\right)_{sliding} + \left(p_{ave} \cdot A\right)_{sticking}$$

Strain hardening (cold - below recrystallization point)

Tresca

$$2\tau_{flow} = Y = K\varepsilon^n$$

Strain rate effect (hot – above recrystallization point)

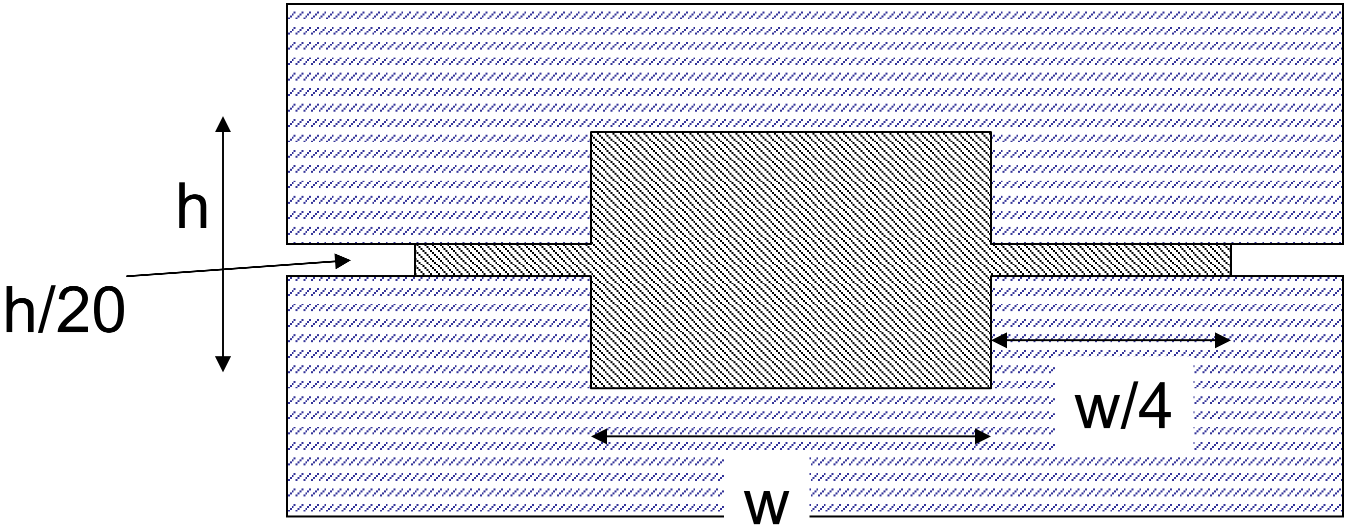
$$\dot{\varepsilon} = \frac{1}{h}\frac{dh}{dt} = \frac{v}{h} = \frac{\text{platen velocity}}{\text{instantaneous height}}$$

Tresca

$$2\tau_{flow} = Y = C\left(\dot{\varepsilon}\right)^{m}$$

Flash for closed die forging
(plane strain)

- Say we have a typical flash with thickness h/20 and length w/4



Average forging pressure

- in forging (Tresca)

$$p_{ave} = 2\tau_{flow} \cdot \left(1 + \frac{\mu w}{2h}\right)$$

- in flash (Tresca)

$$p_{ave} = 2\tau_{flow} \cdot \left(1 + \frac{5\mu w}{h}\right)$$

Flash

- Flash’s high deformation resistance results in filled mold
- Process wouldn’t work without friction

Deformation Work

In general, work done in bulk deformation processes has three components

Total work, $W = W_{ideal} + W_{friction} + W_{redundant}$

Work of ideal plastic deformation, W_{ideal}

= (area under true stress-true strain curve)(volume)

$$= \left(\int_0^{\varepsilon_t} \sigma_t \, d\varepsilon_t \right) \text{(volume)}$$

$$W_{ideal} = (\text{volume}) \left(\frac{K \varepsilon_t^{n+1}}{n+1} \right) = (\text{volume}) \bar{Y}_f \varepsilon_t$$

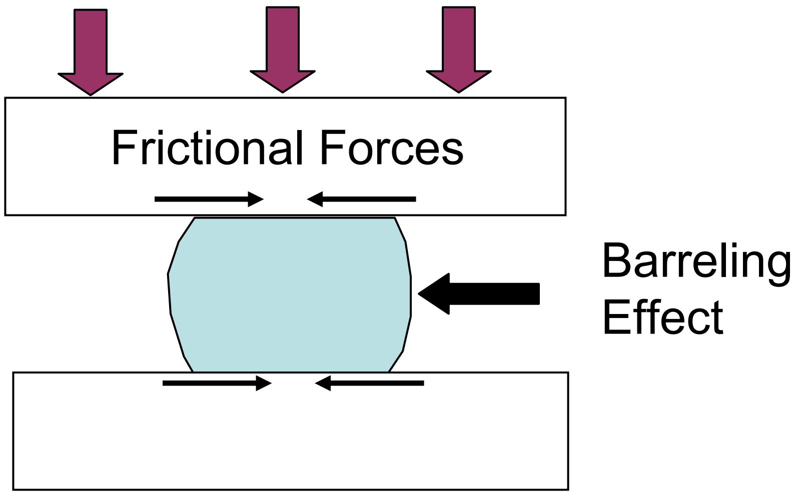
For a true stress-true strain curve :

$$\sigma_t = K \varepsilon_t^n$$

$$\bar{Y}_f = \text{Avg. flow stress}$$

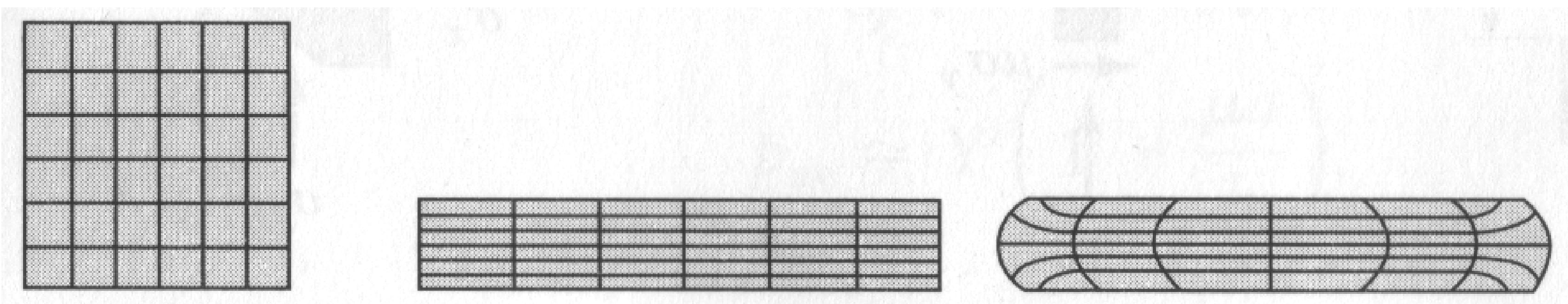
Deformation Work

Friction between dies and workpiece causes inhomogeneous (non-uniform) deformation called barreling



Deformation Work

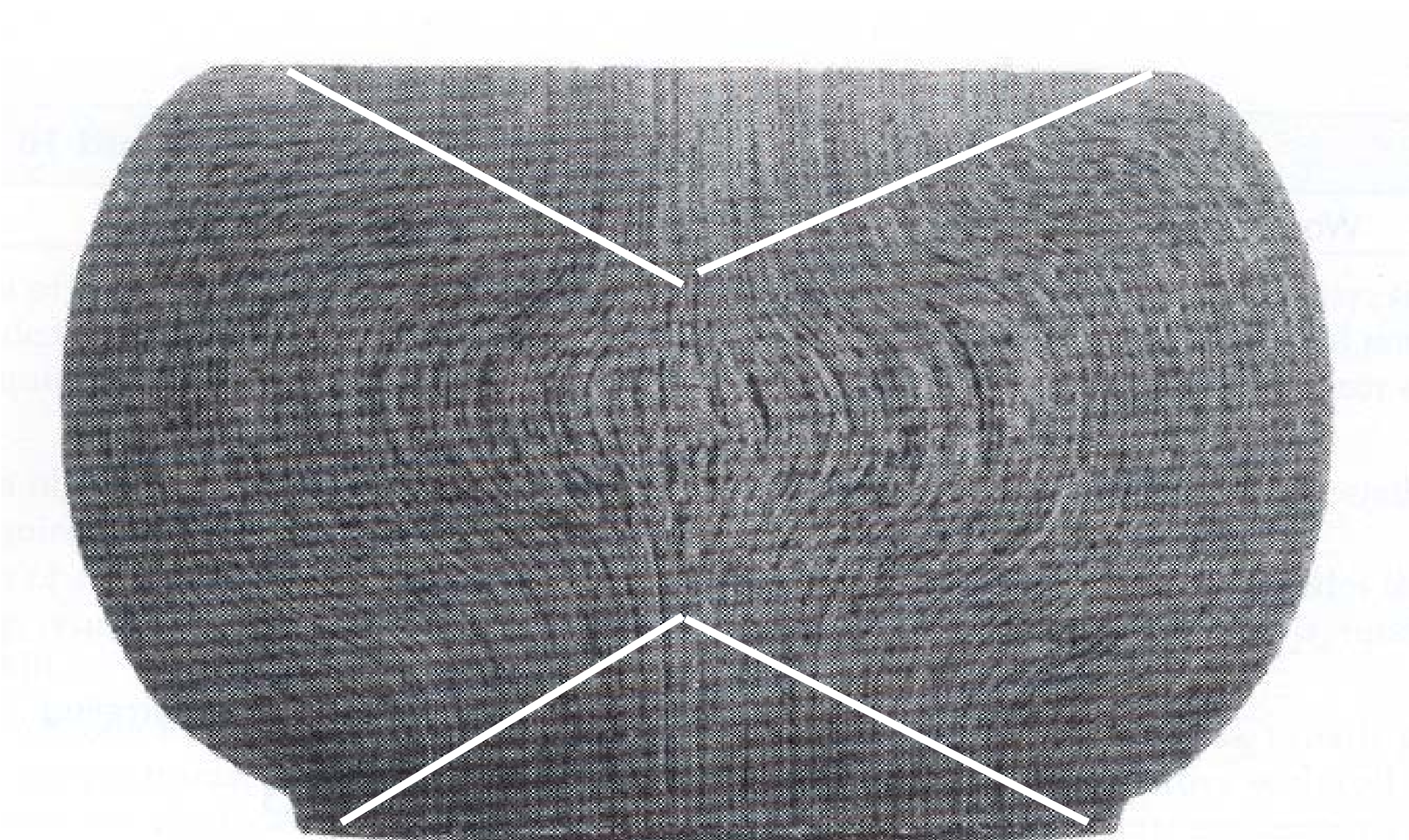
Internal shearing of material requires redundant work to be expended



Ideal Deformation

Redundant Deformation

Redundant Zone



Closed/Impression Die Forging

- Analysis more complex due to large variation in strains in different parts of workpiece
- Approximate approaches
  - Divide forging into simple part shapes e.g. cylinders, slabs etc. that can be analyzed separately
  - Consider entire forging as a simplified shape

Closed/Impression Die Forging

Steps in latter analysis approach

- ***Step 1***: calculate average height from volume *V* and total projected area A_t of part (including flash area)

$$h_{avg} = \frac{V}{A_t} = \frac{V}{Lw}$$

- ***Step 2***: $\varepsilon_{avg} = \text{avg. strain} = \ln\left(\frac{h_i}{h_{avg}}\right)$

$$\dot{\varepsilon}_{avg} = \text{avg. strain rate} = \frac{v}{h_{avg}}$$

Closed/Impression Die Forging

- ***Step 3***: calculate flow stress of material Y_f for cold/hot working

- ***Step 4***:

$$\text{Avg. forging load} = F_{avg} = K_p Y_f A_t$$

K_p = pressure multiplying factor

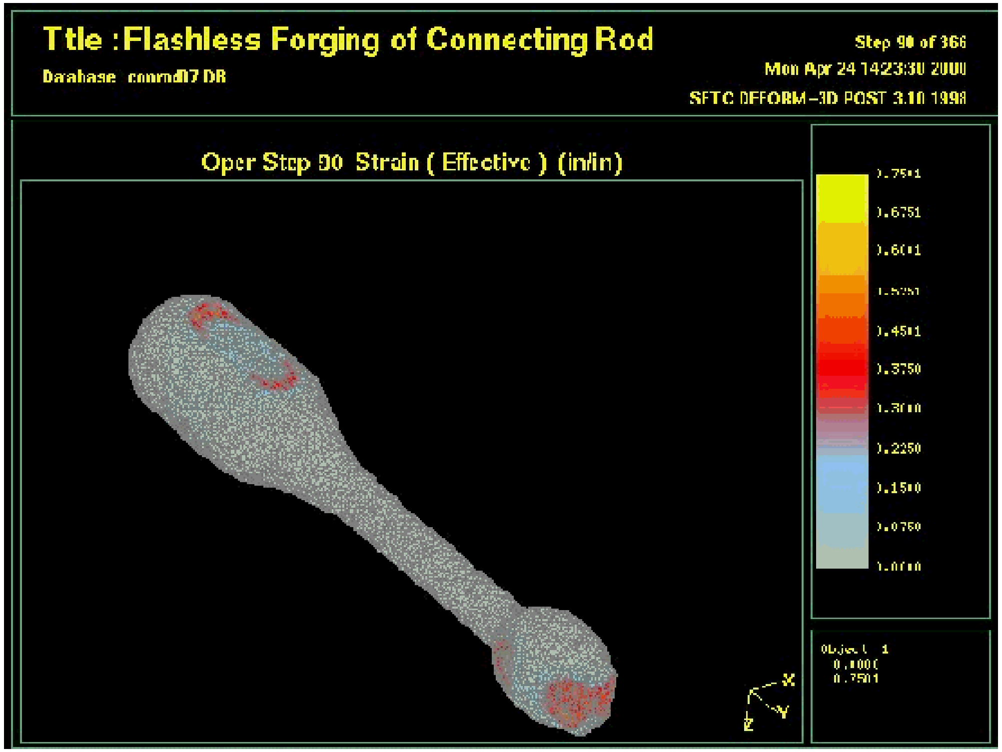
= 3~5 for simple shapes without flash

= 5~8 for simple shapes with flash

= 8~12 for complex shapes with flash

Other Analysis Methods

- Complex closed die forging simulated using finite element software



Source: http://nsmwww.eng.ohio-state.edu/html/f-flashlessforg.html

Upper Bound Theorem

- Any estimate of the collapse load of a structure made by equating the internal rate of energy dissipation to the rate at which external forces do work in some assumed pattern of deformation will be > or = to the correct load.

Upper Bound Theorem Assumptions

- Isotropic and homogeneous
- Neglect strain hardening and strain rate
- Frictionless or constant shear stress condition exists at tool-work piece interface
- 2-D, plane strain with all deformation occurring by shear on a few planes. Elsewhere, material is rigid.

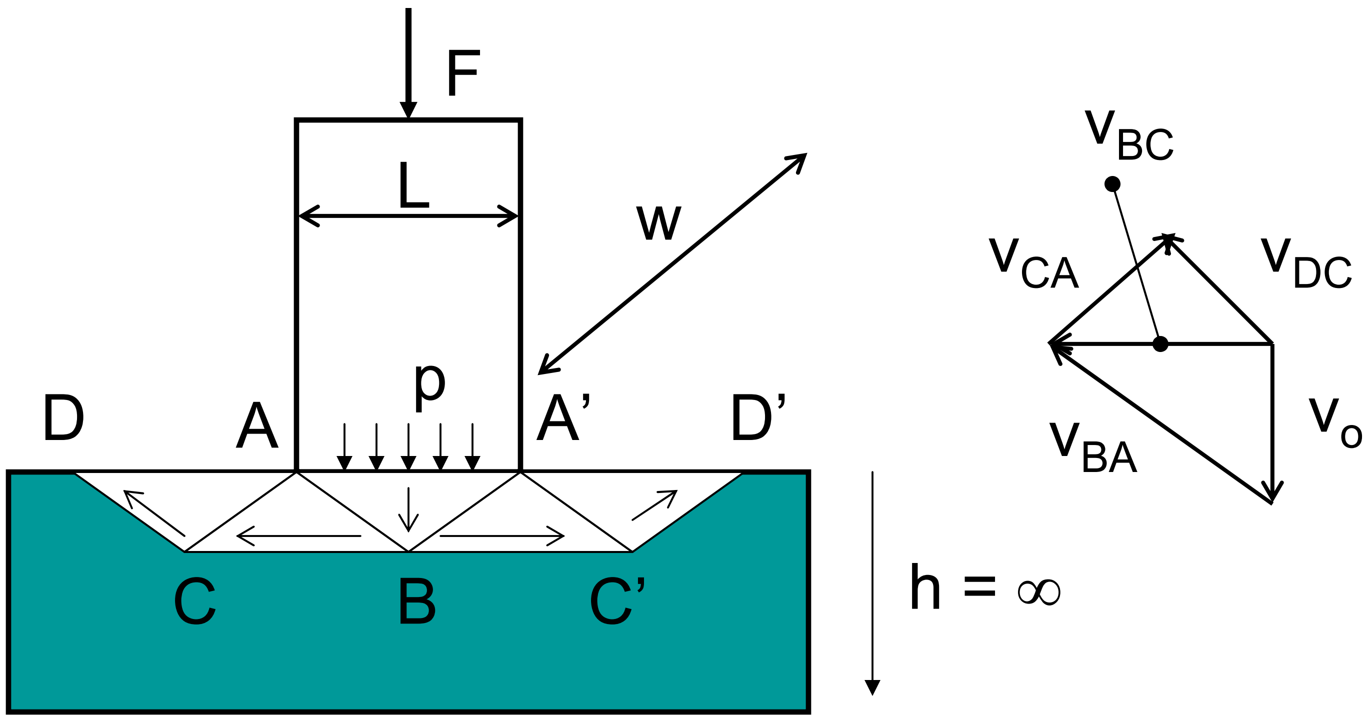
Upper Bound Theorem

$$\frac{dW}{dt} = \sum_{i=1}^{n} kS_i V_i^*$$

- k = shear flow stress
- S_i = length of shear plane
- V_i^* = velocity of shear

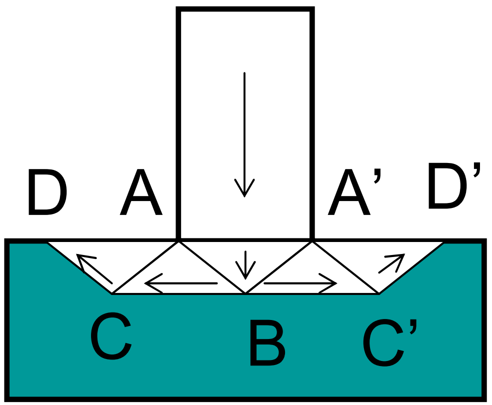
Upper Bound Theorem

- Indentation of a plate (slip-line analysis)



Work, shearing force

- Work is done by shearing along AB, BC, AC, and CD.
  - Lengths calculated from figure at right.
- Shearing force along any boundary, per unit length, w, is k (shear yield stress) times the length of the boundary, L.



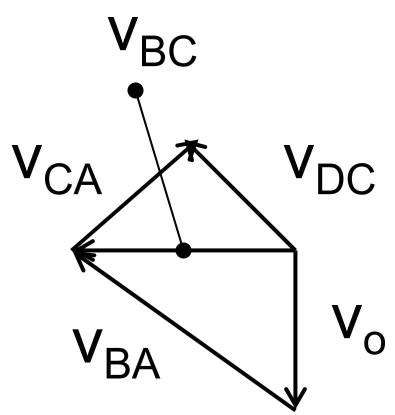
Shearing velocities

$$v_{BA} = v_o\sqrt{2}$$

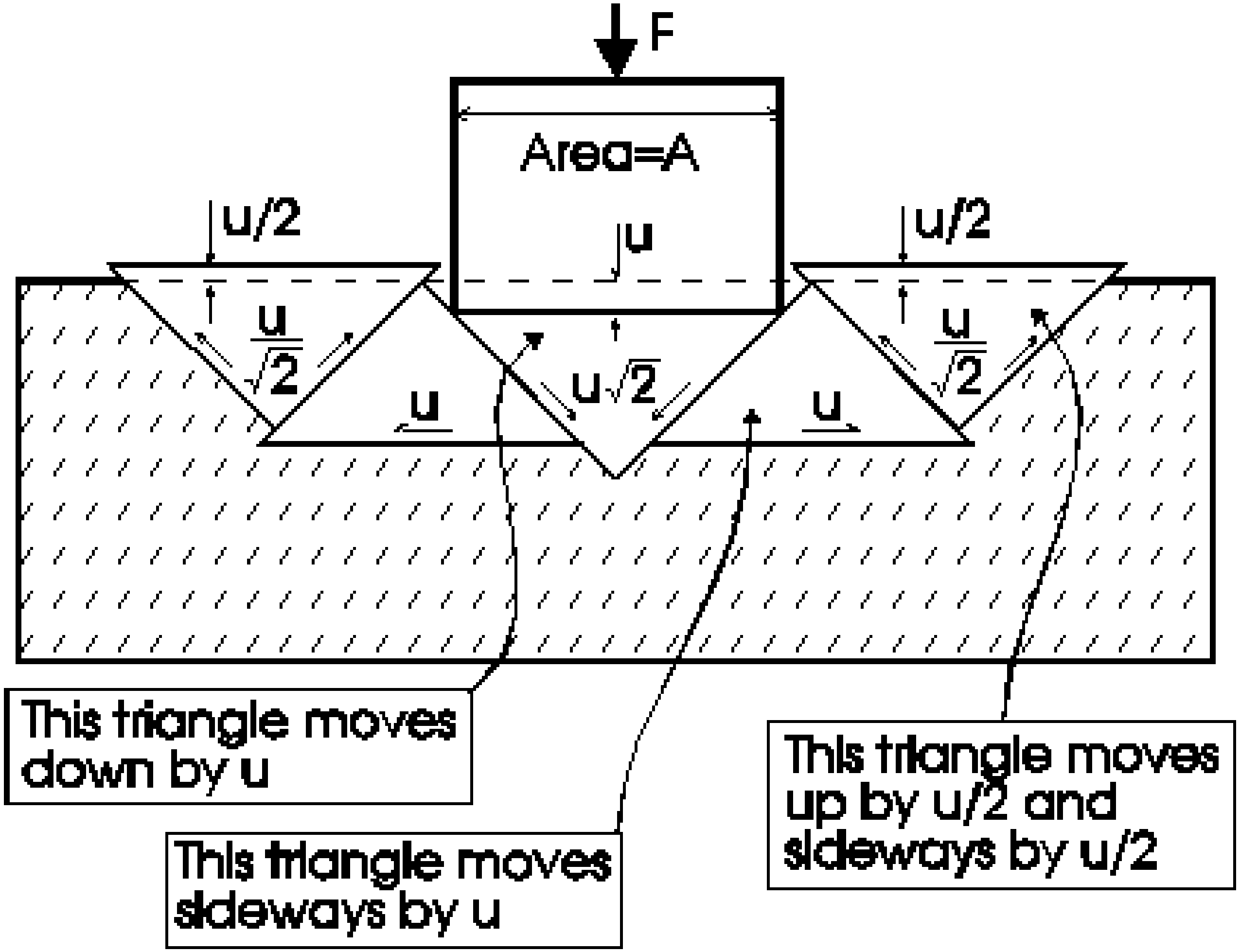
$$v_{BC} = v_o$$

$$v_{CA} = v_o/\sqrt{2}$$

$$v_{DC} = v_o/\sqrt{2}$$



Motions



F
Area=A
u/2
u
u/2
u
√2
u
u√2
u
u
√2
This triangle moves down by u
This triangle moves sideways by u
This triangle moves up by u/2 and sideways by u/2

Total power delivered

$$Lpv_o = 2k\left(\frac{Lv_0\sqrt{2}}{\sqrt{2}} + Lv_o + \frac{Lv_o}{2} + \frac{Lv_o}{2}\right)$$

- each term has been counted twice
  - due to symmetry
- Simplifying

  p = 6k

Total power delivered

p = 6k

- using von Mises

$$k = \frac{Y}{\sqrt{3}} = 0.577 \cdot Y$$

- hence

$$p = \frac{6Y}{\sqrt{3}} = 3.46 \cdot Y$$

Exact solution

$$p = 5.14\ k = 2.97\ Y$$

- Solution above

$$p = 6\ k = 3.46\ Y$$

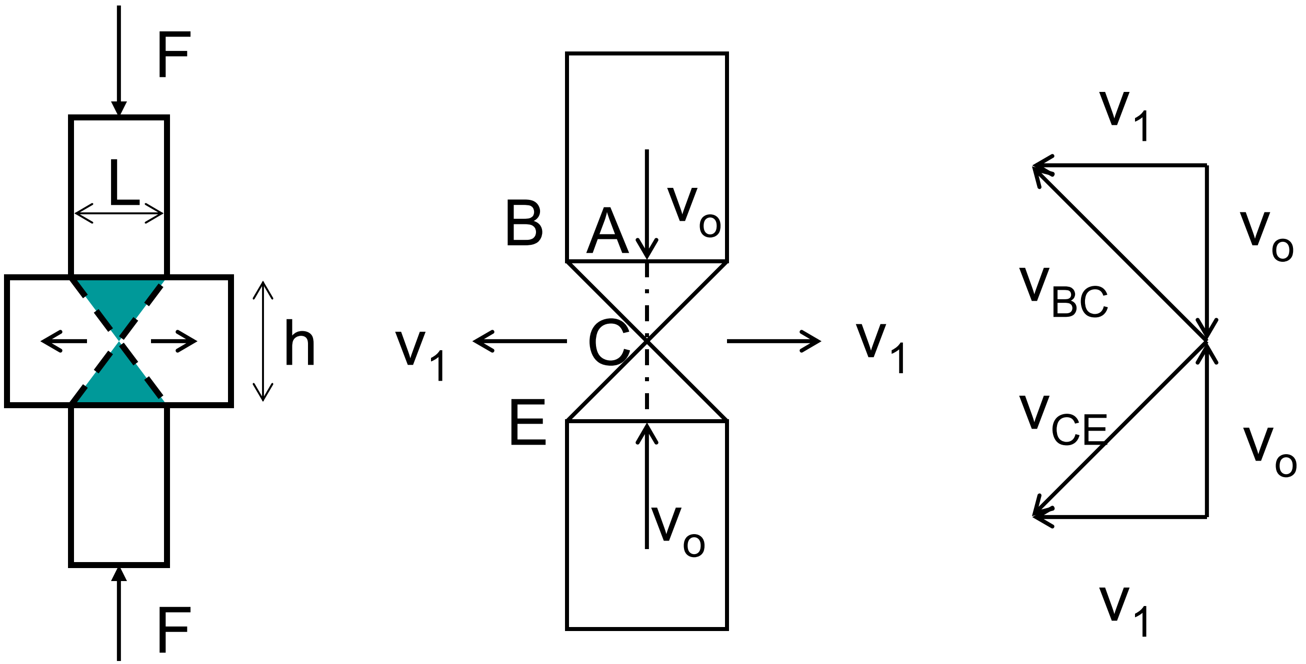
- so we can see the effect of constraint
  - redundant work: higher pressure

Non-homogeneous deformation and Redundant work

- If the slab is thick or friction:
  - non-homogeneous deformation
  - redundant work
- If the slab is thin or unconstrained:

  (e.g., open die forging without friction)
  - no redundant work

Indenting at h/L =1

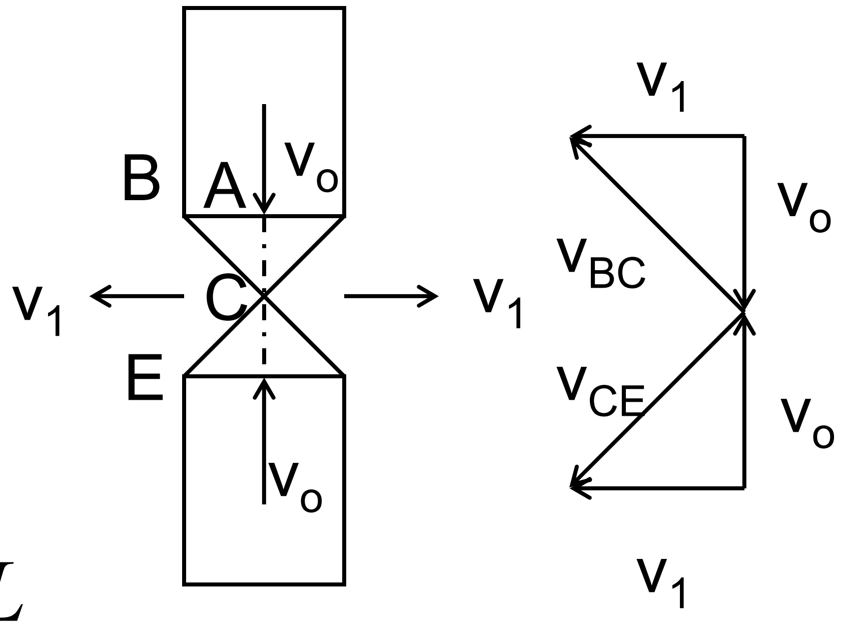


Analysis - power delivered

$$BC = CE = L/\sqrt{2}$$

$$v_{BC} = v_{CE} = \sqrt{2} \times v_o$$

$$\frac{2pv_oL}{2} = 2v_o \times \sqrt{2} \times \frac{kL}{\sqrt{2}}$$



p = 2k = 1.15 Y (plane strain result)

Redundant work limit ($\Delta = h/L$) (plane strain)

- $h/L < 1$: no redundant work
  - $p = 1.15\ Y$
- $1 < h/L < 8.7$: some redundant work
  - $1.15\ Y < p < 2.97\ Y$
- $h/L > 8.7$: redundant work
  - same as infinite plate
  - $p = 2.97\ Y$

Redundant work correction factor (Q_r)

- Can be characterized by:

$$p = Q_r Y$$

or

$$Q_r = p/\sigma_y = p/2\tau_y \text{ (by Tresca)}$$

- where Q_r = correction factor for redundant work

Redundant work factor (Backofen)
(frictionless)

$Q_r = \frac{p}{2\tau_Y}$

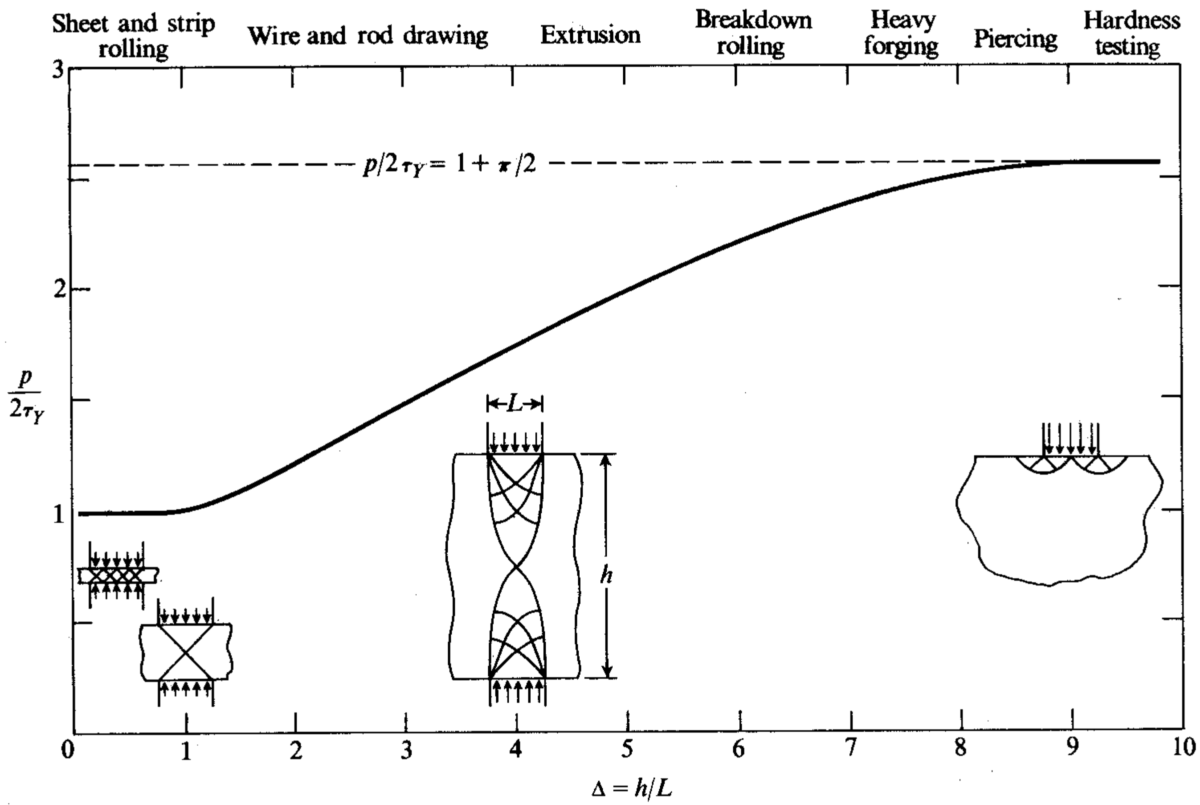


Fig. 7-1. The Δ-dependence of yield pressure for the frictionless plane strain-indentation of a nonstrain-hardening material.

Redundant work factor (Kalpakjian) - (friction)

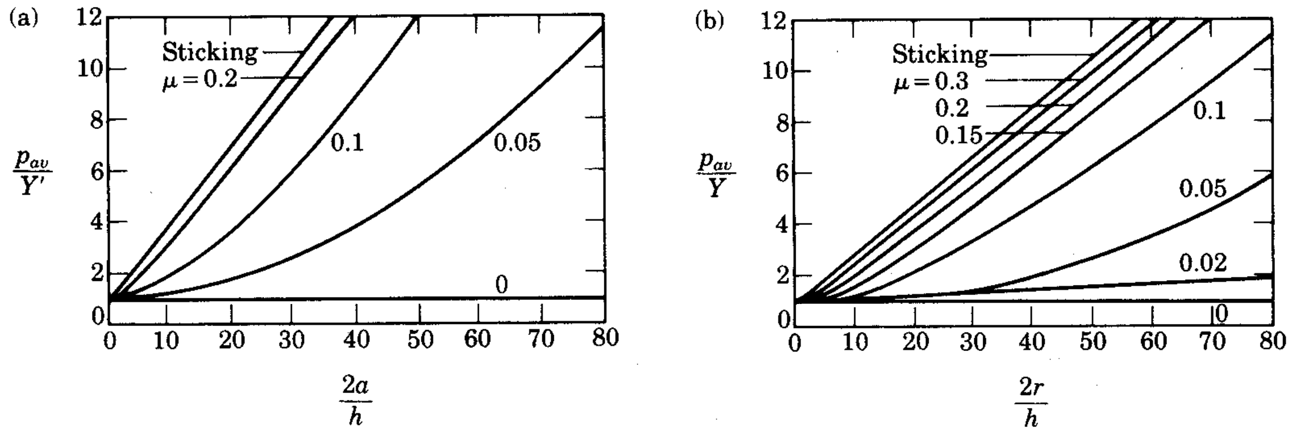


FIGURE 6.9 Ratio of average die pressure to yield stress as a function of friction and aspect ratio of the specimen: (a) plane-strain compression; and (b) compression of a solid cylindrical specimen. Note that the yield stress in (b) is Y, and not Y' as in plane-strain compression in (a).

Summary

- Slab analysis
  - frictionless
  - with friction
  - Rectangular
  - Cylindrical
- Strain hardening and rate effects
- Flash
- Redundant work