Mechanics Review-III

- Elasticity
- Elements of Plasticity
- Material Models
- Yielding criteria, Tresca and Von Mises
- Levy-Mises equations
- Strengthening mechanisms

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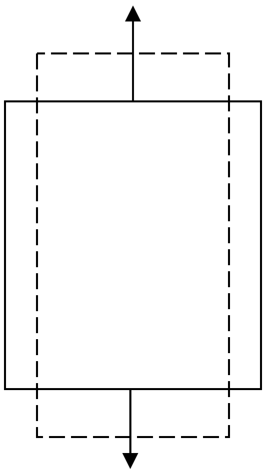
Elastic Stress-Strain

- Linear stress-strain

$$\sigma_x = E\varepsilon_x$$

- The extension in one direction is accompanied by contraction in other two directions, for isotropic material

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x$$



Hooke's Law

- In x direction, strain produced by stresses

Strains in x direction due to various stresses

$$\sigma_x \rightarrow \frac{\sigma_x}{E}$$

$$\sigma_y \rightarrow -\frac{\nu\sigma_y}{E}$$

$$\sigma_z \rightarrow -\frac{\nu\sigma_z}{E}$$

Superimposing,

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\left(\sigma_y + \sigma_z\right)$$

Hooke's Law

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\left(\sigma_y + \sigma_z\right)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}\left(\sigma_x + \sigma_z\right)$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$

$$\tau_{xy} = G\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz}$$

$$\tau_{xz} = G\gamma_{xz}$$

Poisson's Ratio

- Adding the strains

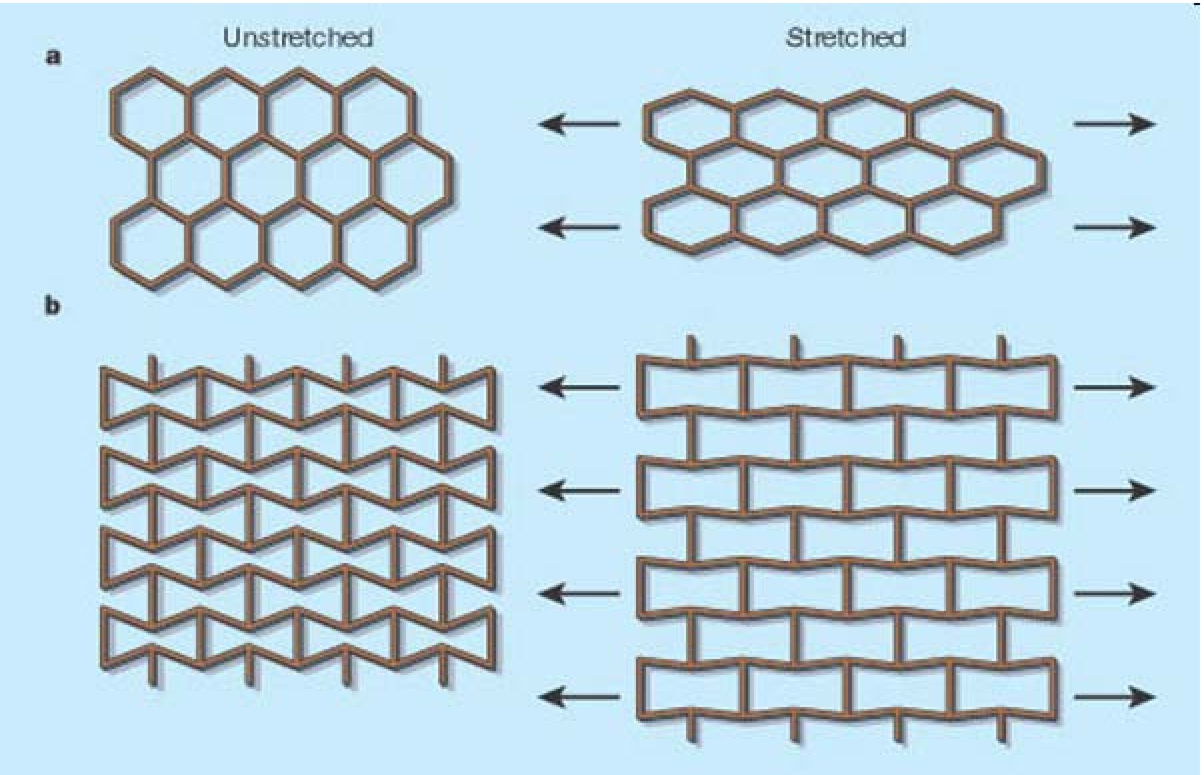
$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}\left(\sigma_y + \sigma_y + \sigma_z\right)$$

$$\frac{\Delta V}{V} = e_x + e_y + e_z, \text{Engineering strains for elastic condition}$$

- For fully plastic,

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$$

$\nu = 0.5$, Typically, $-1<\nu<0.5$, Most metals, 0.3

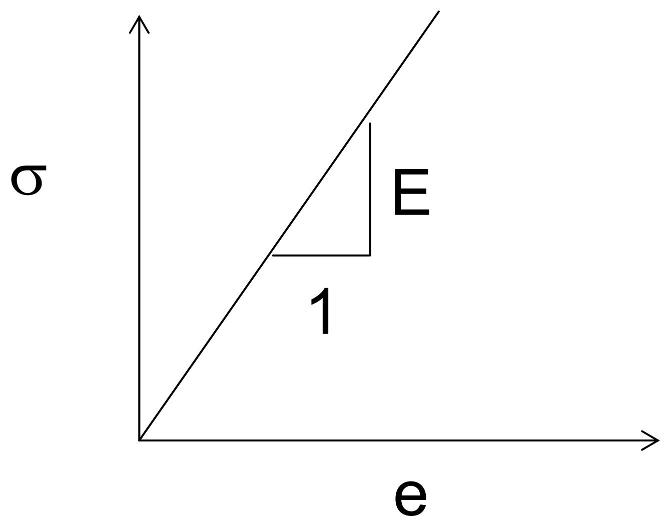


Stretching these two-dimensional hexagonal structures horizontally reveals the physical origin of Poisson's ratio. **a,** The cells of regular honeycomb or hexagonal crystals elongate and narrow when stretched, causing lateral contraction and so a positive Poisson's ratio. **b,** In artificial honeycomb with inverted cells, the structural elements unfold, causing lateral expansion and a negative Poisson's ratio.

Constitutive Behavior Equations

Linear elastic (simplest)

– Young's modulus, $E = \sigma/e$



Thomas Young
1773-1829

Robert Hooke
1635-1703

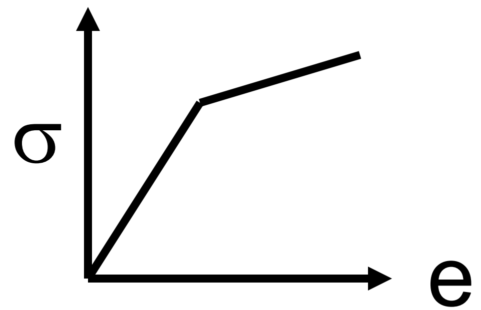
#2 - Match

a) Elastic – plastic material

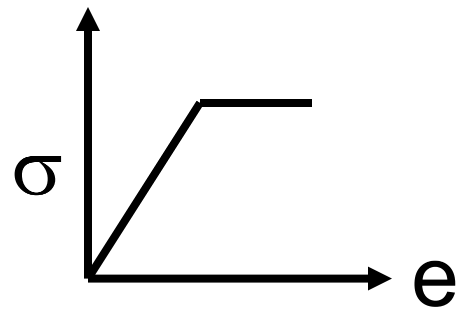
b) Perfectly plastic material

c) Elastic – linear strain hardening material

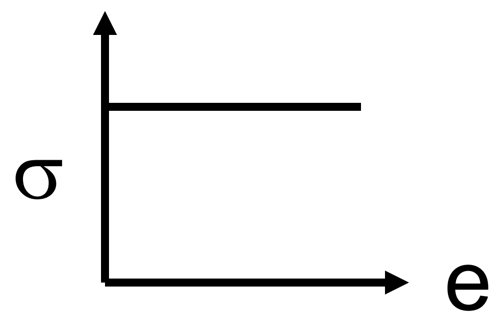
1)



2)

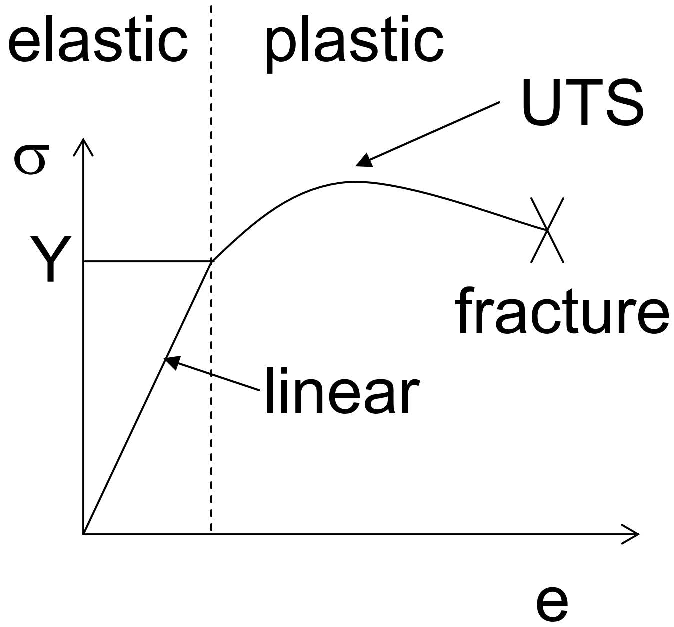


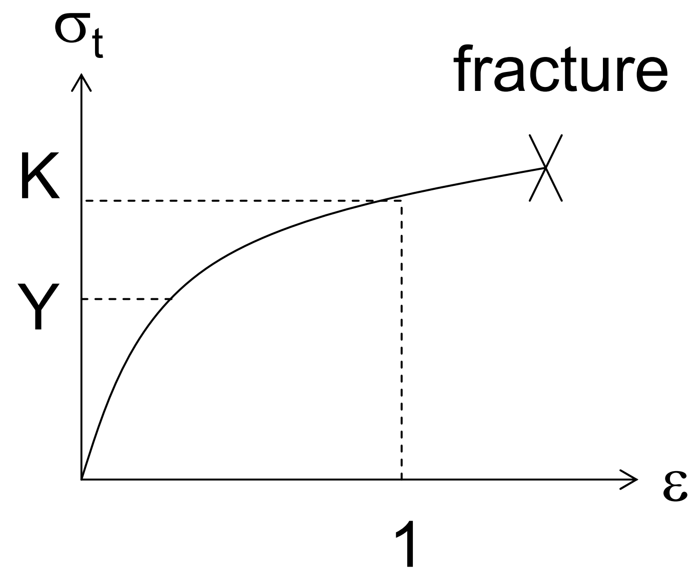
3)



Actual Material Behavior

$$\sigma_t = K\varepsilon^n$$





K = strength coefficient

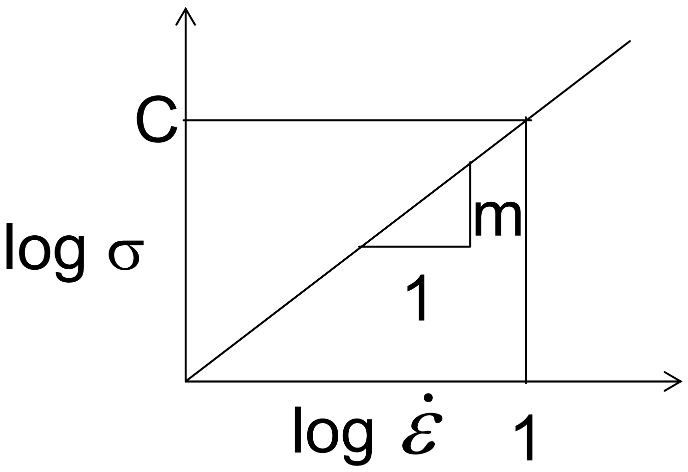
n = strain hardening coefficient

Strain Rate Effect (1)

$$\sigma = C\dot{\varepsilon}^m$$

C = strength coefficient

m = strain rate sensitivity coefficient

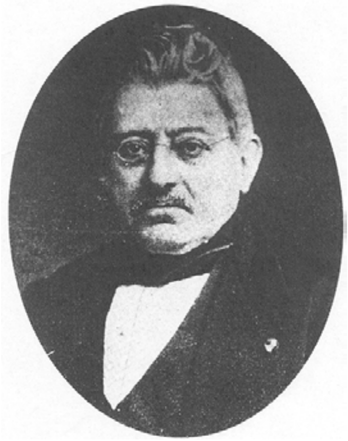


Yield Criteria

- How do you know if a material will fail?

  Compare loading to various yield criteria
  - Tresca
  - von Mises

- Key concepts
  - plane stress
  - plane strain



Henri Tresca
1814-1885

Richard von Mises
1883-1953

#3 - Match

a) Tresca yield criterion
b) Von Mises yield criterion
c) Maximum distortion energy criterion
d) Maximum shear stress criterion

1) $\tau_{max} \geq \frac{1}{2} \sigma_{yield}$

2) $(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 = 2Y^2$

3) $\sigma_3-\sigma_1 = 2\tau_{yield}$

Tresca Yield Criterion

$\tau_{max} \geq k$ or τ_{flow} (shear yield stress) a material property

$\sigma_{max} - \sigma_{min} = Y = 2k = 2\tau_{flow}$

$$\tau = \frac{\sigma_{max} - \sigma_{min}}{2} = k$$

- Simple tension

  $\sigma_1 = Y = 2k = 2\tau_{flow}$

- Under plane stress

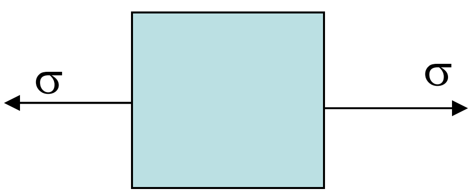
  $\sigma_1 - \sigma_3 = Y$

Importance of Yield Criterion

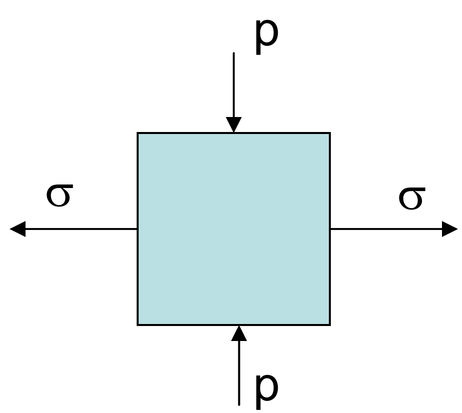
Assume three loading conditions in plane stress:

$\sigma_1 - \sigma_3 = Y$

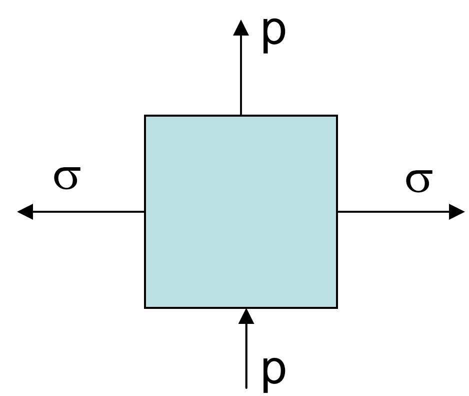
At the onset of yielding,



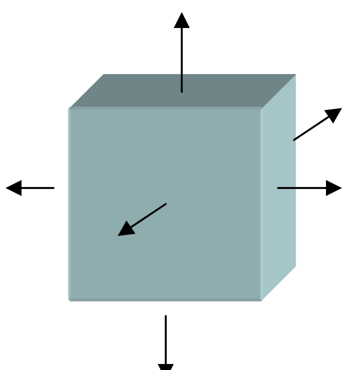
$\sigma = Y$



$\sigma + p = Y$



$\sigma = ?$



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Von Mises Yield Criterion

Based on Distortion Energy

$$(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 = 2Y^2$$

$$(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right) = 2Y^2$$

Y = uni-axial yield stress

VonMises/Tresca Criterion

- The locus of yielding for VonMises

$$\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3 = Y^2$$

Using Tresca Criterion in first quadrant

$$\sigma_1 > 0, \sigma_3 > 0$$

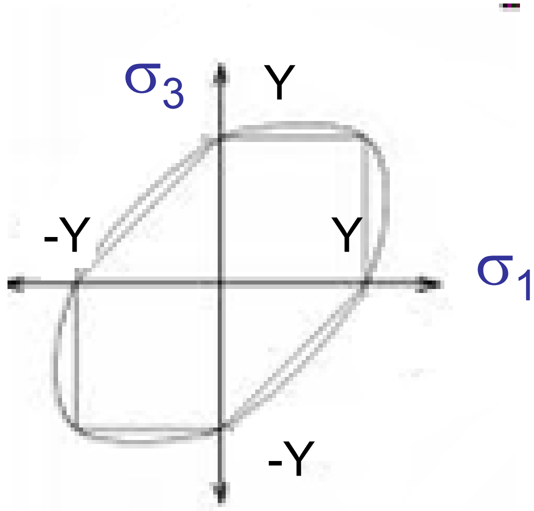
$$\sigma_2 = 0$$

for plane stress maximum value of $\sigma_1, \sigma_3 = Y$ in first and third quadrant

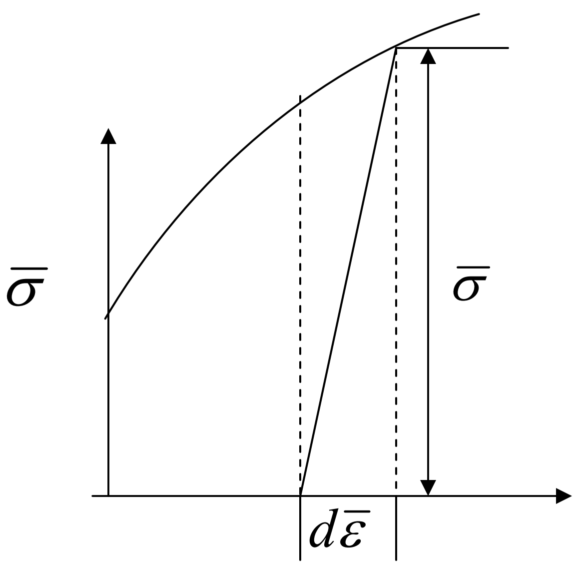
second and fourth it will be a 45 deg line,

$$\sigma_1 - \sigma_3 = Y$$

Locus of Yield



Levy Mises Flow Eqn.



$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\left(\sigma_y + \sigma_z\right)$$

Analogous to Elastic Equation

$$d\varepsilon_1 = \frac{d\bar{\varepsilon}}{\bar{\sigma}}\left[\sigma_1 - \frac{1}{2}\left(\sigma_2 + \sigma_3\right)\right]$$

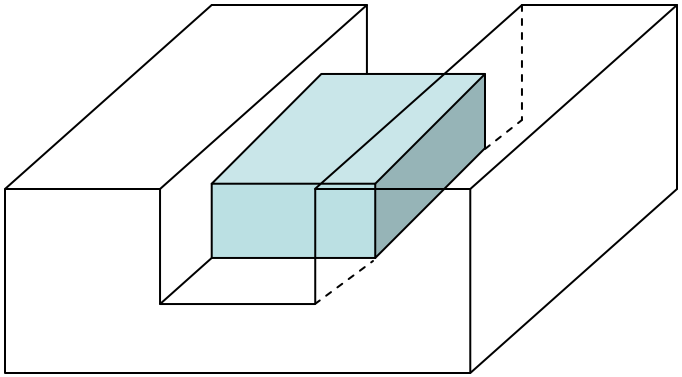
$$d\varepsilon_2 = \frac{d\bar{\varepsilon}}{\bar{\sigma}}\left[\sigma_2 - \frac{1}{2}\left(\sigma_1 + \sigma_3\right)\right]$$

$$d\varepsilon_3 = \frac{d\bar{\varepsilon}}{\bar{\sigma}}\left[\sigma_3 - \frac{1}{2}\left(\sigma_1 + \sigma_2\right)\right]$$

Plane Strain (1)

One pair of faces has NO strain

– each cross-section has the same strain



Material in a groove

Plane Strain (2)

- From VonMises eqn.

$$\sigma_1 - \sigma_2 = \frac{2}{\sqrt{3}} Y = 1.15Y$$