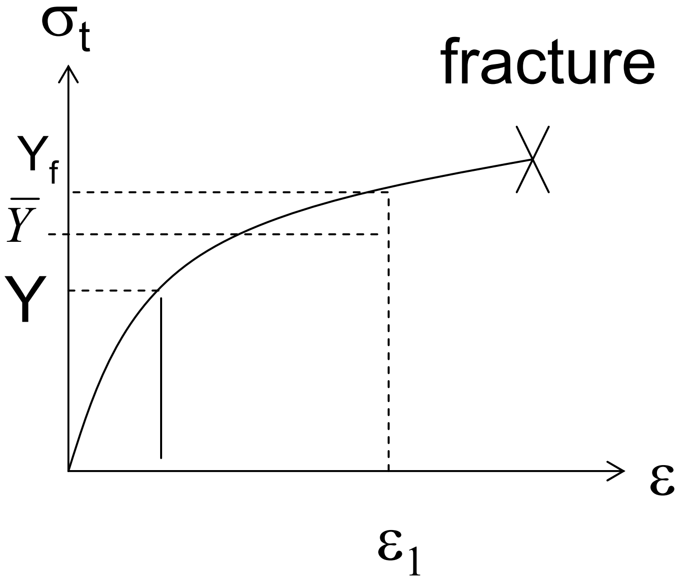
Mechanics Review-IV

- Average yield stresses
- Ideal work analysis
- Material Hardening Mechanism

Average Yield Stress

$$\bar{Y}.\varepsilon_1 = \int_0^{\varepsilon_1} K\varepsilon^n d\varepsilon$$



Analysis of Metal Working Methods

Energy Method ideally gives, if we ignore:

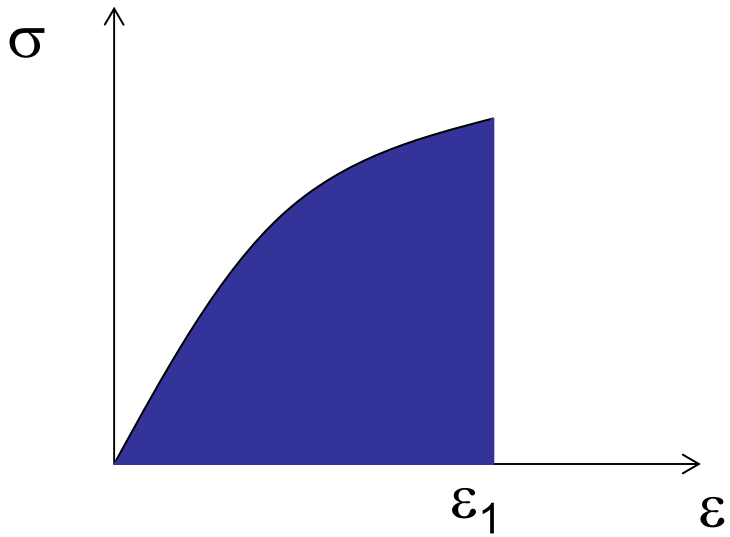
- frictional effects
- redundant deformation

External work = Deformation Work

Energy Method

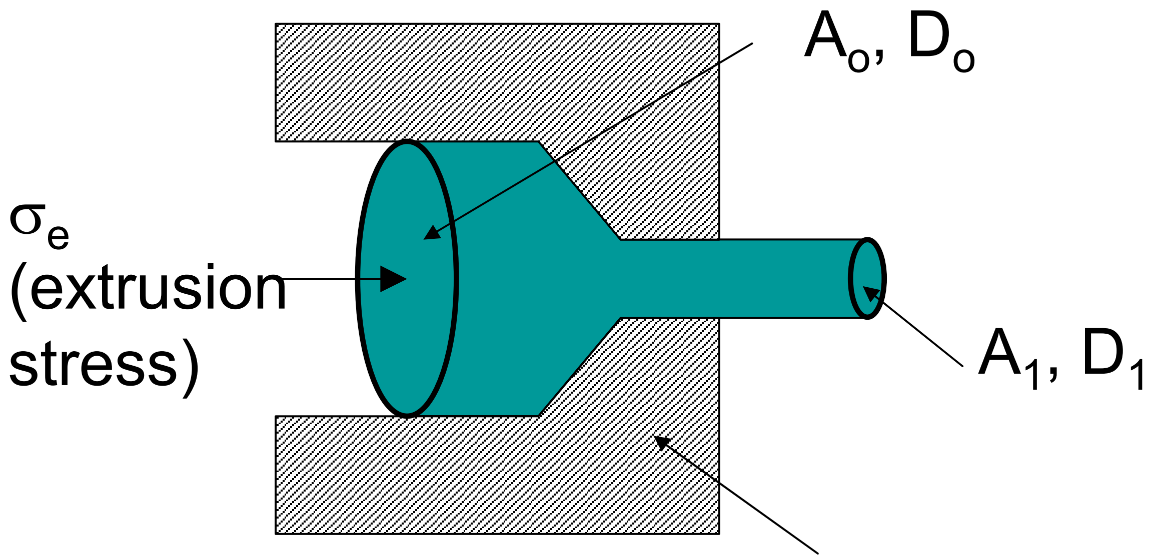
$$W_i * \text{volume of material} = \text{deformation energy}$$

$$= \int_0^{\varepsilon_1} \sigma \mathrm{d}\varepsilon * vol$$



Axi-symmetric Extrusion - Ex. 1-1

(wire drawing is reverse)



Die: hard material
(e.g. WC, diamond)

Extrusion - Ex. 1-2

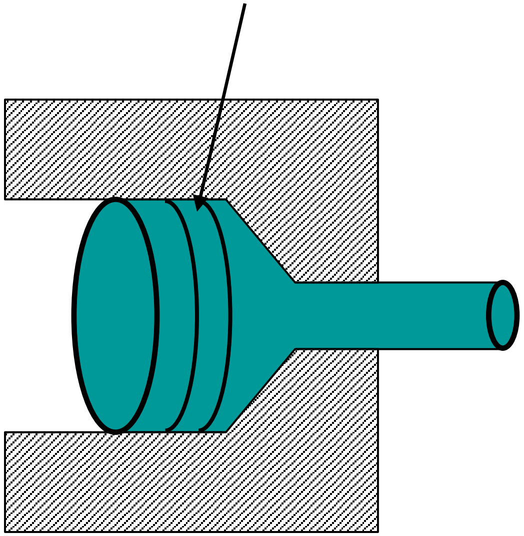
To solve this example, we use:

External Work (W_e) =
Deformation (internal) Work (W_i)

Extrusion - Ex. 1-3

External Work = $\sigma_e(A_o l_o)$

where: $(A_o l_o)$ = original volume being deformed



Extrusion - Ex. 1-4

Strain

(using conservation of volume)

$$\varepsilon_1 = \ln\left(\frac{l_1}{l_o}\right) = \ln\left(\frac{A_o}{A_1}\right)$$

Extrusion - Ex. 1-5

Assuming plastic deformation, then

$$A_o l_o = A_1 l_1$$

is constant during deformation.

Extrusion - Ex. 1-6
Power Law Material

$$\sigma = K\varepsilon^{n}$$

so

$$Deformation\ Energy = W_i$$

$$= \int \sigma d\varepsilon * vol = \frac{K\varepsilon_1^{n+1}}{n+1}\left(A_o l_o\right)$$

Extrusion - Ex. 1-7 Power Law Material

For processing to occur:

$$\text{External Work } (W_e) \geq \text{Deformation Energy } (W_i)$$

$$\sigma_e \left(A_o l_o \right) \geq \frac{K \varepsilon_1^{n+1}}{n+1} \left(A_o l_o \right)$$

Extrusion - Ex. 2-1

What is the extrusion stress (σ_e) for Al 6061-T6 from ϕ = 1 in to ϕ = 0.95 in?

- K = 410 MPa
- n = 0.05

$$\sigma_e \geq \frac{K\varepsilon_1^{n+1}}{n+1} \qquad \varepsilon_1 = \ln\left(\frac{A_o}{A_1}\right)$$

Extrusion - Ex. 2-2

- Here,

$\varepsilon_1 = \ln(1^2/0.95^2) = 0.10$

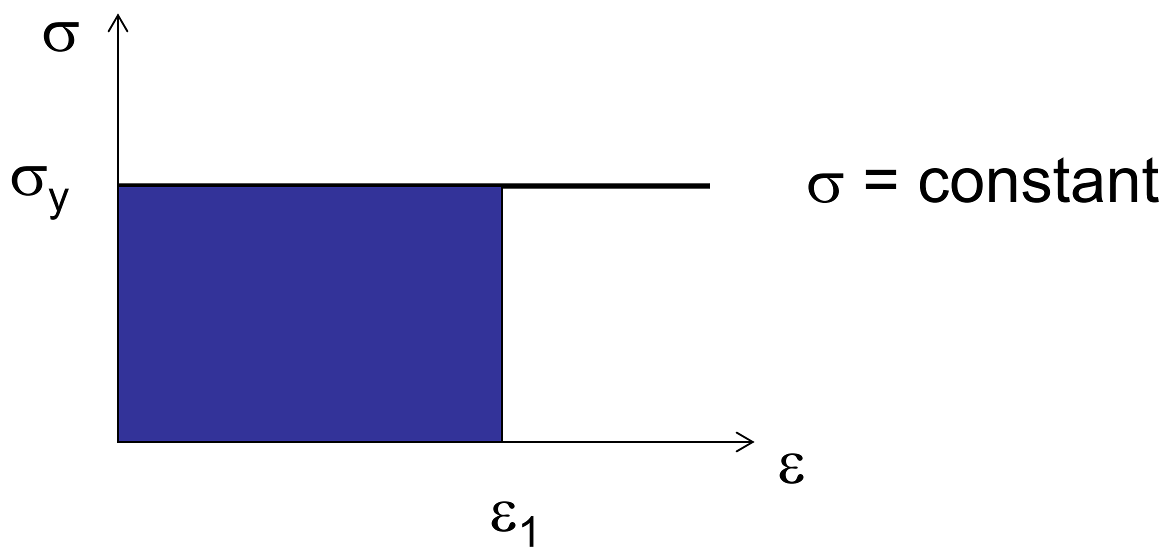
- So

$\sigma_e = 410 * 0.10^{1.05} / 1.05 = 35.8$ MPa

Extrusion - Perfectly Plastic Material (1)

Perfectly plastic = no strain hardening

= no elastic component



Extrusion - Perfectly Plastic Material (2)

External Work = $\sigma_e (A_o l_o)$

Deformation Work = $(\sigma_y \varepsilon_1)(A_o l_o)$

therefore

$$\sigma_e = \sigma_y \varepsilon_1$$

Extrusion - Perfectly Plastic Material (3)

To derive the Perfectly Plastic Material Model behavior:

$$\sigma = K\varepsilon^n$$

n = 0 (no strain hardening)

K = σ_y (once it yields, the stress is constant)

$$\sigma_e = K\varepsilon_1 = \sigma_y\varepsilon_1$$

Extrusion - Ex. 3-1

What is the extrusion stress for 6061-T6 aluminum, if we assume that it is perfectly plastic, with the conditions before?

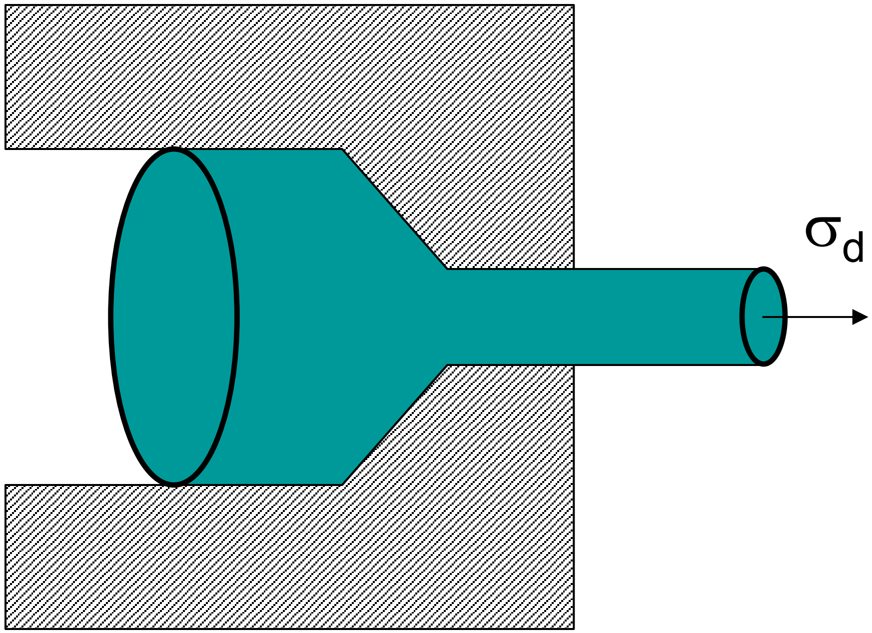
$\sigma_e = \sigma_y \varepsilon_1 = 275 \times 0.1 = 27.5\ \text{MPa}$

A bit below the 35.8 MPa found before.

Wire Drawing

Analogously:

$$\sigma_d = \frac{K\varepsilon_1^{n+1}}{n+1}$$



Strengthening

- Strain hardening
  - Dislocations present can become entangled
  - Impeded by grain boundaries, inclusions
- Grain size
  - Large grain size: low strength, low hardness and high ductility produces rough surface

$$\sigma_y = \sigma_0 + \frac{k_y}{\sqrt{d}}$$

Strengthening

- Grain boundary effect
  - Impede the movement of dislocation
  - At elevated temperature, grain boundary sliding giving rise to creep

Summary

- Completed a brief mechanics review
- Mechanics are very important for materials processing
- You will use them a lot in this class